ISER Working Paper Series

Pareto models, top incomes, and recent trends in UK income inequality

Stephen P. Jenkins
LSE, Institute for Social and Economic Research
(University of Essex), and IZA

No. 2016-07
August 2016

INSTITUTE FOR SOCIAL
& ECONOMIC RESEARCH

E·S·R·C
ECONOMIC
& SOCIAL
RESEARCH
COUNCIL

www.iser.essex.ac.uk

Pareto models, top incomes, and recent trends in UK income inequality

Summary

Statistical agencies and other researchers typically estimate income inequality levels and trends from either household survey data or tax return data, but rarely combine the information in the two types of data source. The result is that very different impressions about how inequality is changing over time may arise. Estimates from tax return data show a substantial rise in inequality over the last two decades in both the UK and USA, whereas survey-based estimates of inequality show much less change. For the UK, for example, the share of total income held by the richest 1% increased by 29% between fiscal years 1996/97 and 2007/08 whereas the Gini coefficient for household income increased by 7%. For the USA, the corresponding increases over the same period are 30% and 2%.

Research users may reasonably ask what the 'true' picture of inequality trends is. There is a good case for providing them with answers using methods that combine information from survey and tax data in order to take advantage of the strengths of each source, and this is what I do. Tax return data provide better coverage of top incomes than do survey data, and survey data provide the ability to create income variables with the same definitions, so that combination is done on a like-for-like basis.

I analyse income inequality levels and trends for the UK by combining inequality estimates from survey and tax data. As part of this analysis, I also provided new findings about survey under-coverage of top incomes in UK survey data: the problem becomes apparent at around the 99^{th} percentile in the 1990s but at around the 95^{th} percentile in the 2000s. In addition, I provide new results about how to summarise the distribution of top incomes using Pareto models, arguing in favour of a Pareto II model rather than the Pareto I and for using modelling thresholds rather higher than often employed.

My conclusions about aggregate UK inequality trends since the mid-1990s are broadly robust to the way in which I employ the information about top incomes in the tax data. For example, the Gini coefficient for gross individual income rose by around 7% to 8% between 1996/97 and 2007/08, with most of the increase occurring after 2003/04. When I use only survey data, with tax data not exploited at all, the Gini coefficient is estimated to decrease by around 5% over the same period.

Pareto models, top incomes, and recent trends in UK income inequality

Stephen P. Jenkins
(LSE, ISER (University of Essex), and IZA)

Revised version, 1 August 2016

Abstract

I determine UK income inequality levels and trends by combining inequality estimates from tax return data (for the 'rich') and household survey data (for the 'non-rich'), taking advantage of the better coverage of top incomes in tax return data (which I demonstrate) and creating income variables in the survey data with the same definitions as in the tax data to enhance comparability. For top income recipients, I estimate inequality and mean income by fitting Pareto models to the tax data, examining specification issues in depth, notably whether to use Pareto I or Pareto II (generalised Pareto) models, and the choice of income threshold above which the Pareto models apply. The preferred specification is a Pareto II model with a threshold set at the 99th or 95th percentile (depending on year). Conclusions about aggregate UK inequality trends since the mid-1990s are robust to the way in which tax data are employed. The Gini coefficient for gross individual income rose by around 7% or 8% between 1996/97 and 2007/08, with most of the increase occurring after 2003/04. The corresponding estimate based wholly on the survey data is around –5%.

Keywords: inequality, top incomes, Pareto distribution, generalized Pareto distribution, survey under-coverage, HBAI, SPI

JEL Classifications: C46, C81, D31

Acknowledgements

This paper is inspired by and draws on Frank Cowell's contributions to the analysis using Pareto models, income data problems, and inequality decomposition (see, inter alia, Cowell 1977, 1980, 1989, 2011, 2013; Cowell et al. 1998; Cowell and Flachaire 2007; Cowell and Kuga 1980; Cowell and Victoria-Feser 1996, 2007). Frank: saluto tu! My research is part supported by an Australian Research Council Discovery Grant (award DP150102409, with Richard Burkhauser, Nicolas Hérault, and Roger Wilkins) and core funding of the Research Centre on Micro-Social Change at the Institute for Social and Economic Research by the University of Essex and the UK Economic and Social Research Council (award ES/L009153/1). I wish to thank Nicolas Hérault for preparing the individual-level income data; Tony Atkinson, Facundo Alvaredo, and Christian Schluter for helpful discussions about Pareto distributions and top incomes; David Roodman and Philippe Van Kerm for sharing their Stata programs; and the handling editor, two anonymous referees, and audiences at Aix-Marseilles University, KU Leuven, and Ca' Foscari University of Venice for comments on a first draft.

Correspondence: Stephen P. Jenkins, Department of Social Policy, London School of Economics, Houghton Street, London WC2A 2AE, UK. Email: s.jenkins@lse.ac.uk.

1. Introduction

There is a bifurcation in the literature on income inequality levels and trends. On the one hand, most official statistics and academic analysis utilise data from household surveys and report estimates of the inequality of family or household disposable income summarised using Gini coefficients and other inequality indices calculated using all incomes from poorest to richest. (See e.g. OECD 2008, 2011, 2015, on cross-national comparisons, and Belfield et al. 2015 and Department of Work and Pensions 2015 on UK trends.) On the other hand, there is the 'top incomes' literature that uses administrative record data on personal income tax returns, reporting estimates of top income shares – the share of total income received by the richest 1% or richest 10%, and so on. (See e.g. Alvaredo et al. 2013, Atkinson and Piketty 2007 on cross-national comparisons, and Atkinson 2005 on UK trends.)

The two literatures differ in their findings about recent inequality trends: estimates from tax return data show a substantial rise in inequality over the last two decades in both the UK and USA, for instance, whereas survey-based estimates of inequality show much less change. For the UK, for example, the share of total income held by the richest 1% increased by 29% between fiscal years 1996/97 and 2007/08 whereas the Gini coefficient increased by 7% (Alvaredo et al. 2015, Belfield et al. 2015). For the USA, the corresponding increases over the same period are 30% and 2% (Alvaredo et al. 2015, De-Nevas Walt and Proctor 2015).

The divergent findings about inequality trends from the two data sources arise partly because different inequality indices and income definitions are employed (more on this later). However, another important explanation is that household surveys do not capture top incomes very well, whereas tax data do a much better job of this.

In this paper, I determine UK income inequality levels and trends since the mid-1990s by combining estimates from tax return data (for the 'rich') and household survey data (for the 'non-rich'), taking advantage of the better coverage of top incomes in tax return data (which I demonstrate) and creating income variables in the survey data with the same definitions as in the tax data to enhance comparability. I also analyse how estimates of inequality trends differ by inequality index.

There are multiple sources of under-coverage of top incomes in survey data. The first is under-reporting among high-income respondents or top-coding of their responses by survey administrators. In these cases, survey data are right-censored. A second source of under-coverage is the sampling of high-income respondents per se. Respondents may provide

sparse coverage of the top income ranges and, in addition, there may be no respondents at all from the extreme right-hand tail, because the survey organisation does not target potential high income respondents by design, or it is unable to contact them, or there is contact but refusal to participate. In this case, the observed income data are a right-truncated sample of the 'true' distribution. Both types of under-coverage contribute downward bias to survey estimates of inequality for a given year because there is not enough income observed in the very top income ranges. A by-product of sparse coverage of the top income ranges is that the high-income observations present in the survey data have the characteristics of outliers (even if they are genuine rather than an error), and have substantial influence on the conventional non-parametric estimate of an inequality measure for a given year: see Cowell and Victoria-Feser (1996, 2007) and Cowell and Flachaire (2007). This sensitivity can also introduce spurious volatility in a time series of inequality estimates.

There are three approaches to estimating inequality measures that address these under-coverage problems: see Figure 1 for a schematic summary. Approach A is based entirely on survey data. It derives an inequality estimate for the poorest p% using non-parametric methods applied to survey unit-record data, and derives an inequality estimate from the richest $(1–p)$% by fitting a Pareto Type I distribution to the top income observations from the same source. The estimate of total inequality, mostly summarised using the Gini coefficient, is calculated by adding together three components: inequality within the top group, inequality within the non-top group, and between-group inequality.

<Figure 1 near here>

Cowell and Flachaire (2007) provide a thorough examination of the properties of Approach A motivated by, and focusing on, the problem of sparse coverage of top income ranges. Their headline conclusion is that such 'use of appropriate semiparametric methods for modelling the upper tail can greatly improve the performance of those inequality indices that are normally considered particularly sensitive to extreme values' (2007: 1044). Alfons et al. (2013) also motivate their application of Approach A, using EU-SILC survey data for Austria and Belgium, with reference to sensitivity issues. Neither article refers to under-coverage per se. By contrast, Ruiz and Woloszko motivate their application to survey data for OECD countries in terms of 'correcting household survey data for underreporting in the upper-tail of income distributions' (2015: 6). Burkhauser et al. (2012) use Approach A to adjust for the systematic under-coverage of high incomes in public use Current Population Survey datasets introduced by US Census Bureau top-coding. In both applications, the idea is that the upper

tail to the income distribution implied by the parametric model estimates will capture more income than non-parametric estimates.

There is evidence that Approach A's ability to address survey under-coverage at the top is limited. For example, survey-based estimates of the share of total income held by the top 1% are several percentage points less than the estimates from tax return data according to the analysis of Atkinson et al. (2011) and Burkhauser (2012) for the USA. Put differently, fitting a parametric upper tail may obviate the sparsity problem (there is density mass at all points of the distribution's support, by assumption), but the estimate of the 'true' upper tail based on model-based extrapolation from the observed survey observations may not be reliable. This motivates the use of tax return data, as they have better coverage of the upper tail.

Approaches B and C both use tax return data but take different routes to addressing under-coverage issues. Approach B replaces the highest incomes in the survey with cell-mean imputations based on the corresponding observations in the tax return data. The 'SPI adjustment' to Family Resource Survey income data – used to derive the UK's official income distribution statistics since the early 1990s – is an example of this approach (see e.g. Department for Work and Pensions 2015). Burkhauser et al. (2016) apply Approach B in a more extensive and comprehensive manner and use World Top Incomes Database (Alvaredo et al. 2015) estimates of top income shares as a benchmark. Bach et al. (2009) is an application to Germany.

Approach C, used in this paper, *combines estimates* from the two types of data source rather than *combining data* per se as Approach B does. It is thus identical to Approach A except that it uses both survey and tax data rather than only the former; it is this feature that addresses the under-coverage problem. Approach C was developed by Atkinson (2007: 19–20) with an application to the USA by Atkinson et al. (2011), and extended by Alvaredo (2011) who also included applications to Argentina and the USA. Subsequent applications include those by Alvaredo and Londoño Vélez (2015) and Diaz-Bazan (2015) to Colombia, and by Lakner and Milanovic (2016) and Anand and Segal (2016) to global income inequality. Each of the applications cited uses a Pareto I model to describe the upper tail of the income distribution. In principle, researchers could employ non-parametric estimates of inequality indices for the top incomes in the tax data, but there is then the issue of whether these would be subject to the sensitivity problems mentioned earlier. The issue has not been studied before using tax data: I do so in this paper.

To perform well, Approaches B and C both rely on the researcher using the same 'income' definition in both data sources and ensuring that calculations refer to the same population. Otherwise, there is an 'apples + bananas' problem: non-comparability introduces bias. To avoid this, we may exploit a comparative advantage of survey data. The ability to change income definitions in tax return data is limited but, with access to unit record survey data, we can do a cross-walk from survey to tax data definitions. That is what I do in this paper, employing the same harmonized income variables for the survey and tax data as Burkhauser et al. (2016). For more details, see below.

This paper makes several contributions. First, there is the substantive application to UK inequality trends since the mid-1990s. How much income inequality has been growing is of much public interest. Second, related, there is question of whether Approaches C and B tell the same story about trends when applied to the same data sources. I contrast my Approach C estimates with the Approach B estimates provided by the official statistics (Department for Work and Pensions 2015; see also Belfield et al. 2015) and Burkhauser et al. (2016). Third, I provide new evidence about the extent to which there is under-coverage by survey data of the UK income distribution, using comparable tax data as the benchmark.

Fourth, I provide new analysis of issues that arise when fitting a Pareto model to the upper tail of the income distribution, and hence of direct relevance to researchers applying the semiparametric Approaches A and C. My findings are relevant to analysis of other heavy-tailed distributions such as wealth (Shorrocks et al. 2015, Vermeulen 2014), and city and firm size (Eeckhout 2004; Gabaix 2009, 2016). I use unit record tax return data rather than grouped (bracketed) data and so have flexibility to explore a number of econometric issues. (On estimation issues that arise with grouped tax return data – the only source available for deriving very long historical series – see Atkinson 2005, 2007 and references therein.) For instance, for the Pareto Type I model, I compare the performance of ordinary least squares, maximum likelihood, and maximum likelihood-robust estimators. I also address two implementation questions.

The first question is: what model should be fitted to top incomes? To date, researchers have invariably used the Pareto Type I model. This has a single shape parameter and there are simple formulae for calculating mean income and inequality indices from parameter estimates. There is also a widespread view that Pareto Type I models fit top income data well (Atkinson et al. 2014: 14). However, many of the goodness of fit checks that researchers have employed do not reliably distinguish Pareto distributions from other heavy-tailed distributions. In addition, most of the goodness of fit approaches used can only check whether

data are consistent with a distribution in the Pareto family, i.e. not with the Pareto Type I specifically (Cirillo 2013). I provide the first systematic comparison of the goodness of fit of Pareto Type I and Pareto Type II ('generalised Pareto') models to top income data, and show that the latter outperforms the former except at extremely high thresholds – thresholds that are well above those typically employed.

The second and related implementation question is: if we assume that incomes are described by a Pareto model above some threshold, what should that threshold be? In particular, when implementing Approaches C or A, what is the cut-off to use to distinguish between top incomes and non-top incomes? Is the top income group the top 10% (Ruiz and Woloszko 2015), or the top 5% (Atkinson 2016), or the top 1% (Alvaredo 2011)? There is some evidence that a higher cut-off decreases the estimate of the Pareto Type I shape parameter, i.e. increases inequality among top incomes, other things being equal (see e.g. Burkhauser 2012: Appendix A). However, the impact on total inequality estimated using Approach C of changing the threshold is unclear, because inequality and the mean among non-top incomes and between-group inequality also change.

Several criteria have been proposed for choosing Pareto thresholds (see e.g. Clauset et al. 2009, Coles 2001) and I employ them. However, I also argue that there is an additional issue to be taken into account when applying Approach C. That is, because non-coverage issues motivate the approach, it is important to ascertain precisely where along the top income range it is that survey non-coverage occurs. There is little evidence about this for the UK. I show that survey non-coverage is apparent from around the 99th percentile upwards in the mid- to late-1990s or from around the 95th percentile in the 2000s. I use the 99th and 95th percentiles as the Pareto threshold when deriving my inequality estimates, as well as the 90th percentile as a robustness check.

I introduce in Section 2 the UK tax return and survey data that I use, and explain the creation of income variables using harmonized definitions and hence on a comparable basis. Section 3 provides evidence about under-coverage of the survey data using the tax data as the benchmark. I analyse the fitting of Pareto models to top incomes in tax return data in Section 4, and present estimates of overall inequality levels and trends since the mid-1990s in Section 5. Section 6 provides a summary and conclusions. Applying Approach C, I show that choosing different Pareto models and different thresholds has noticeable impacts on estimates of inequality among the rich. However, my conclusions about overall inequality trends are broadly robust to the choice of Pareto model and percentile threshold, and there are similar results if upper tail inequality and mean income are estimated non-parametrically. The

estimated inequality trends from Approach C are also similar to those derived using Approach B (Burkhauser et al. 2016). For example, the Gini coefficient for gross individual income rose by around 7% or 8% between 1996/97 and 2007/08, with most of the increase occurring after 2003/04. The corresponding estimate based wholly on the survey data is around –5%.

2. Survey and tax data, and the definition of income

The income tax return data are from the public-release files of the *Survey of Personal Incomes* (SPI) for each year 1995/96 through 2010/11, with the exception of 2008/09 for which no data have been released. Atkinson (2005) uses these data, as well as published tabulations from the SPI and from supertax and surtax returns for earlier years, in his pioneering analysis of trends in UK top income shares since 1908. (See also Atkinson 2016 for Pareto I parameter estimates back to 1799.) The SPI data underlie the UK top income share estimates in the World Top Incomes Database (WTID) (Alvaredo et al. 2015). Each year's SPI is a stratified sample of the universe of tax returns. The number of individuals in the data has increased from around 57,000 individuals in 1995/96 to nearly 677,500 in 2010/11, corresponding to around 32 million taxpayers. For further details, see HM Revenue and Customs KAI Data, Policy and Co-ordination (2014) and Burkhauser et al. (2016). The data are comparable over time, except for a small discontinuity between 1995/96 and later years (the effect of which I show later). Self-assessment was introduced that year and there were changes to the SPI methodology (personal communication with HMRC). Hence, I use 1996/97 as the base year for analysis of inequality trends rather than 1995/96.

Throughout the period of my analysis (and since 1990), the unit of assessment in the UK income tax system has been the individual. For this reason, the SPI income variables are all individual-level variables, rather than referring to the incomes of families or households (as in the survey data and official income distribution statistics). The SPI income variable I use is *individual gross income* (total taxable income from the market plus taxable government transfers, and before the deduction of income tax), i.e. the same variable that the WTID and the top income shares literature focuses on.

In addition, and to further align my research with the WTID and top income shares literature, I restrict analysis to the population of individuals aged 15 years or more. Because the SPI does not cover all individuals in the UK population or all of their income, the WTID

uses external population and income control totals for each year, i.e. estimates of the total number of individuals aged 15 or more, and of the total income held by them. I use the WTID control totals throughout. In practice, I accomplish this by introducing some observations with zero income into each year's unit record data and adjusting the grossing-up weights supplied with the data.

The unit record survey data I employ come from the *Family Resources Survey* (FRS), and the accompanying subfiles of derived income variables called the *Households Below Average Income* (HBAI) dataset (Department for Work and Pensions 2013, Department for Work and Pensions et al. 2014). I use data for the same period as the SPI data, 1995/96–2010/11. The FRS is a large continuous cross-sectional survey with data released annually for around 20,000 respondent households and the individuals within them. The Department for Work and Pensions (DWP) administers the FRS, and DWP staff produce the HBAI subfiles that they use to derive the UK's official income distribution statistics published annually using a variant of Approach B, i.e. the 'SPI adjustment'. (Despite its label, the HBAI provides information about the income distribution as a whole.) In essence, the HBAI subfiles contain a set of FRS income variables that DWP statisticians have cleaned.

Because the DWP's focus is on family and household post-tax post-transfer income variables (reflecting the needs of official statistics), there is a definitional mismatch between the income variables in the HBAI and the SPI. As it happens, the DWP's public-use files do contain an individual-level gross income variable but only from 2005/06 onwards. Burkhauser et al. (2016) create a complete time series for the period 1995/96–2010/11 (as for the SPI data) from FRS variables and show that their derived individual-level gross income variable is virtually identical to the DWP's for the years for which they can make comparisons. I use Burkhauser et al.'s individual gross income variables derived from the HBAI in this paper. (None of these variables are SPI-adjusted in the sense described earlier.) Burkhauser et al. (2016) go on to create a second set of individual-level income variables when implementing Approach B. These data reflect a more extensive 'SPI adjustment' procedure than employed by the DWP for the official statistics, and Burkhauser et al. (2016) label it 'SPI2' accordingly.

In sum, there are two main individual-level gross income data series employed in the paper to implement Approach C: that from the tax data ('SPI') and from the DWP's cleaned-up survey data ('HBAI'). In Section 5, I contrast my results for overall inequality based on the SPI and HBAI series (combining estimates) with those derived using Approach B (combining data). I refer to the DWP's (2015) inequality series as 'HBAI-SPI' and the

Burkhauser et al. (2016) series as 'HBAI-SPI2'.

To fully align the survey data with the tax return data, I restrict attention to individuals aged 15 years or more. I use the FRS weights in all calculations with the survey data and SPI weights with the tax data ones. All income variables (from tax and survey data) are expressed in pounds per year in 2012/13 prices.

3. Under-coverage of top incomes by household survey data

Ascertaining the point on the income range at which survey under-coverage of top incomes begins is an integral part of implementing Approaches A and C and of independent interest as well.

Table 1 shows estimates of percentiles $p90$, $p95$, $p99$, $p99.5$ and $p99.9$ derived from the survey and tax data as well as the ratio of each corresponding survey and tax data estimate (in %), by year. (For brevity, henceforth I refer to tax years 1995/96 as '1995', 1996/97 as '1996', and so on.) Real incomes at the top of the distribution generally rose over the period according to either source (look down each column of Table 1), except that there is fall in the uppermost percentiles after 2007, especially in the tax data estimates.

<Table 1 near here>

There are two explanations for the post-2007 fall in the uppermost percentiles. One is the recession at that time. The second, particularly relevant here, is the incentive for high income taxpayers to declare income in tax year 2009/10 rather than 2010/11 in order to avoid the increase in top marginal tax rate from 45% to 50% with effect from April 2010. The subsequent reintroduction of the 45% top marginal rate with effect from April 2013 provided an incentive to defer declaration of income. On these issues of 'forestalling' and 'reverse forestalling', see HM Revenue and Customs (2012). Because of these issues (and having no SPI data for 2008), although I provide annual estimates for the full period between 1995 and 2010, I mostly focus discussion on inequality trends through to 2007.

Table 1 provides clear evidence of under-coverage in top incomes and that its nature changed over the period. Survey estimates of the very top percentiles are more volatile over time than are the tax data estimates, which is indicative of the sparsity aspect of under-coverage. Regarding under-coverage per se, look at the 'ratio' columns: values less than 100% suggest under-coverage. Throughout the period, there is a broad correspondence between survey and tax incomes up to around $p99$. In the mid- to late-1990s, one might refer

to ‘over-coverage’ of the survey up to *p*95. However, in the 2000s, there is a substantial uplift in the very highest incomes shown by the tax data. This is not picked up by the survey data. Between 2000 and 2007, the ratio of survey *p*99 to the tax data *p*99 fell from around 100% to 82%. There is a similar decline in the corresponding ratio for *p*99.5 starting from around 1997 (when it was 100%), down to 78% in 2007. These changes in under-coverage over time suggest that it may be inappropriate to use the same percentile cut-off to define the top income group for all years. I return to this issue. This aside, the table also suggests that the optimal threshold for application of Approach C (or A) should not be lower than *p*95, because survey coverage is adequate up to this point.

Figure 2 provides a complementary perspective on the nature of survey under-coverage. It focuses on 1996 and 2007; the full series for all years is shown in Appendix A. I show densities derived from a histogram for the full distribution of log(income) in the survey data and for the tax data for each year. (I use the logarithmic scale in order to focus on the upper tail.) There are three plots for each year. The leftmost one shows densities plotted for log(income) greater than 10 (i.e. income > £22,026), and the vertical dotted lines mark *p*90, *p*95, *p*99, and *p*99.5 for the relevant year. The other two graphs provide more detailed views on the upper tail by plotting the same densities by plotted only for log(income) greater than 11 (i.e. income > £59,874; middle graph) and log(income) greater than 12 (i.e. income > £162,755; rightmost graph). Histogram areas reflect survivor function proportions, and so comparisons of areas provide information about under-coverage in the sense of how much of top income being captured by the survey data. The histograms also provide information about sparsity and ‘outlierness’ in top income ranges. Sensitivity issues are likely to be more important, the more that the histograms do not approximate a continuous function and show clumping of density mass.

<Figure 2 near here>

The leftmost plots suggest that the concentration of incomes in the tax and survey data is quite similar for most of the income range if one focuses on the top 5% to 10% of the distribution as a whole. Coverage, summarised by differences in histogram areas, is not so different – though it is clearly worse in 2007 than 1996. Both survey and tax densities appear quite smooth and continuous, though the tax data distribution has a long tail that is not present in the survey data, especially in 2007.

However, differences in income concentration across data sources are much more apparent if one focuses on the extreme top: look at the middle and rightmost plots. In 1996, both densities are discontinuous: extreme incomes are spread sparsely across the top income

range, and this range is much greater for tax data. There are greater proportions at the very top in the tax data than in the survey data (the total area of the dark bars is greater than the total area of the light bars). By 2007, and with the secular growth of incomes over the previous decade (Table 1), the survey data are even more clumpy and the proportion with extremely high incomes is more markedly less than for the tax data. In the tax data, the density is relatively continuous up to extremely high incomes.

Overall, Figure 2 and Appendix A suggests that, from the point of view of survey undercoverage of top incomes, the cut-off used to implement Approach C (or A) should lie at around $p95$ or higher, depending on the year. In addition, the sparse spread of incomes along the very top income range means that there are potentially 'high leverage' outliers (Cowell and Flachaire 2007) even in the tax data, and these could bias non-parametric estimates of inequality indices and also estimation of Pareto model parameters. I address these issues below.

4. Fitting Pareto models to top incomes

An integral part of inequality estimation using Approaches A and C is to fit a parametric model to top income data, but there are implementation issues concerning the choice of model and the top income range over which they are fitted. There is also a prior question of whether top incomes are described better by a model other than a Pareto one. This issue has rarely been addressed though one exception is Harrison (1979, 1981) who compares the fit to UK men's top earnings data of Pareto I, lognormal, and sech^2 distributions. Addressing all these issues is complicated by a chicken and egg problem: most methods for choosing the appropriate model are conditional on a given threshold; and most methods for choosing the threshold have been applied to a single model. One can use multiple models and thresholds but there can be an information overload, and this is and potentially worsened by having 15 years of data covering a period when the income distribution changed. What is appropriate for one year's data may not be appropriate for another. To address the implementation issues, I have had to make some judicious choices regarding empirical strategy and what I report. A full set of estimates is provided in appendices.

My analysis focuses on comparisons of Pareto I and Pareto II models fitted to SPI tax data. In this section, first I explain the model properties and different parameter estimation methods. (Important references on Pareto distributions include Arnold 2008, 2015, Coles

2001, Cowell 2011, 2015, and Kleiber and Kotz 2003.) Next, I report on tests checking whether Paretianity is an appropriate assumption, and whether answers depend on the income threshold used. Then I consider the relative goodness of fit of Pareto I and II models using two methods and multiple thresholds. Finally, I address the choice of threshold issue using both rule-of-thumb and more formal statistical methods. Overall, I demonstrate that the choice of model and threshold is not as clear cut as typical practice might suggest.

Pareto Type I and Type II models

If income x is characterised by a Pareto Type I model, the survivor function showing the fraction of the population with incomes greater than x, $S(x)$, i.e. one minus the cumulative distribution function, $F(x)$, is:

$$S(x) = 1 - F(x) = \left(\frac{x}{x_m}\right)^{-\alpha} \tag{1}$$

where $x \geq x_m > 0$, and $x_m > 0$ is the lower bound on incomes. Parameter α is the shape parameter ('tail index') describing the heaviness of the right tail of the distribution, with smaller values corresponding to greater tail heaviness. The k^{th} moment exists only if $k < \alpha$.

The survivor function for the Pareto Type II model is:

$$S(x) = \left[1 + \xi\left(\frac{x - \lambda}{\sigma}\right)\right]^{-\frac{1}{\xi}}, \xi > 0 \tag{2}$$

where $x > \lambda$ (a location parameter), and $\sigma > 0$ is a scale parameter. Parameter ξ is the shape parameter. In principle, ξ can take on any real value (including the limiting case of $\xi = 0$, which implies an exponential distribution), but the restriction $\xi > 0$ yields heavy-tailed distributions of the 'Pareto' kind. The k^{th} moment exists only if $k < 1/\xi$. The Pareto Type I and II models are equivalent when $\xi = 1/\alpha$, $\lambda = x_m$, and $\sigma = x_m/\alpha$. With one additional parameter, the Pareto Type II model has the potential to fit real-world top incomes better. But the improvement in goodness-of-fit may be negligible and this has be balanced against the greater simplicity of the Pareto I model.

To implement Approaches A and C, we need formulae for the mean and inequality for the top income group (those with incomes greater than x_m or, equivalently, λ) expressed in terms of the model parameters. I display the formulae for these statistics in Table 2, and clearly they are simpler for the Pareto I model.

<Table 2 near here>

Estimation

Estimation of the two Pareto models proceeds by assuming x_m or μ is a threshold pre-specified by the researcher (not estimated) with its choice determined by a simple rule-of-thumb (such as the 95th or 99th percentile) or other means. I return to this issue below.

There are two methods commonly used to estimate the Pareto I shape parameter α. The first is an Ordinary Least Squares (OLS) regression of the log of empirical survivor function on the log of income and a constant term. The idea is that, if eq. (1) holds, then the Zipf plot – a plot of the log of the survivor function against logarithms of income (for incomes in ascending order and greater than x_m) – is a straight line with slope equal to $-\alpha$. Atkinson (2016) explains that α may be estimated by OLS in two other ways. (The Zipf approach uses data on income and the survivor function; the other two approaches utilize information about the total income received by income units.) I have estimated α using all three methods, but find that the Zipf method performed best, and so report only estimates from this in the main text. For the full set of estimates for all years, see Appendix B.

The OLS estimate of α is consistent (Quandt 1966) but the standard error is incorrect because no account is taken of the positive autocorrelation in the residuals introduced by the ranking of incomes. In contrast, the Maximum Likelihood (ML) estimator of α and its standard error is consistent, efficient, and asymptotically normal (Hill 1975, Quandt 1966). I implement the ML estimator using software by Jenkins and Van Kerm (2015). Both OLS and ML estimators are potentially biased in small samples, but the sample sizes in the tax return data employed in this paper are never 'small' – an advantage of using this source.

The ML estimator of α is susceptible to bias when there are a few high outlier incomes, the values of which may be potentially genuine or may reflect error and data contamination in the sense of Cowell and Victoria-Feser (1996, 2007) and Cowell and Flachaire (2007). The influence function for the ML estimator is unbounded in this situation. Figure 2 (and Appendix A) suggest that this issue may be relevant, even for tax data. I address this potential problem by using the ML 'Optimal b-robust estimator' (ML-OBRE) of Ronchetti and Victoria-Feser (1994). (The software implementation is by Van Kerm 2007.) The idea is to use the ML score function for most of the data (and exploit the efficiency of the ML estimator) but to place an upper limit c on the score function for high values in the interests of robustness. Ronchetti and Victoria-Feser (1994) show that, with 95% efficiency, the optimal value in the Pareto case is $c = 3$, and this is what I use. I use both ML and ML-OBRE estimators because only the former can be used for likelihood ratio tests of Pareto I

versus Pareto II models. Differences between their estimates are indicative of the empirical importance of the robustness problem.

There are several estimators of the Pareto Type II model: see e.g. Singh and Guo (2009) for a review. However, ML is the most commonly used and provides consistent, efficient and asymptotically normal estimates. The software implementation is by Roodman (2015); software for an ML-OBRE estimator is not available.

Are top incomes Pareto distributed?

Researchers commonly check for Pareto properties by inspecting whether Zipf plots are linear above some income threshold (while perhaps discounting apparent non-linearity in the very highest income range given the sparsity of observations there). However, Cirillo (2013) argues persuasively that we should not check Paretianity in this way: our eyes are unreliable detectors of linearity, and what we see as linearity is also consistent with non-Pareto distributions including lognormal distributions that do not have a heavy tail. As it happens, Zipf plots for each year of SPI data do appear roughly linear above a threshold (with the exception of 1995 – see below). However, given Cirillo's critique, I relegate these plots to Appendix C.

Mean excess plots are another tool used for checking Pareto properties. They plot mean income above a threshold against a series of thresholds. For Pareto distributions, the graph is a positively-sloped straight line above some minimum income; deviations from linearity are evidence of non-Paretianity. I show mean excess plots for selected years in Figure 3, using thresholds ranging from £10,000 per year to £600,000 per year. The graphs also show pointwise 95% confidence bands. The estimates for all years are shown in Appendix D.

<Figure 3 near here>

It is difficult to draw definitive conclusions from the mean excess plots. On the one hand, the plots are roughly linear at thresholds above approximately £50,000 per year though perhaps accompanied by some small decrease in slope at extremely high thresholds. On the other hand, in every plot, confidence intervals (CIs) become very wide as the income threshold increases (there are few observations at extremely high incomes), and so it is difficult to cite non-linearities with confidence. The plot for 1995 is an exception because non-linearity is much clearer. However, this is no doubt due to the SPI discontinuities cited in the previous section. The non-linearity in the 1996 plot arises at thresholds of £300,000 or more and hence relates to a tiny number of incomes.

Cirillo (2013: 5983) also points out that mean excess plots provide a reliable means of differentiating between Pareto distributions and lognormal distributions only if the number of observations is very large (he mentions 10,000). The most reliable conclusion that we can draw from the mean excess plots (and Zipf plots) is that there is no decisive rejection of Paretianity.

Zenga curves provide a much better means for discriminating between different types of model (Cirillo 2013). A Zenga curve, $Z(u)$, is a transformation of the Lorenz curve:

$$Z(u) = \frac{u - L(u)}{u[1 - L(u)]}, 0 < u < 1, \quad (3)$$

where $L(u)$ is the Lorenz curve for the distribution of incomes above a pre-specified threshold. For Pareto distributions, the Zenga curve is positively-sloped and rises as $u \to 1$ and, the higher the curve, the more heavy-tailed the distribution is. By contrast, for a lognormal distribution, the Zenga curve is horizontal. Figure 4 shows plots for 1996 and 2007 for thresholds of £60,000 and £120,000 (the higher threshold provides greater resolution over the top income range). See Appendix E for other years and thresholds.

The Zenga plots provide strong evidence in favour of Paretianity for all years (with the exception of 1995 for the reasons cited earlier.) At the same time, the location and precise shape of the curves changes over time and with threshold. This suggests that not only do Pareto tail indexes vary from year to year but also with the threshold chosen. I return to these issues below.

<Figure 4 near here>

Which distributional model for top incomes? Pareto I or Pareto II?

We cannot reliably differentiate between Pareto Type I and Type II models with these graphical checks. To do this, I use two approaches. The first is a straightforward likelihood ratio test. The second is comparisons of probability plots, specifically 'PP' plots graphing values of $p = F(x)$ predicted from each model against the values of p in the data, with a different plot for each threshold. Plots that lie wholly along the 45° line from the origin indicate perfect goodness of fit. The better fitting model is the one with less deviation from the 45° line.

Figure 5 summarizes likelihood ratio test statistics – equal to twice the difference in estimated log-likelihoods of ML-estimated Pareto I and II models – for thresholds up to £300,000 for 1996, 2001, 2007, and 2010. I cap the test statistics at 100 for plotting purposes. The dotted lines show critical values of the $\chi^2(1)$ distribution at significance levels 0.05, 0.01,

and 0.001. (Plots for other years are in Appendix F.) Regardless of the critical value chosen, the findings are clear. Using a likelihood criterion, we should choose the Pareto I model over Pareto II only if the threshold used to fit the models is extremely high. For 1996, the balance in favour of Pareto II is at all thresholds below around £100,000, which lies between *p*99 and *p*99.5. For the other three years shown in Figure 5, the cut-off threshold is at the same high level or even higher, and hence above the income level at which survey non-coverage starts (Table 1, Figure 2). The plots for other years confirm this general finding.

<Figure 5 near here>

The PP plots shown in Figure 6 compare model goodness of fit over the full range of incomes above the pre-specified threshold. Plots for the Pareto I model are on the left and for the Pareto II model on the right. For brevity, I show results only for 2007 and thresholds of £60,000 and £80,000 (between p95 and *p*99 in 2007), with plots for other years and thresholds in Appendix G. The fit of each model is good: the curves shown are closer to the 45° line than most textbook illustrations of PP plots. However, there is evidence that the Pareto II model fits better than Pareto I at the lower of the two thresholds (consistent with the likelihood ratio test findings). Below the median of the left-truncated distribution, Pareto I under-predicts empirical probabilities. More evidence in favour of Pareto II is apparent for other years and thresholds (see Appendix G). Overall, probability plots provide evidence in favour of the Pareto II model over the Pareto I model, but the differences in goodness of fit are generally not large.

<Figure 6 near here>

The results from the two types of goodness of fit check suggest that the choice between Pareto models is threshold-contingent. What, then, is the optimal threshold?

What is the optimal high income threshold?

Clauset et al. (2009) and Coles (2001) review methods for determining the threshold. The most commonly-used approaches are reviews of Zipf plots or minimum excess plots, as discussed above. Another intuitively attractive approach is to plot estimated parameters against thresholds and to choose as optimal threshold, the minimum income above which the plot is horizontal. For the Pareto I model the plot is of fitted α against threshold t; for the Pareto II model, the plots are of fitted ξ and 'modified scale parameter' $\sigma^* = \sigma\xi - t$ against t (Coles 2001: 83).

Clauset et al. argue against these 'subjective' approaches and in favour of a 'more objective and principled approach based on minimizing the "distance" between the power-law model and the empirical data' (2009: 670). After reviewing alternatives, they favour measuring distance between fitted and empirical distributions using the Kolmogorov-Smirnov (KS) statistic, i.e. the maximum distance between their cumulative distribution functions, *D*:

$$D = \max_{x \geq x_m} [F(x) - P(x)] \quad (3)$$

where $F(x)$ is the empirical CDF for incomes at the threshold x_m or above and $P(x)$ is the model-predicted CDF over the same range. (*D* is thus a numerical summary of information shown in a PP plot.) The optimal threshold is the value of x_m that minimizes *D*.

Figure 7 displays plots of estimated parameters against thresholds for both models, for 1996 and 2007. (Plots for other years are in Appendix H.) The vertical dashed lines show, from left to right, the percentiles *p*90, *p*95, *p*99, and *p*99.5 in the SPI data.

The figure shows that the choice of estimator matters when fitting a Pareto I model. On the one hand, the OLS estimator produces estimates of α that are distinctly smaller than those derived from ML and ML-OBRE estimators, except at extremely high thresholds. On the other hand, the ML and ML-OBRE estimates are remarkably similar.

<Figure 7 near here>

Regardless of estimator, the choice of threshold for the Pareto I model is not clear cut if the information in Figure 7 and Appendix H is used as the guide. The graphs are relatively flat only at extremely high thresholds, though the flattening out occurs at thresholds that are lower in later years – but they are very high nonetheless. The pattern for 2007 is also apparent from the start of the 2000s (Appendix H). Put differently, if we restrict the range of thresholds to between *p*95 and *p*99.5, i.e. in the range commonly used, then in 1996 the estimate of α varies between around 2.5 and 2. This is a wide range: it corresponds to Gini coefficients between 0.25 and 0.33 (according to the formula in Table 2). For 2007 and over the same range, the α estimates vary between 2.2 and 1.8, and hence Gini coefficients between 0.29 and 0.38.

In contrast, this sensitivity of parameter estimates is not apparent for the Pareto II model for thresholds in the range of *p*95 and *p*99.5. The curves are relatively flat and there is evidence for an optimal threshold lying between *p*95 and *p*99, with the precise range depending on the year.

Figure 8 displays optimal thresholds derived using the KS minimum distance criterion for both Pareto models. For the Pareto I case, the optimal thresholds are very similar for each year for ML and ML-OBRE estimators, with the exception of 1996 and 2004. It is striking that the optimal thresholds for the Pareto I model are typically much larger than those for the Pareto II model (except in 2007). For the Pareto I model, the optima are at around $p99.5$ or higher; for the Pareto II model, they are at about £50,000 which corresponds to around $p99$ in the mid- to late-1990s or $p95$ in 2000. Although there is variation in the estimated optimal threshold from year to year, there is much less variability in the optima derived for the Pareto II model than for those derived for the Pareto I model.

<Figure 8 near here>

The general lesson of this analysis is that Pareto I model estimates from top income data are sensitive to the choice of threshold, and perhaps more so than has been appreciated by researchers to date. Put differently, the range of thresholds for which the Pareto I model estimates are stable is well above the thresholds commonly used. Pareto II model estimates are more robust to the choice of threshold.

The specific lesson for applications of Approach C (and A) to determining total inequality is that estimates may be sensitive to choice of both the model of top incomes and the threshold. The criterion regarding threshold choice discussed earlier – that it should be in the income range at which survey under-coverage becomes apparent – further complicates matters. For the period considered here, this criterion implies a threshold somewhere between $p95$ and $p99$, with the former more appropriate in later years, the latter more appropriate in earlier years. This income range is broadly consistent with optimal thresholds derived for the Pareto II model but not those for the Pareto I model. In the light of these results, and in order to check the robustness of findings about overall inequality, my implementation of Approach C uses both Pareto models and multiple thresholds.

5. UK income inequality: estimates from combining estimates and combining trends

To implement Approaches C and A, we exploit the properties of inequality indices that are additively decomposable by population subgroup. For all such indices, we may write:

Total inequality = inequality among the top incomes group
+ inequality among the non-top incomes group (4)

+ between-group inequality

where between-group inequality is the inequality that would arise if each individual is attributed the mean of his or her income group. Additively decomposable indices include all members of the generalized entropy class I_a, including the mean logarithmic deviation (I_0 or '*L*'), the Theil index (I_1, '*T*'), and half the squared coefficient of variation (I_2, HSCV), that were cited in Table 2. The larger that a is, the more sensitive is I_a to income differences at the top of the distribution compared to the bottom. HSCV is particularly top-sensitive. Because the incomes of the top income group and the non-top income group do not overlap (by construction), the Gini coefficient is also additively decomposable in this context. For further discussion of decomposable inequality indices, see inter alia, Cowell (1980) and Cowell and Kuga (1981).

The decomposition formula for the Gini coefficient, *G*, derived by Atkinson (2007) and Alvaredo (2011), is also set out clearly by Cowell (2013: 43):

$$G = P_R S_R G_R + P_N S_N G_N + G_B. \quad (5)$$

P_R is the proportion of the population in the top income group ('Rich') in a given year; $P_N = 1–P_R$ is the proportion of the population in the non-rich group; $S_R = P_R \mu_R / \mu$ and $S_N = P_N \mu_N / \mu$ are the shares in total income of each group; μ_R and μ_N are the group mean incomes; and $\mu = P_R \mu_R + P_N \mu_N$ is the overall mean. Between-group inequality $G_B = S_R – P_R$.

Pareto I and Pareto II models fitted using the same threshold and data provide different estimates of total inequality *G* in a given year because they imply different estimates of G_R and μ_R. (G_R and μ_R may also be estimated non-parametrically: see below.) A higher estimate of μ_R from one model implies larger S_R and G_B. That model's estimate of *G* will be greater as well unless the higher μ_R coincides with a sufficiently lower value of G_R. For either model, what happens to estimates of *G* when one changes the threshold (and thence P_R) is less clear cut because there are changes in G_N and μ_N as well as in G_R and μ_R.

The researcher has to choose the value of P_R. In the light of the analysis in previous sections, I use three thresholds for each year, *p*99, *p*95, and *p*90, estimating them non-parametrically from the *survey* data. (Although *p*90 is substantially below the thresholds discussed earlier, I include it as a robustness check; it has been used by Ruiz and Woloszko 2015.) Because the survey estimates differ from their tax data counterparts (Table 1), P_R in the *tax* data is close to but not exactly equal to 1%, 5%, or 10% respectively (see Appendix I for the values for each year). I also estimate μ_N and G_N non-parametrically from the survey data, and μ_R and G_R, L_R, and T_R from the estimates of the two Pareto models using the

formulae shown in Table 1. (I report estimates for Pareto I derived using the ML-OBRE estimator.)

I calculate the combined estimate G using the formula in (5) and employ analogous steps to calculate estimates of L and T. I could not derive T for the Pareto II model (there were numerical integration problems) and I did not calculate HSCV because of its strong top-sensitivity and because the requisite moments of the fitted Pareto distribution do not always exist (Figure 7, Appendix H). Appendix I contains the estimates derived from the SPI data of the Pareto model parameters and their standard errors; μ_R, G_R, L_R, T_R and their standard errors (derived from the Pareto parameters using Table 2 formulae; and also calculated non-parametrically), plus μ_N and G_N, L_N, and T_N (derived non-parametrically from HBAI data). Appendix I also contains the combined estimates G, L and T, for all years, and for each of the three sets of estimates of mean income and inequality among the Rich. I focus discussion initially on the Pareto model-based estimates for the Gini coefficient, and later compare them with the fully non-parametric estimates, together with corresponding estimates for L and T.

Figure 9 charts the Pareto-based estimates of mean income among the Rich (μ_R), the share of total income held by the Rich (S_R), inequality among the Rich (G_R), and the overall combined estimate (G), for each of the three percentile thresholds. The Pareto I estimates are on the left; the Pareto II estimates are on the right.

The headline finding is that income inequality summarized by the Gini coefficient distinctly increased between the mid-1990s and 2007: see panel (a). It then fell back to late-1990s levels by 2010, though assessment of the fall is complicated by the forestalling issues mentioned earlier. Most of the inequality increase occurred between 2004 and 2007. These conclusions hold regardless of which Pareto model and threshold is used.

<Figure 9 near here>

Using a higher threshold leads to higher estimates of μ_R, S_R, and G_R in each year, for both Pareto models, and especially going from P_R = 5% to P_R = 1%. The S_R estimates closely track those shown in the World Top Incomes Database for the UK (based exclusively on SPI data), though there are some differences in levels (the S_R depend also on survey data).

However, when looking at overall inequality summarised by G, the Pareto II estimates are less sensitive to the choice of threshold than are the Pareto I estimates: see panel (a). Each yearly Pareto II estimate of G differs by at most one percentage point across series for the three thresholds (in the mid-1990s), and the series for P_R = 5% and P_R = 10% are virtually identical up to 2006. For the Pareto I model, the corresponding range is larger, reaching a

maximum of around 2.5 percentage points (2009). Otherwise, again, the largest differences are between the series for P_R = 1% on the one hand, and P_R = 5% or 10% on the other hand. The variation in estimates relates back to the earlier findings regarding choice of the optimal threshold. The thresholds used in this section correspond to range of optimal thresholds for the Pareto II model, but well below those for the Pareto I model.

I now contrast my estimates of inequality trends derived using Approach C with estimates derived using other approaches. For brevity, I show only the results for P_R = 5%: see Figure 10. Conclusions are largely insensitive to choice of threshold in any case: for the corresponding graphs for the other two thresholds, see Appendix I.

The three series of Approach C estimates ('HBAI & SPI' variants) differ according to whether top incomes are summarised using the Pareto I or Pareto II models or non-parametrically. There are two HBAI series showing trends in inequality for the poorest 95% and the poorest 100% of the survey data, i.e. not including any information from the tax data. The HBAI-SPI series uses the estimates in the UK official income statistics derived using a variant of Approach B (the DWP's SPI adjustment, cited earlier). It is important to note that the HBAI-SPI series uses a different income definition and refer to a different population than all of the other series shown in the figure. It refers to inequality of equivalized household net income among all individuals (adults and children) rather than to individual gross income among adults. Estimates of inequality *levels* based on HBAI-SPI definitions are smaller than estimates based on the tax data definition, but the differences in definitions have little effect on estimates of inequality *trends* (Burkhauser et al. 2016). The HBAI-SPI2 series is from Burkhauser et al. (2016) and uses a different variant of Approach B (see earlier). I summarise inequality not only using the Gini coefficient (panel a), but also using *L* and *T* (panels b and c). The DWP does not publish estimates for *L* or *T*: I derived them non-parametrically from public use HBAI unit record data.

<Figure 10 near here>

Figure 10 shows that if one restricts attention to the poorest 95% in the survey data each year, all three indices show a marked decline in inequality over the period as a whole. (These estimates are unlikely to be contaminated by the 'forestalling' issue.) Inequality also appears to be falling according to the series that uses 100% of the survey data observations (the Gini fell by around 5% between 1996 and 2007), but another distinctive feature of the series is its volatility. This is particularly acute for the Theil index, which is unsurprising because it is the most top-sensitive of the three indices. Thus Figure 10 illustrates well the sensitivity problems analysed by Cowell and Flachaire (2007) and also their conclusion that

in terms of performance in finite samples there is little to choose between the Gini coefficient and the mean logarithmic deviation (L). But what happens if one utilizes information about top incomes from tax data?

According to all three Approach C variants, and all three inequality indices, inequality increased between 1996 and 2007. The Gini coefficient increased by around 5% according to the Pareto I estimates, by around 8% according to the Pareto II estimates, and by around 7% according to the non-parametric ones. For L, the corresponding increases are 1%, 5%, and 4%. For T, the increase in the Pareto I estimate is 24% and 33% for the non-parametric estimate. These results indicate that using a Pareto II model for top incomes leads to larger estimates of the rise in inequality over this period than does a Pareto I model, not only according to the Gini coefficient (Figure 9) but also according to L. In addition, the Pareto II estimates of trends in G and L are quite similar to the non-parametric ones. This is reassuring evidence for analysts that the Pareto II model provides a parsimonious but good description of distributions of top incomes. Using a more top-sensitive index (T rather than L) leads to a greater estimated increase in inequality, reflecting the marked increase is top income shares over the period.

Figure 10 also shows how the three Approach C estimates of inequality trends compare with the two Approach B series. The Burkhauser et al. (2016) HBAI-SPI2 series is very similar to the Pareto I-based Approach C series for all three inequality indices.

By contrast, trends in the DWP's official statistics series (HBAI-SPI) appear at first sight to differ markedly from those of all three Approach C estimates and for all three inequality indices. However, closer inspection of the figure reveals that the differences in trends arise almost entirely between 1996 and 1998. The official series shows a sharp increase in inequality over those two years; trends are much more similar across series in subsequent years. It is difficult to explain the sub-period inconsistency across series. One possible source is the way in which the DWP's SPI adjustment derives the cell mean estimates for top income groups. According to Department for Work and Pensions (2015*a*: 11), values to be used in year t of the HBAI data are derived by HMRC statisticians by 'projections' from SPI data for year t–1 (because year t's SPI data are not yet available). No further details of the projection method are given. By contrast, all my Approach C estimates and Burkhauser et al.'s (2016) Approach B estimates combine information HBAI data for year t and SPI data for year t. I conjecture that a larger than usual difference between projected cell-means and actual out-turns for 1996–1998 are the source of the inconsistency

observed for that period. The public-use SPI data do not contain the variables that would allow me to check if this is the case.

In sum, apart from the exceptional and short sub-period just discussed, there is substantial consistency across the different methods for combining survey data and tax data about top incomes. Compared to the estimates that are wholly survey-based, all show a rise in income inequality over the decade prior to the onset of the Great Recession, whereas the estimates that are wholly survey-based show no increase in inequality.

6. Summary and conclusions

Statistical agencies and other researchers typically estimate income inequality levels and trends from either survey data or tax return data, but rarely combine the information in the two types of data source. The result is that very different impressions about how inequality is changing over time may arise, as the examples for the UK and the USA in the Introduction show. Research users may reasonably ask what the 'true' picture of inequality trends is. There is a good case for providing them with answers using methods that combine information from survey and tax data in order to take advantage of the strengths of each source. In particular, tax return data provide better coverage of top incomes than do survey data; and survey data provide the ability to create income variables with the same definitions, so that combination is done on a like-for-like basis.

I have analysed income inequality levels and trends for the UK by combining inequality estimates from survey and tax data (Approach C), contrasting these estimates with those derived by combining data per se (Approach B). As part of this analysis, I have also provided new findings about survey under-coverage of top incomes in UK survey data (Section 3). The problem becomes apparent at around the 99th percentile in the 1990s but at around the 95th percentile in the 2000s.

I have found that that conclusions about aggregate UK inequality trends since the mid-1990s are broadly robust to the way in which tax data are employed in Approach C. One may conclude for example that the Gini coefficient for gross individual income rose by around 7% to 8% between 1996/97 and 2007/08, with most of the increase occurring after 2003/04. When I use only survey data, with tax data not exploited at all, the Gini coefficient is estimated to decrease by around 5% over the same period.

The result that combining information about top incomes from tax data with information about the rest of the distribution in survey data leads to an estimated increase in inequality is unsurprising given knowledge of survey under-coverage of top incomes and the marked rise in top income shares in the UK over the last two decades. But I have shown how we may go beyond the qualitative conjecture and provide specific quantitative estimates of inequality trends, and for a range of inequality indices.

The analysis highlights the continuing importance of normative judgements for inequality analysis. Different inequality indices incorporate different assumptions about how to evaluate income differences in different parts of the income distribution (Cowell 1977, 2011). A focus on the income share of the top 1% measure means that zero weight is placed on income differences among the poorest 99% or on differences between the rich and non-rich groups. This paper has considered inequality indices that give a non-zero weight to everyone. It is because of this, and because the rich are assumed to form such a small proportion of the population, that I estimate the increase in UK income inequality over the last two decades to be substantially smaller than the rise in the income share of the top 1% whether shown by the tax data alone or by the combined estimate (Figure 9).

The portfolio of inequality indices is also constrained by practical considerations: the indices used cannot be too top-sensitive. Application of the semiparametric Approach C is problematic if the distribution of top incomes is particularly heavy-tailed. Various moments of the fitted Pareto distributions do not exist in this case, and hence nor do many top-sensitive inequality indices (Figure 7; Appendix H). Cowell and Flachaire (2007) make this argument in the context of Approach A; I have shown that it also applies even if one uses tax data to describe top incomes. One might instead consider implementing Approach C using non-parametric estimates of top-sensitive inequality measures for the top income group, but I have found that such estimates are non-robust and volatile in the sense described in the Introduction, even using SPI tax data rather than HBAI survey data. (See the nonparametric estimates of the HSCV for the rich that are reported in Appendix I.)

In this paper, I have focused on inequality estimation issues related to data quality and ignored issues of statistical significance (as has virtually all previous work using Approaches B and C). It is relatively straightforward to estimate standard errors or the various elements in the inequality decomposition equation (1) using standard asymptotic formulae (these estimates are provided in Appendix I). However, there are non-trivial challenges to overcome in providing reliable inference for the overall inequality estimate. Cowell and Flachaire (2007, especially Section 3.3) discuss these issues with reference to generalised entropy

inequality indices. The case of the Gini coefficient appears not to have been discussed in the literature to date.

As part of deriving the substantive results about inequality levels and trends, this paper has also provided new evidence about which Pareto model to fit to the upper tail of a heavy-tailed distribution, and which threshold to use when doing this. Although the application has been to income, the analysis should be of broad interest because Pareto distributions are commonly used in many other contexts,. In his recent review of power laws in economics and finance, Gabaix argued that 'the Pareto law has survived the test of time: It fits still quite well. The extra degree of freedom allowed by a lognormal might be a distraction from the essence of the phenomenon' (2009: 285). He might have substituted 'Pareto II' for 'lognormal'. My analysis has shown that there is a good case for exploiting the extra degree of freedom provided by the Pareto II model, especially given the top income thresholds that are typically used (*p*99 or less). Put differently, the Pareto I model is as good as Pareto II only at extremely high incomes, beyond the range of thresholds usually considered. My conclusions refer to income rather than other variables such as wealth or city and firm size, and to the UK rather than to other countries, so checking the robustness of my findings in other contexts would be a useful topic for future research.

References

Alfons, A., Templ, M. and Filzmoser, P. (2013). Robust estimation of economic indicators from survey samples based on Pareto tail modelling. *Journal of the Royal Statistical Society, Series C*, 62 (2), 271–286.

Alvaredo, F. (2011). A note on the relationship between top income shares and the Gini coefficient. *Economic Letters*, 110 (3), 274–277.

Alvaredo, F., Atkinson, A. B., Piketty, T., and Saez, E. (2013). The top 1 percent in international and historical perspective. *Journal of Economic Perspectives*, 27 (3), 3–20.

Alvaredo, F., Atkinson, A. B., Piketty, T., Saez, E. (2015). The World Top Incomes Database. http://g-mond.parisschoolofeconomics.eu/topincomes/.

Alvaredo, F. and Londoño Vélez, J. (2013). High incomes and personal taxation in a developing economy: Colombia 1993–2010. CEQ Working Paper 12. New Orleans, LA: CEQ Institute, Tulane University.

http://www.commitmentoequity.org/publications_files/CEQWPNo12%20HighTaxationDevEconColombia1993-2010_19March2013.pdf

Anand, S. and Segal, P. (2016). Who are the global top 1%? International Development Institute Working Paper 2016-02. London: Kings College London. http://www.kcl.ac.uk/sspp/sga/idi/Research/IDI-Working-Papers.aspx

Arnold, B. C. (2008). Pareto and generalized Pareto distributions. In: D. Chotikapanich (ed.), *Modeling Income Distributions and Lorenz Curves*. New York: Springer Science+Business Media.

Arnold, B. C. (2015). *Pareto Distributions*, second edition. Boca Raton, FL: Chapman and Hall/CRC.

Atkinson, A. B. (2005). Top incomes in the UK over the twentieth century. *Journal of the Royal Statistical Society, Series A*, 168 (2), 325–343.

Atkinson, A. B. (2007). Measuring top incomes: methodological issues. In: A. B. Atkinson and T. Piketty (eds), *Top Incomes over the Twentieth Century: A Contrast Between Continental European and English-Speaking Countries*. Oxford: Oxford University Press.

Atkinson, A. B. (2016). Pareto and the upper tail of the income distribution in the UK: 1799 to the present. Unpublished paper, Nuffield College, Oxford. Forthcoming in *Economica* special issue in honour of Frank Cowell.

Atkinson, A. B., and Piketty, T. (eds). (2007). *Top Incomes over the Twentieth Century: A Contrast Between Continental European and English-Speaking Countries*. Oxford: Oxford University Press.

Atkinson, A. B., Piketty, T., and Saez, E. (2011). Top incomes in the long run of history. *Journal of Economic Literature*, 49 (1), 3–71.

Bach, S., Corneo, G., and Steiner, V. (2009). From bottom to top. The entire income distribution in Germany, 1992–2003. *Review of Income and Wealth*, 55 (2), 303–330.

Belfield, C., Cribb, J., Hood, A., and Joyce, R. (2015). *Living Standards, Poverty and Inequality in the UK: 2015*. Report 107. London: Institute for Fiscal Studies. http://www.ifs.org.uk/publications/7878

Burkhauser, R.V., Feng, S., Jenkins, S. P., and Larrimore, J. (2012). Recent trends in top income shares in the USA: reconciling estimates from March CPS and IRS tax return data, *Review of Economics and Statistics*, 94 (2), 371–388.

Burkhauser, R. V., Hérault, N., Jenkins, S. P., and Wilkins, R. (2016). What has been happening to UK income inequality since the mid-1990s? Answers from reconciled and

combined household survey and tax return data. ISER Working Paper 2016-03. Colchester: Institute for Social and Economic Research. https://www.iser.essex.ac.uk/research/publications/working-papers/iser/2016-03

Cirillo, P. (2013). Are your data really Pareto distributed? *Physica A*, 392, 5947–5962.

Clauset, A., Shaliza, C. R., and Newman, M. E. J. (2009). Power-law distributions in empirical data. *SIAM Review*, 51 (4), 661–703.

Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. London: London.

Cowell, F. A. (1977). *Measuring Inequality*. Oxford: Philip Allan.

Cowell, F. A. (1980). On the structure of additive inequality measures. *Review of Economic Studies*, 47, 521–531.

Cowell, F. A. (1989). Sampling variance and decomposable inequality measures. *Journal of Econometrics*, 42, 27–41.

Cowell, F. A. (2011). *Measuring Inequality*, third edition. Oxford: Oxford University Press.

Cowell, F. A. (2013). UK wealth inequality in international context. In: J. Hills, F. Bastagli, F. Cowell, H. Glennerster, E. Karagiannaki, and A. McKnight, *Wealth in the UK, Distribution, Accumulation, and Policy*. Oxford: Oxford University Press.

Cowell, F. A., Ferreira, F. H. G., and Litchfield, J. A. (1998). Income distribution in Brazil 1981–1990. Parametric and non-parametric approaches. *Journal of Income Distribution*, 8, 63–76.

Cowell, F. A. and Flachaire, E. (2007). Income distribution and inequality measurement: the problem of extreme values. *Journal of Econometrics*, 141, 1044–1072.

Cowell, F. A. and Kuga, K. (1981). Inequality measurement: an axiomatic approach. *European Economic Review*, 15, 287–305.

Cowell, F. A. and Victoria-Feser, M.-P. (1996). Robustness properties of inequality measures. *Econometrica*, 64 (1), 77–101.

Cowell, F. A. and Victoria-Feser, M.-P. (2007). Robust stochastic dominance: A semi-parametric approach. *Journal of Economic Inequality*, 5, 21–37.

De-Nevas Walt, C. and Proctor, B. D. (2015). *Income and Poverty in the United States: 2014*. Washington, DC: US Bureau of the Census.

Department for Work and Pensions (2013). *Households Below Average Income, 1994/95-2012/13* [computer file]. *7th Edition.* Colchester, Essex: UK Data Archive [distributor], October 2013. SN: 5828.

Department for Work and Pensions (2015). *Households Below Average Income An Analysis of the Income Distribution 1994/95 – 2013/14*. London: Department for Work and Pensions.

Department for Work and Pensions (2015*a*). *Households Below Average Income. Quality and Methodology Information Report – 2013/14*. London: Department for Work and Pensions.

Department for Work and Pensions, National Centre for Social Research, and Office for National Statistics Social and Vital Statistics Division (2014), *Family Resources Survey, 2010-2011* [computer file]. *2nd Edition.* Colchester, Essex: UK Data Archive [distributor], October 2014. SN: 7085, http://dx.doi.org/10.5255/UKDA-SN-7085-2

Diaz-Bazan, T. (2015). Measuring inequality from top to bottom. World Bank Policy Research Paper 7237. Washington, DC: World Bank. http://documents.worldbank.org/curated/en/2015/04/24353317/measuring-inequality-top-bottom

Eeckhout, J. (2004). Gibrat's law for (all) cities. *American Economic Review*, 94 (5), 1429–1451.

Gabaix, X. (2009). Power laws in economics and finance. *Annual Review of Economics*, 1, 255–293.

Gabaix, X. (2016). Power laws in economics: an introduction. *Journal of Economic Perspectives*, 30 (1), 185–206.

Harrison, A. (1979). The upper tail of the earnings distribution. Pareto or lognormal? *Economic Letters*, 2, 191–195.

Harrison, A. (1981). Earnings by size: a tail of two distributions. *Review of Economic Studies*, 48 (4), 621–631.

Hill, B. M. (1975). A simple general approach to inference about the tail of a distribution. *Annals of Statistics*, 3 (5), 1163–1174.

HM Revenue and Customs (2012). The Exchequer effect of the 50 per cent additional rate of income tax. London: HMRC.

HM Revenue and Customs KAI Data, Policy and Co-ordination (2014), *Survey of Personal Incomes, 2010-2011: Public Use Tape* [computer file]. Colchester, Essex: UK Data Archive [distributor], November 2014. SN: 7569, http://dx.doi.org/10.5255/UKDA-SN-7569-1.

Jenkins, S. P. and Van Kerm, P. (2015). PARETOFIT: Stata module to fit a Type 1 Pareto distribution. Statistical Software Components archive. Boston, MA: Boston College. https://ideas.repec.org/c/boc/bocode/s456832.html

Kleiber, C. and Kotz, S. (2003). *Statistical Size Distributions in Economics and Actuarial Sciences*. Hoboken, NJ: John Wiley & Sons.

Lakner, C. and Milanovic, B. (2016). Global income distribution. From the fall of the Berlin Wall to the Great Recession. *World Bank Economic Review*, 30 (2), 203–232.

OECD (2008). *Growing Unequal? Income Distribution and Poverty in OECD Countries*. Paris: OECD Publishing.

OECD (2011). *Divided We Stand: Why Inequality Keeps Rising*. Paris: OECD Publishing.

OECD (2015). *In It Together: Why Less Inequality Benefits All*. Paris: OECD Publishing.

Roodman, D. (2016). EXTREME: Stata module to fit models used in univariate extreme value theory. Statistical Software Components archive. Boston, MA: Boston College. https://ideas.repec.org/c/boc/bocode/s457953.html

Ruiz, N. and Woloszko, N. (2015). What do household surveys suggest about the top 1% incomes and inequality in OECD countries? Economics Department Working Paper No. 1265. Paris: OECD. http://dx.doi.org/10.1787/5jrs556f36zt-en

Shorrocks, A. F., Davies, J. and Lluberas, R. (2015). *Credit Suisse Global Wealth Databook 2015*. Zurich: Credit Suisse Institute.

Singh, V. P. and Guo, H. (1995). Parameter estimation for 3-parameter generalized Pareto distribution by the principle of maximum entropy (POME). *Hydrological Sciences Journal*, 40 (2), 165–181.

Van Kerm, P. (2007). Extreme values and robust distribution analysis. Presentation at 13th UK Stata User Group, London. http://www.stata.com/meeting/13uk/vankerm-uksug_slides.pdf

Vermeulen, P. (2014). How fat is the top tail of the wealth distribution? ECB Working Paper 1962. Frankfurt am Main: European Central Bank. https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1692.pdf

Victoria-Feser, M.-P. and Ronchetti, E. (1994). Robust methods for personal income distribution models. *Canadian Journal of Statistics*, 22 (2), 247–258.

Table 1. Percentiles of individual gross income (£ p.a., 2012/13 prices), survey and tax data estimates

Year	*p*90			*p*95			*p*99			*p*99.5		
	HBAI	SPI	Ratio	HBAI	SPI	Ratio	HBAI	SPI	Ratio	HBAI	SPI	Ratio
1995	35,551	30,964	115	45,602	40,056	114	83,362	78,340	106	107,080	105,630	101
1996	37,028	31,209	119	47,623	41,043	116	85,141	80,869	105	109,948	113,469	97
1997	37,317	32,381	115	48,084	42,133	114	89,995	86,330	104	122,952	122,466	100
1998	38,285	33,595	114	50,485	44,541	113	95,853	92,694	103	127,830	131,499	97
1999	39,075	35,333	111	51,065	46,776	109	96,053	98,250	98	126,214	138,335	91
2000	40,371	37,273	108	53,048	49,786	107	106,413	105,722	101	142,586	150,871	95
2001	41,854	38,370	109	54,721	51,117	107	105,741	108,620	97	140,193	153,585	91
2002	41,360	38,498	107	54,401	51,370	106	103,532	108,742	95	136,152	153,379	89
2003	41,932	38,174	110	54,644	50,860	107	100,354	108,587	92	135,495	153,474	88
2004	42,460	40,575	105	55,847	53,533	104	107,367	114,855	93	144,244	165,249	87
2005	42,691	42,191	101	56,158	56,172	100	108,183	125,574	86	140,916	183,723	77
2006	43,335	42,885	101	56,101	57,369	98	109,910	131,012	84	157,894	193,230	82
2007	42,735	43,994	97	56,007	59,149	95	111,282	136,392	82	156,536	201,972	78
2008	43,312			56,703			109,812			143,624		
2009	42,564	41,917	102	56,839	55,782	102	113,420	124,753	91	156,384	183,777	85
2010	41,036	40,690	101	54,377	53,777	101	106,812	114,368	93	147,956	163,524	90

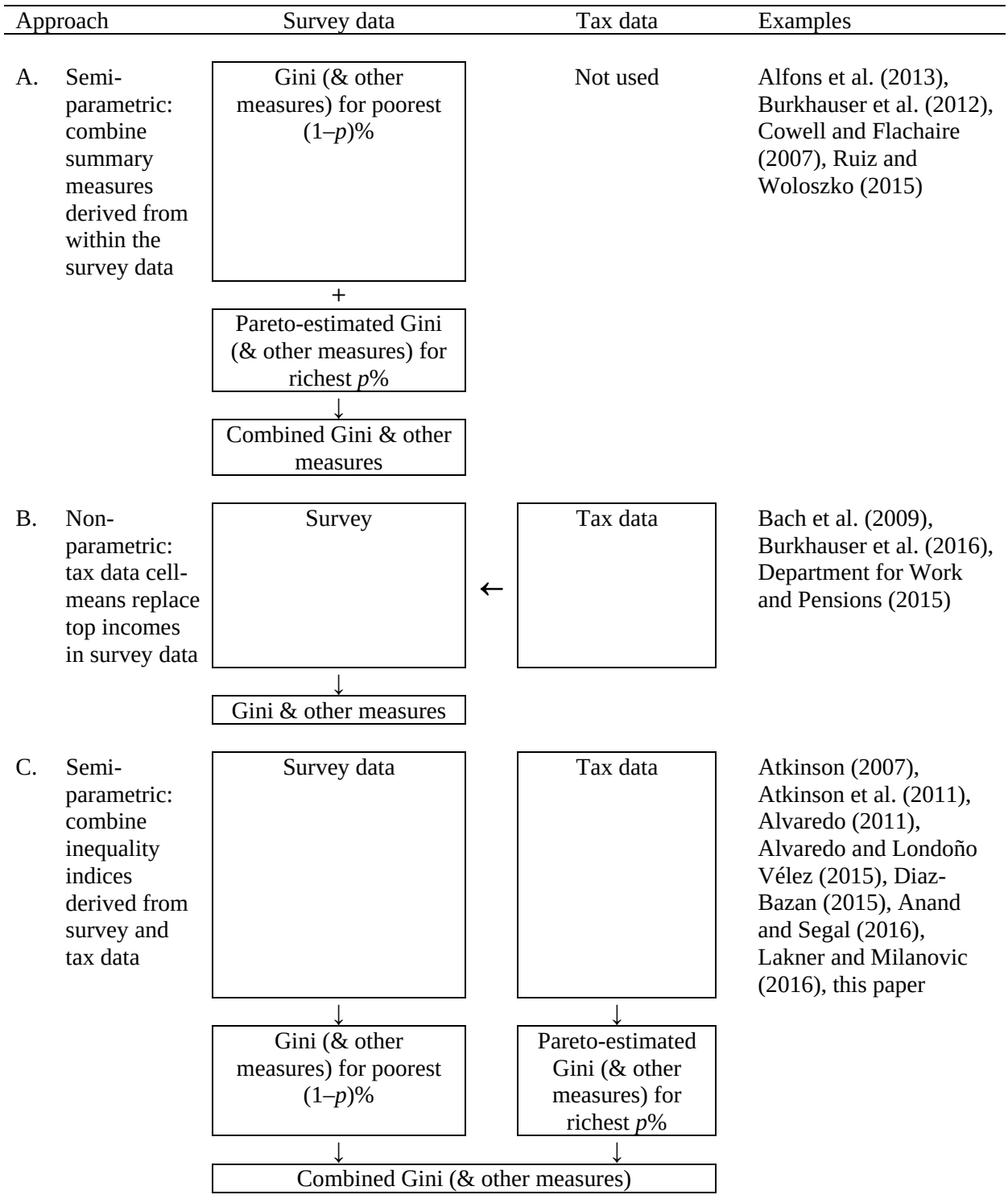
Notes. Author's estimates from HBAI (survey) and SPI (tax) data. Years refer to fiscal years ('1995' means 1995/96, and so on). No SPI unit record data have been released for 2008. Ratio: ratio of HBAI estimate to SPI estimate, in percent.

Table 2. Pareto Type I and Type II models: mean and inequality indices

Statistic	Pareto Type I	Pareto Type II
Mean	$\frac{\alpha x_m}{\alpha - 1}, \alpha > 1$	$\lambda + \left(\frac{\sigma}{1-\xi}\right), \xi < 1$
Gini coefficient (G)	$\frac{1}{2\alpha - 1}, \alpha > 1$	$1 - \left[\frac{\lambda + 2\left(\frac{\sigma}{\xi^2}\right) B\left(\frac{2-\xi}{\xi}, 2\right)}{\lambda + \left(\frac{\sigma}{\xi^2}\right) B\left(\frac{1-\xi}{\xi}, 2\right)}\right], \xi < 1$
Mean logarithmic deviation (L)	$\log\left(\frac{\alpha}{\alpha - 1}\right) - \left(\frac{1}{\alpha}\right), \alpha > 1$	No closed form expression
Theil index (T)	$\left(\frac{1}{\alpha - 1}\right) - \log\left(\frac{\alpha}{\alpha - 1}\right), \alpha > 1$	No closed form expression
Half the coefficient of variation squared (HSCV)	$\frac{1}{2\alpha(\alpha - 2)}, \alpha > 2$	$\frac{\sigma^2}{2(1-2\xi)[\lambda(1-\xi) + \sigma]^2}, \xi < \frac{1}{2}$

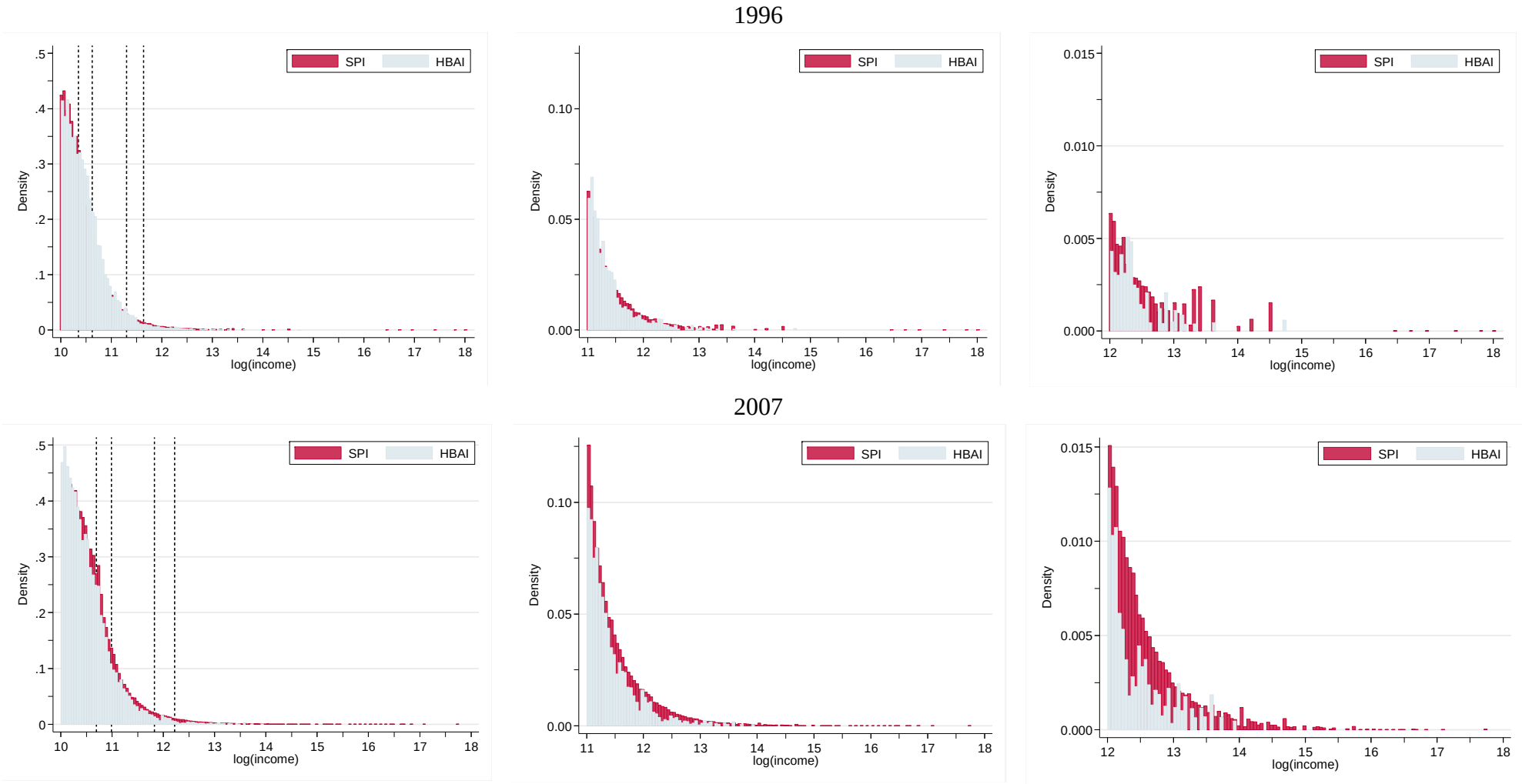
Notes. For the formulae for the survivor functions of the Pareto I and II models, see the main text. $B(.)$ is the Beta distribution. Sources for formulae: Arnold (2008), Cowell (2007), Kleiber and Kotz (2003), and Singh and Guo (1995). L, T, and HSCV are members of the generalized entropy family of inequality indices, $I(a)$, with a = 0, 1, and 2 respectively. Values of the L and T for the Pareto II distribution may be derived by numerical integration using the formulae for generalized moments in Cowell (1989), if the relevant moments exist.

Figure 1. Estimating inequality: approaches to addressing top income issues in survey data



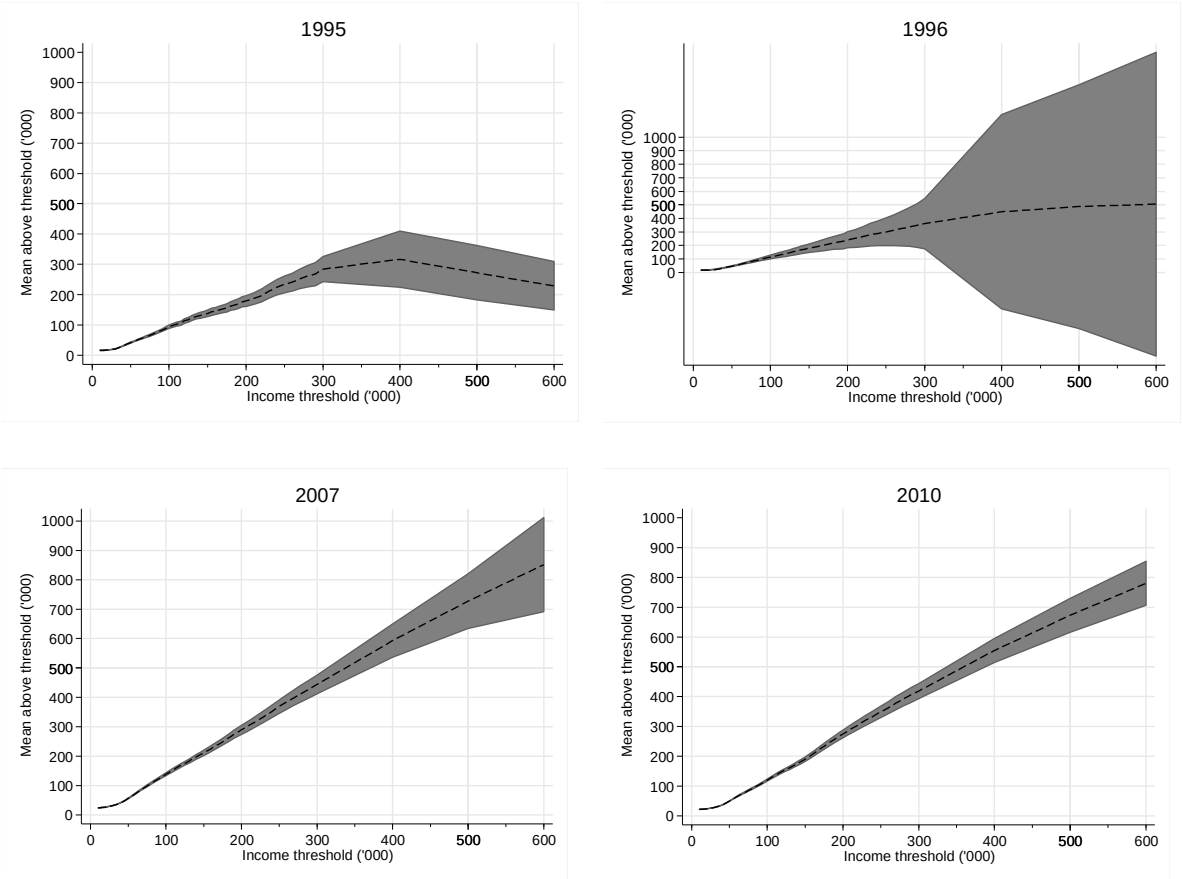
Note. Approach C may also be implemented using non-parametric estimates from tax data: see main text.

Figure 2. The concentration of income in high and extremely high income ranges: survey and tax return data compared, 1996 and 2007



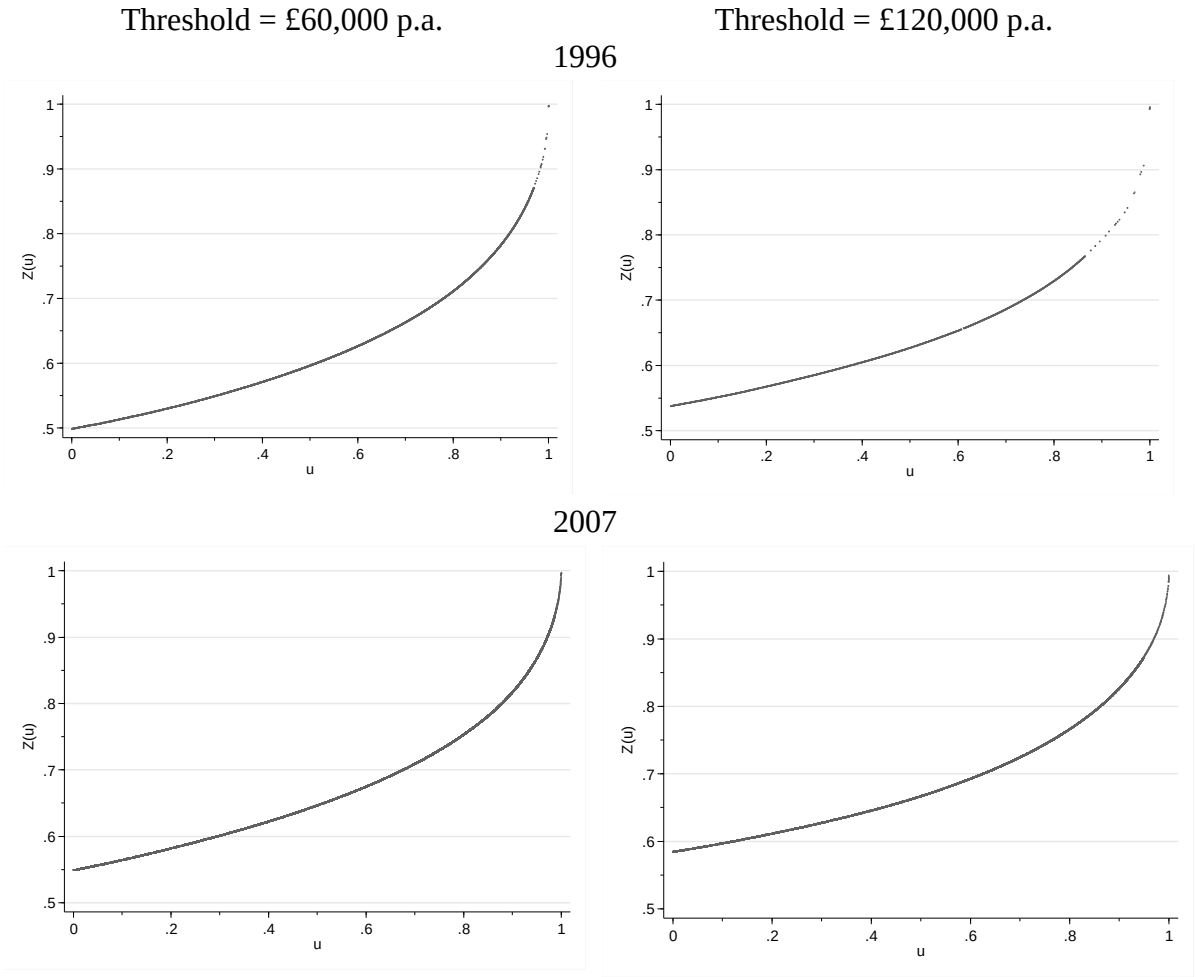
Notes. Author's estimates from SPI (tax) and HBAI (survey) data. Income is in £ per year, 2012/13 prices. Vertical dashed lines show (from left to right) *p*90, *p*95, *p*99, *p*99.5. For plots for other years, see Appendix A.

Figure 3. Mean excess plots for top incomes, tax return data, by year



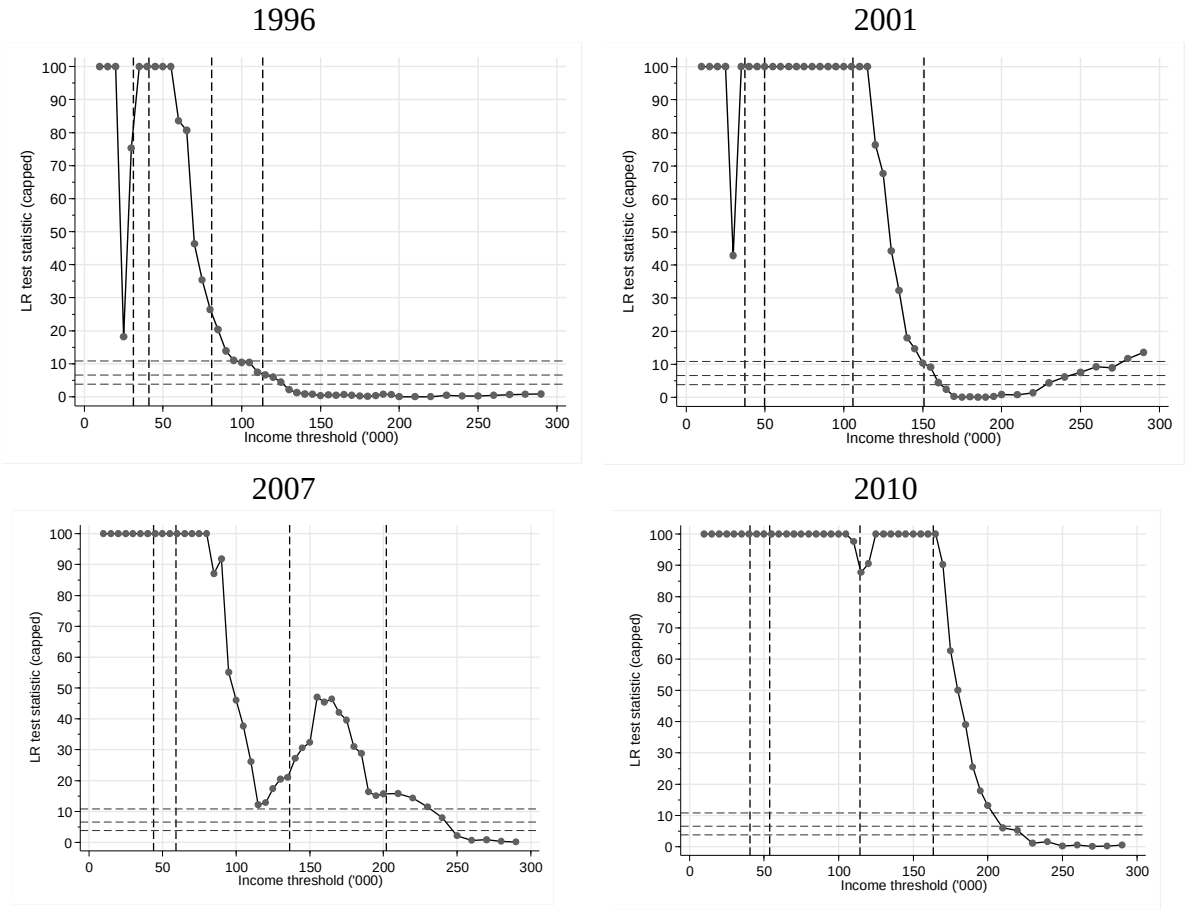
Notes. Author's estimates from SPI data. For plots for other years, see Appendix C. The shaded areas represent pointwise 95% confidence bands. Thresholds are in £ per year, 2012/13 prices. Plots estimated at intervals of £5,000 for thresholds between £10,000 and £200,000, £10,000 between £210,000 and £300,000, and £100,000 thereafter.

Figure 4. Zenga plots for top incomes, tax return data, by threshold and year



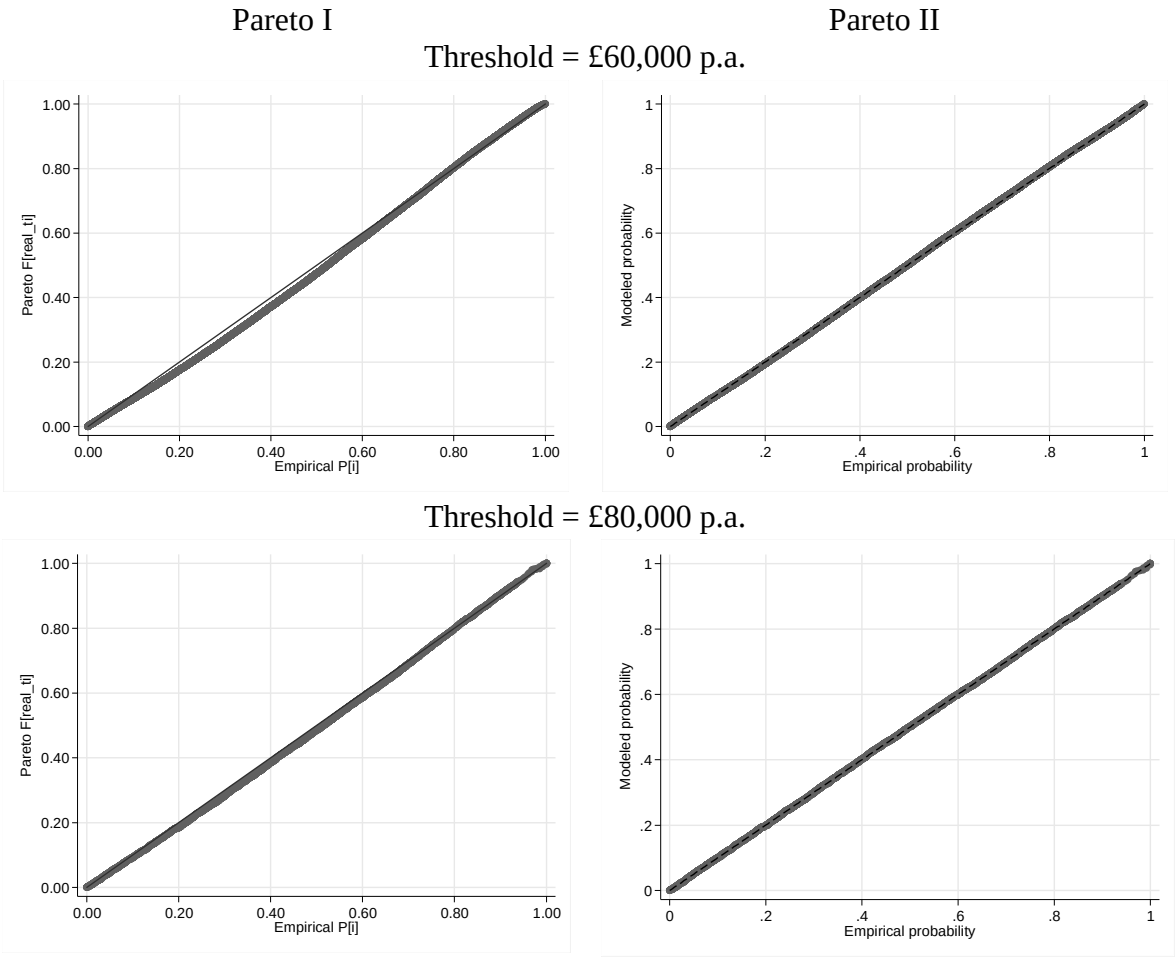
Notes. Author's estimates from SPI data. For plots for other years and thresholds, see Appendix D. On the Zenga plot, see the main text and Cirillo (2013).

Figure 5. Likelihood ratio test statistics (Pareto I versus Pareto II), by threshold: tax return data for 1996, 2001, 2007, and 2010



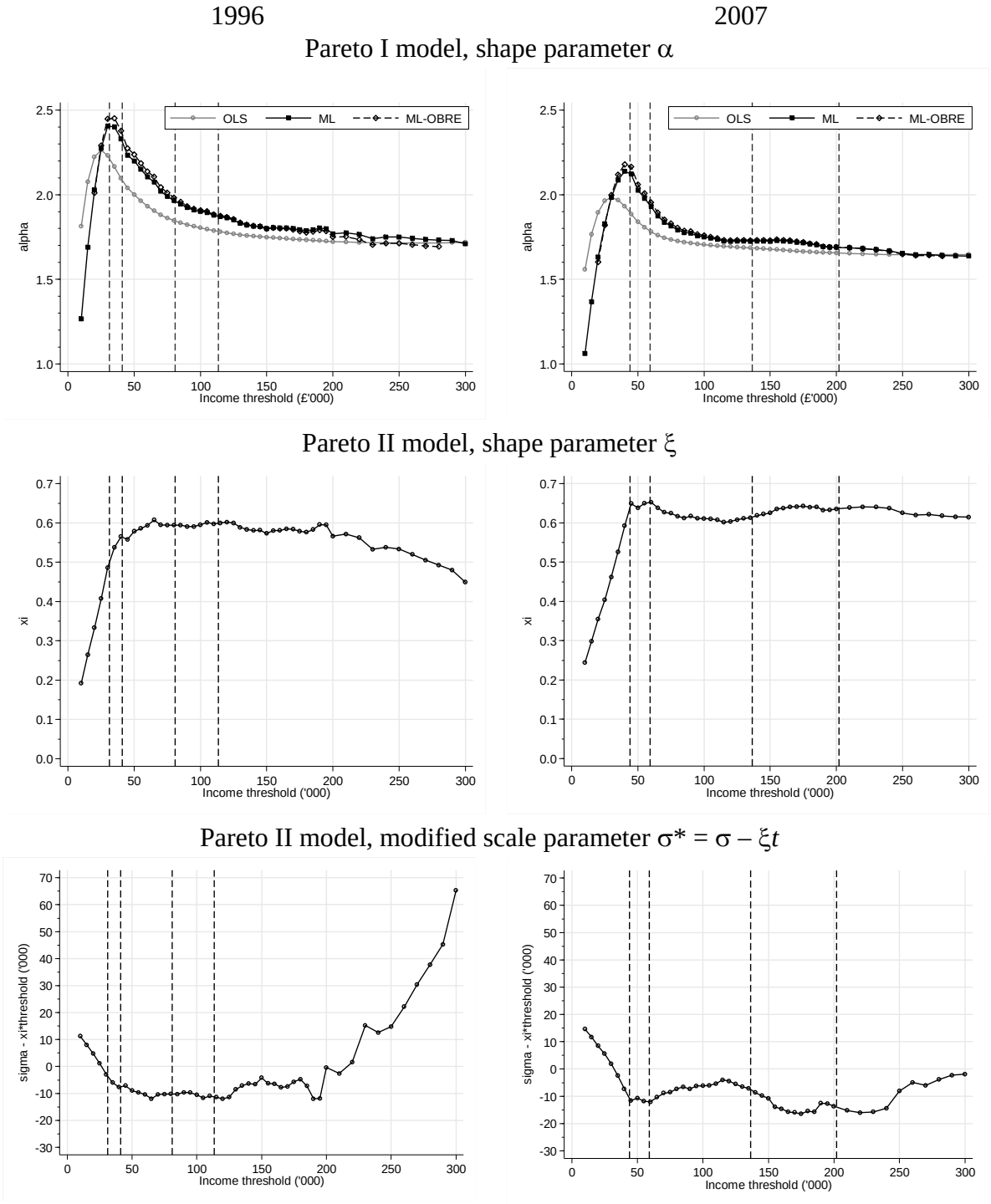
Notes. Author's estimates from SPI data. The figures plot twice the difference in log-likelihood for Pareto I and II models (each fitted using ML). Test statistics are capped at 100 for plotting purposes. Dotted horizontal lines show critical values of the $\chi^2(1)$ distribution at significance levels 0.05, 0.01, and 0.001. Vertical dashed lines show (from left to right) *p*90, *p*95, *p*99, *p*99.5. For plots for other years, see Appendix E.

Figure 6. PP plots for top incomes, by threshold: tax return data for 2007



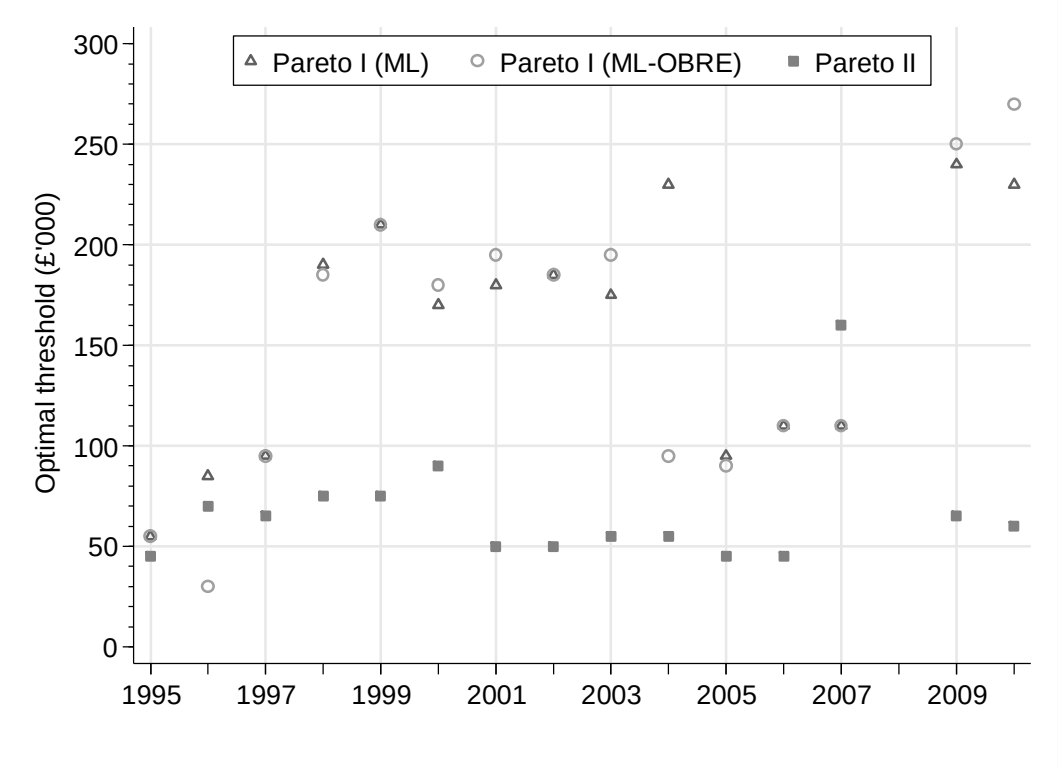
Notes. Author's estimates from SPI data. The charts plot modelled (cumulative) probabilities against empirical probabilities: see text. For plots for other years and thresholds, see the Appendix F. ML estimator used for both models.

Figure 7. Pareto I and II parameter estimates, by threshold, tax return data for 1996 and 2007



Notes. Author's estimates from SPI data. Vertical dashed lines show (from left to right) *p*90, *p*95, *p*99, *p*99.5. For plots for other years, see Appendix G.

Figure 8. Optimal Pareto threshold (KS criterion), tax return data, by estimator and year

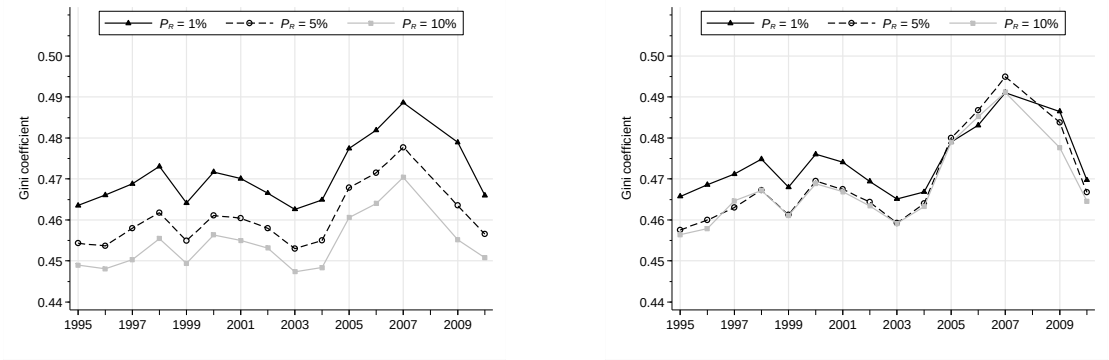


Notes. Author's estimates from SPI data. The figure plots the thresholds selected using the Kolmogorov-Smirnov criterion described in eq. (3) and main text.

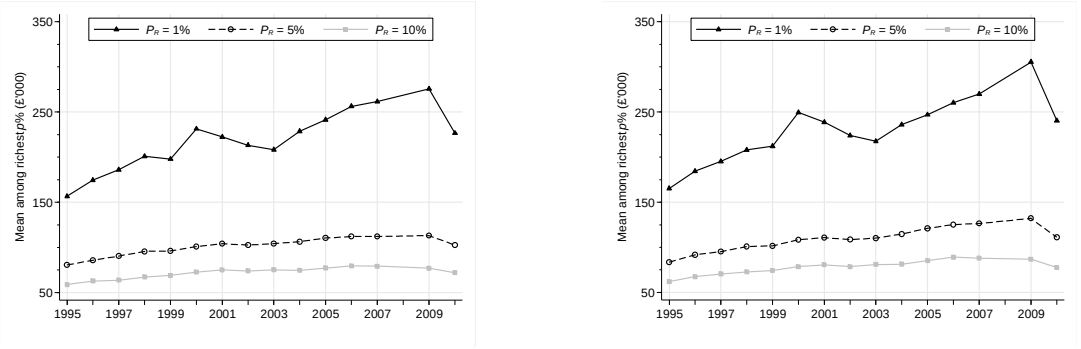
Figure 9. Combined data estimates (Approach C): Gini coefficient overall, mean income, income share, and Gini coefficient of the Rich, by Pareto model and high-income threshold

Pareto I estimates Pareto II estimates

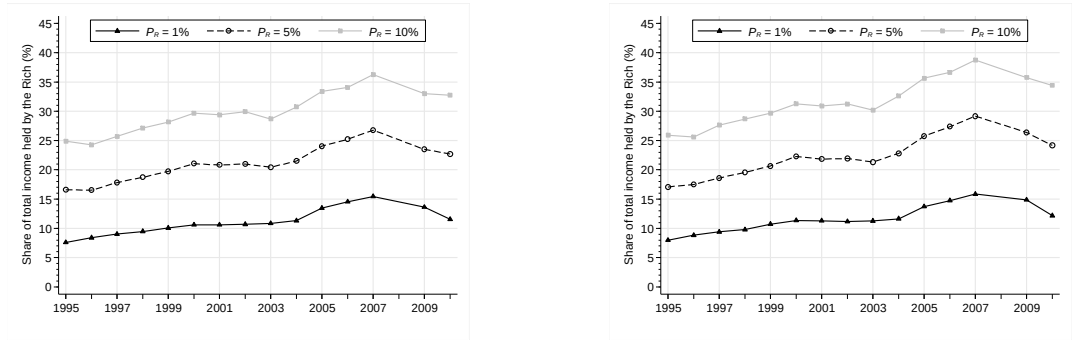
(a) Gini coefficient (all adults, Rich and Non-rich), *G*



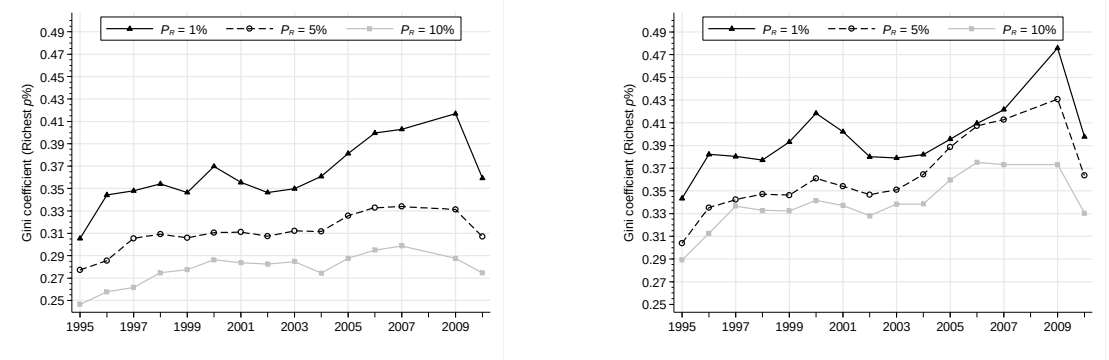
(b) Mean income of the Rich, μ_R



(c) Share of total income held by the Rich, S_R (%)



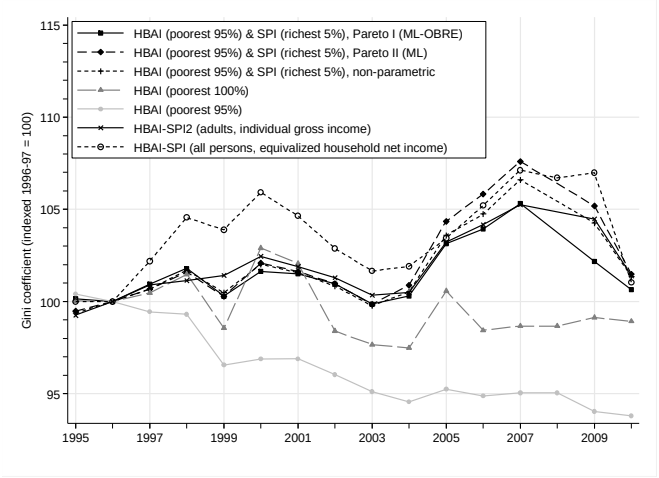
(d) Gini coefficient among the Rich, G_R



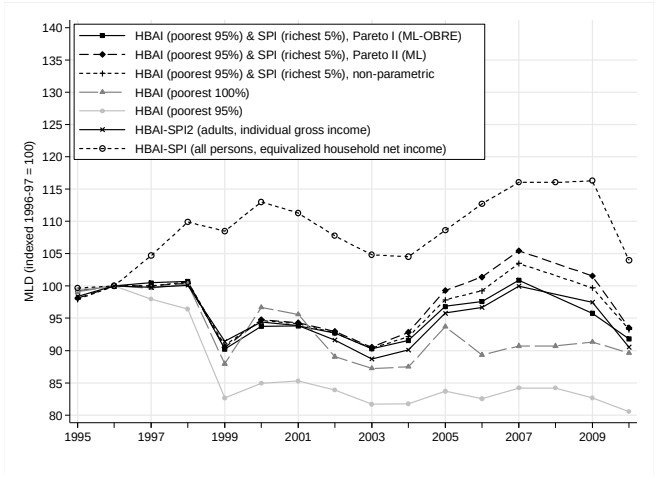
Notes. Author's derivations from SPI and HBAI data using eqn. (5). Pareto I (ML-OBRE) and II models fitted using each of three thresholds to define the Rich group: P_R = 1%, 5%, and 10% (cut-offs derived from survey data – see text for further explanation). All series refer to distributions of individual gross income among adults. Estimates of μ_R, μ_N, S_R, S_N, G_R, and G_N are listed in Appendix H.

Figure 10. UK inequality (indexed 1996 = 100), by series and inequality index

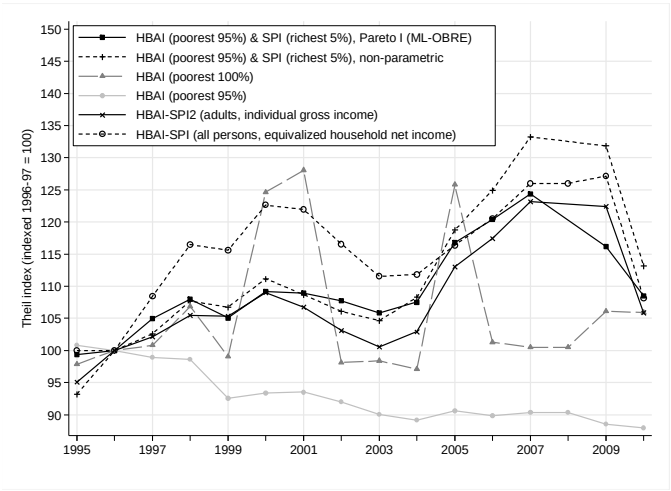
(a) Gini coefficient (*G*)



(b) Mean logarithmic deviation (*L*)



(c) Theil index (*T*)



Notes. Author's derivations from SPI, HBAI, HBAI-SPI (Department for Work and Pensions 2015), and HBAI-SPI2 data (Burkhauser et al. 2016). All series shown are based on the distribution of individual gross income among adults, with the exception of the HBAI-SPI series which refers to equivalized household net income among all individuals (see main text). There are no Pareto II estimates for the Theil index. Threshold: *p*95 in the HBAI data (see main text). The corresponding graphs for the other two percentile thresholds are shown in Appendix I.

Online Supplementary Material (Appendices)

to accompany

Pareto models, top incomes, and recent trends in UK income inequality

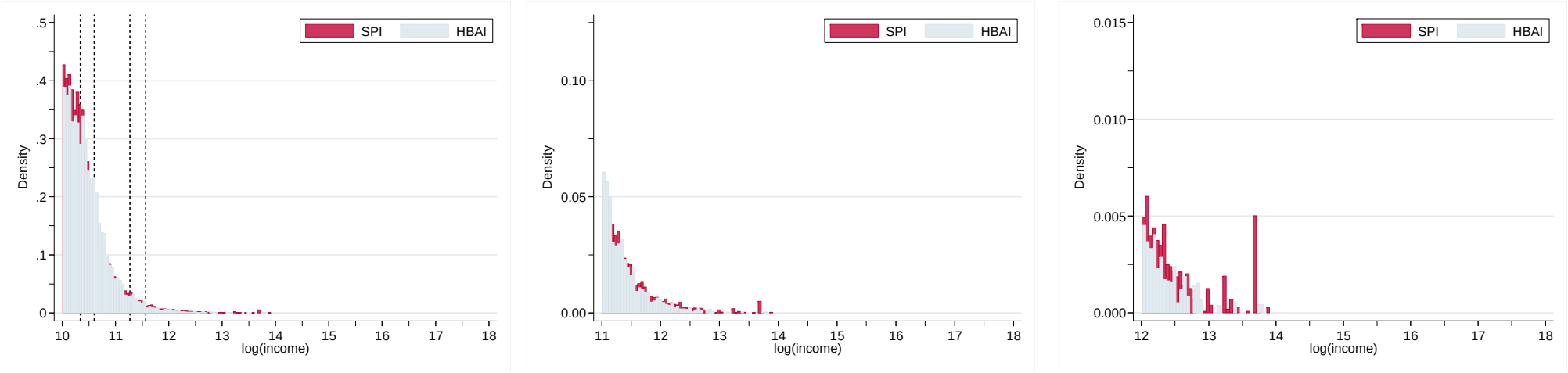
Stephen P. Jenkins
(LSE, ISER (University of Essex), and IZA)

Appendices:

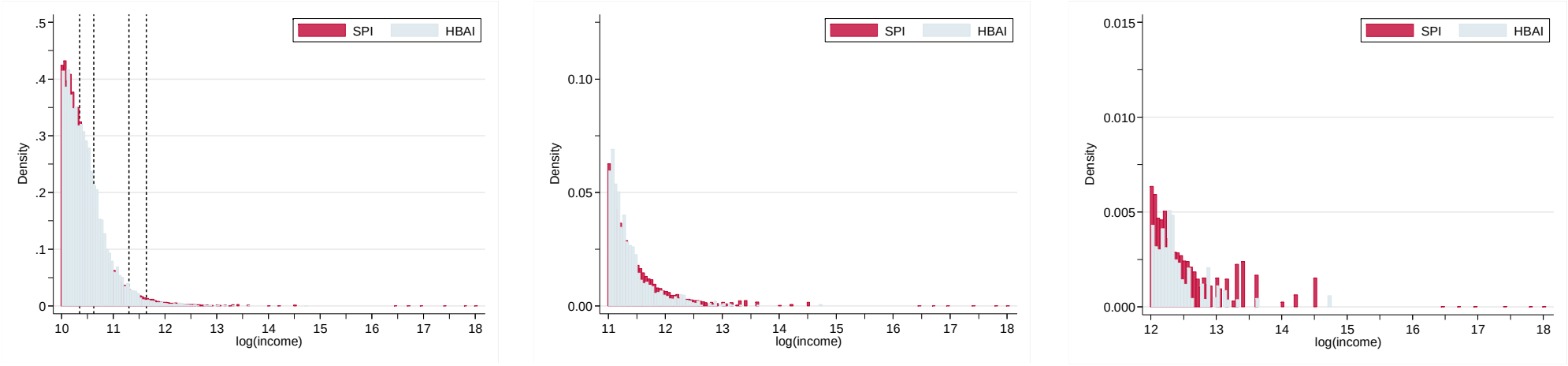
The concentration of income in high and extremely high income ranges: survey and tax return data, by year

Vertical dashed lines show (from left to right) *p*90, *p*95, *p*99, *p*99.5. Income is in £ per year (2012/13 prices)

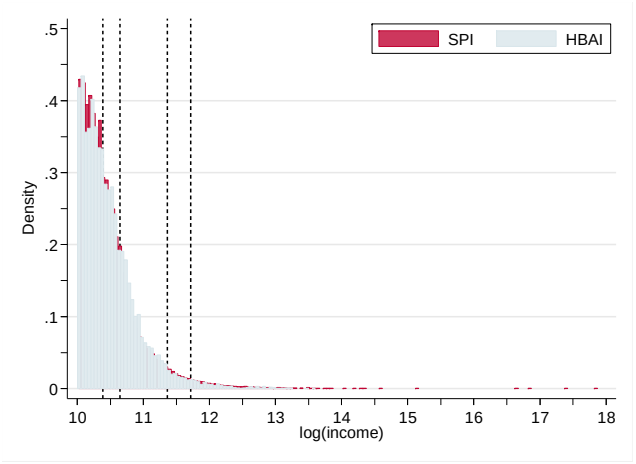
1995

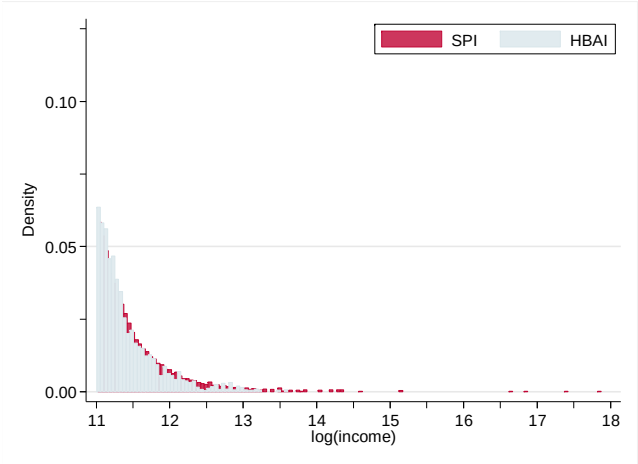


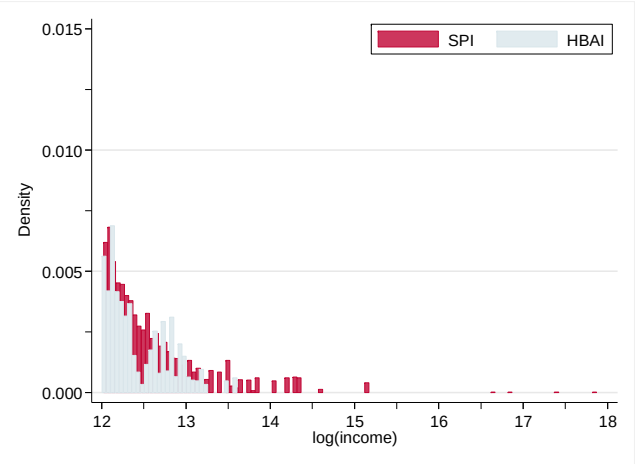
1996



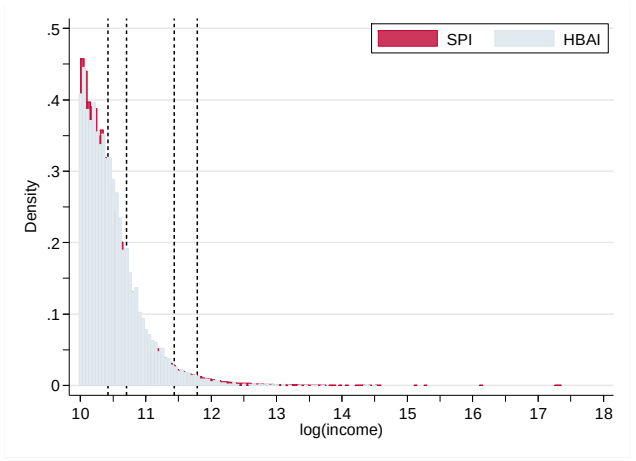
1997

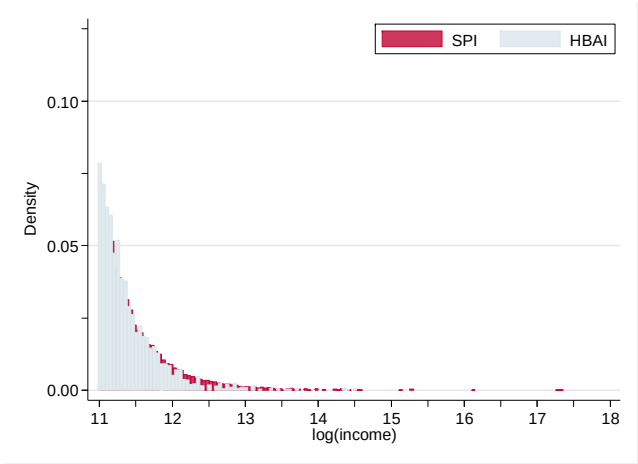

SPI
HBAI
Density
log(income)

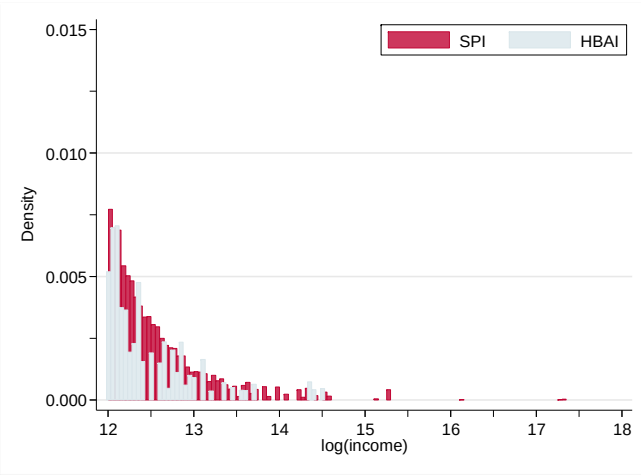

SPI
HBAI
Density
log(income)


SPI
HBAI
Density
log(income)

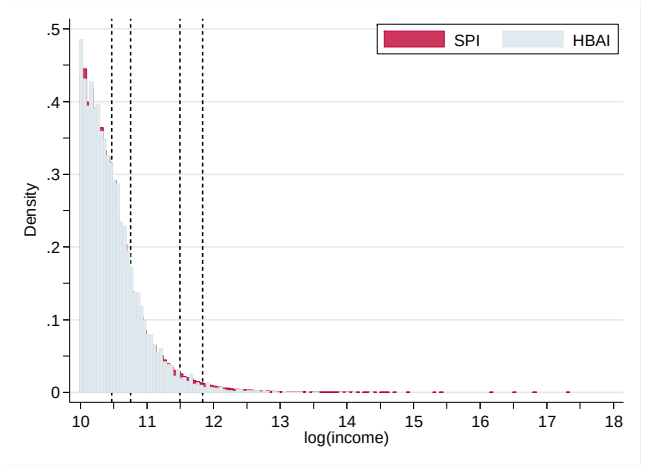
1998

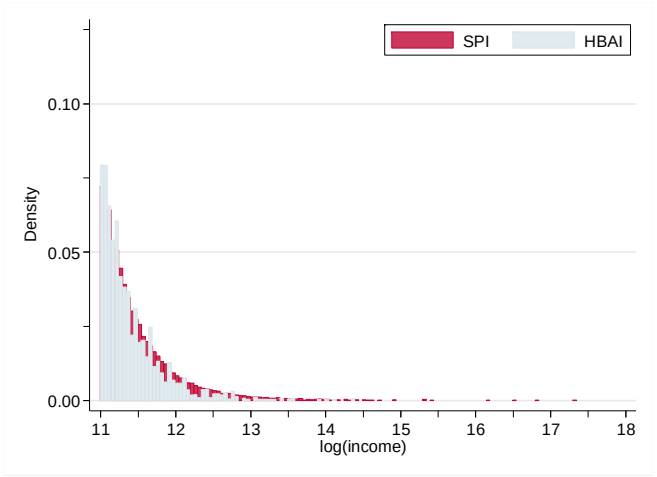

SPI
HBAI
Density
log(income)

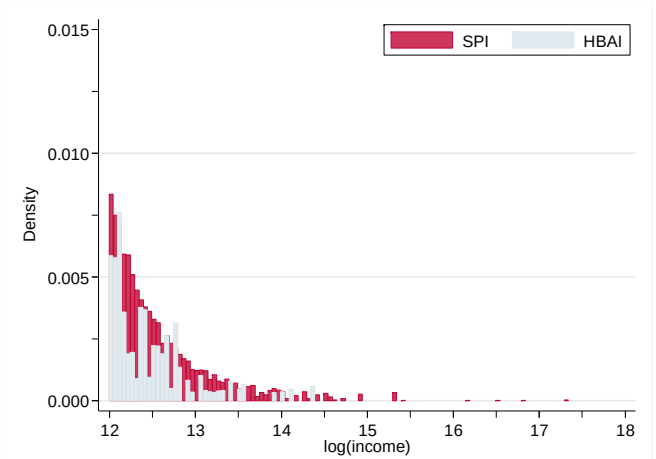

SPI
HBAI
Density
log(income)


SPI
HBAI
Density
log(income)

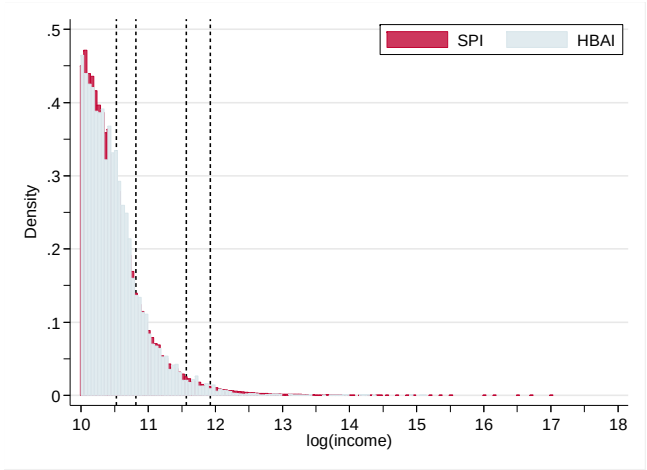
1999

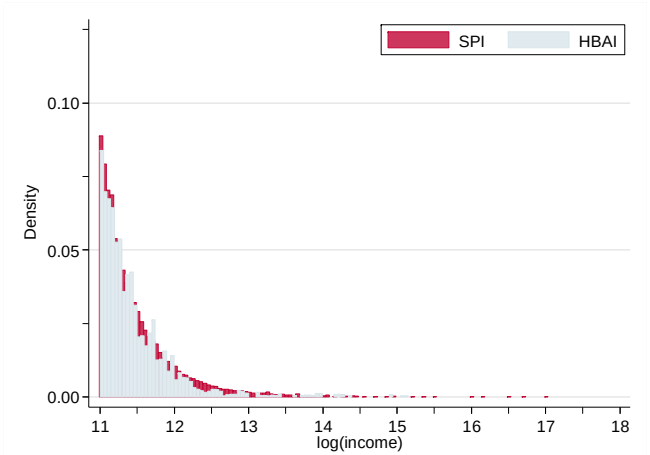

SPI
HBAI
Density
log(income)

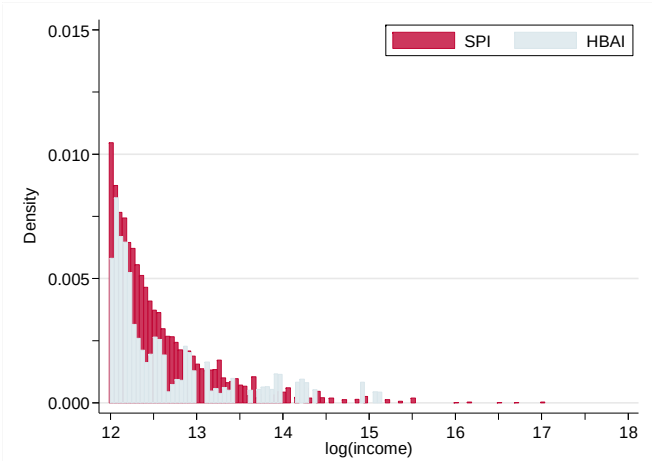

SPI
HBAI
Density
log(income)


SPI
HBAI
Density
log(income)

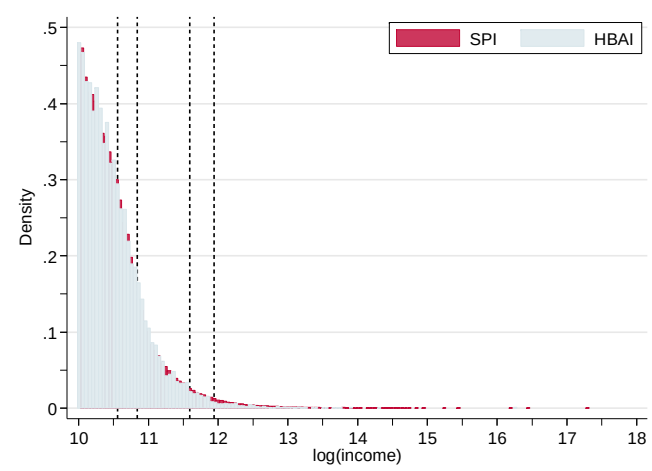
2000

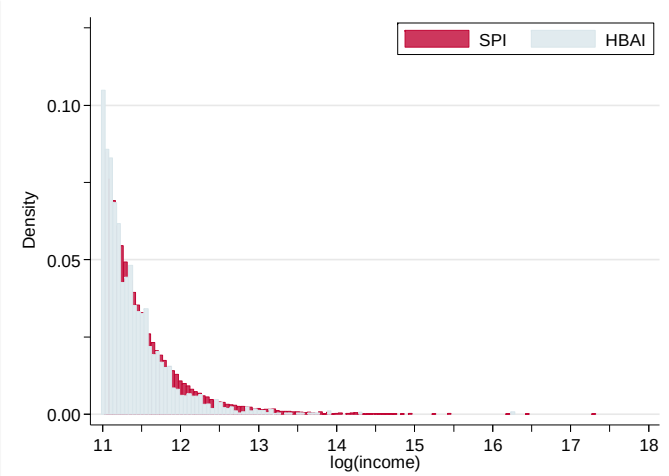

SPI
HBAI
Density
log(income)

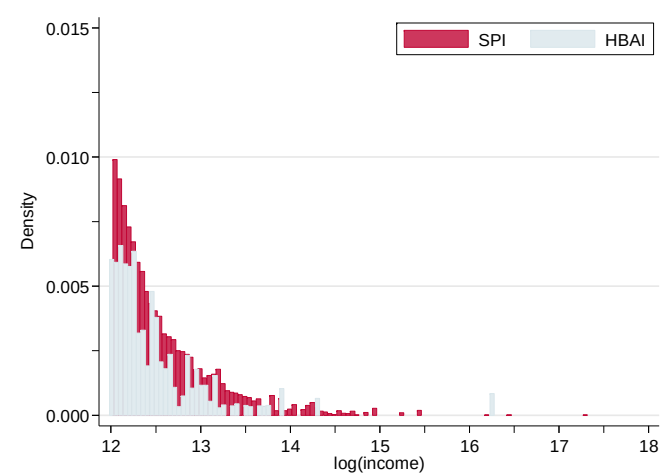

SPI
HBAI
Density
log(income)


SPI
HBAI
Density
log(income)

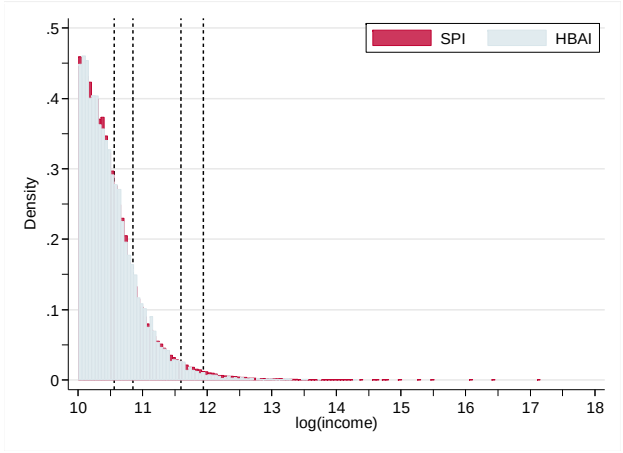
2001

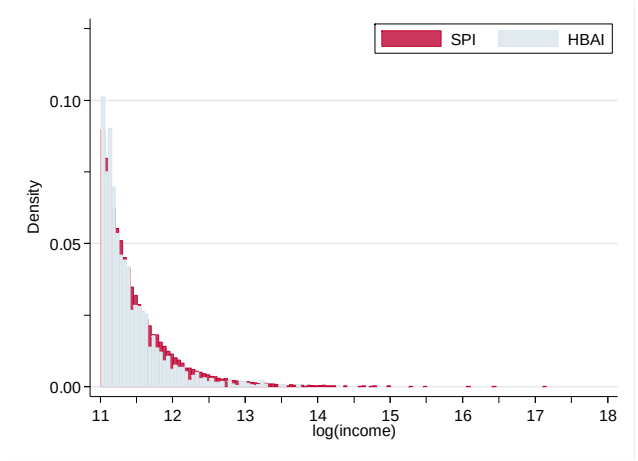

SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)

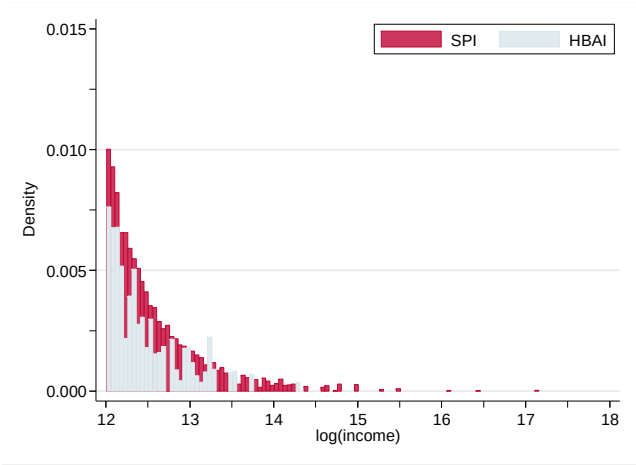

SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

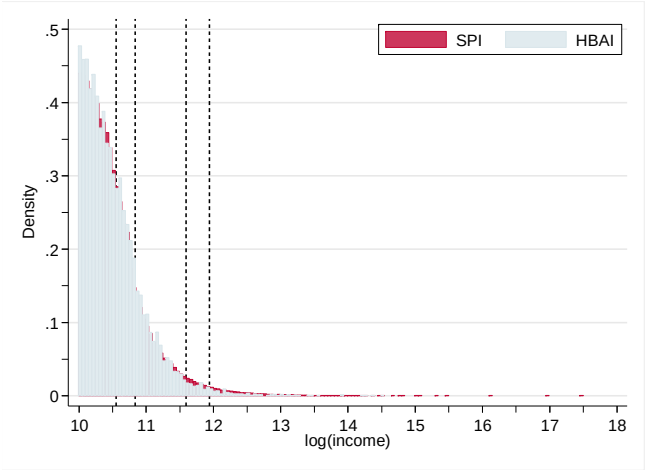
2002

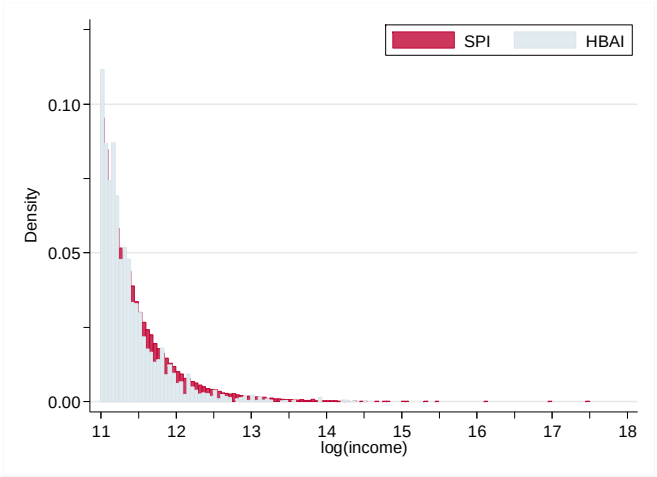

SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)

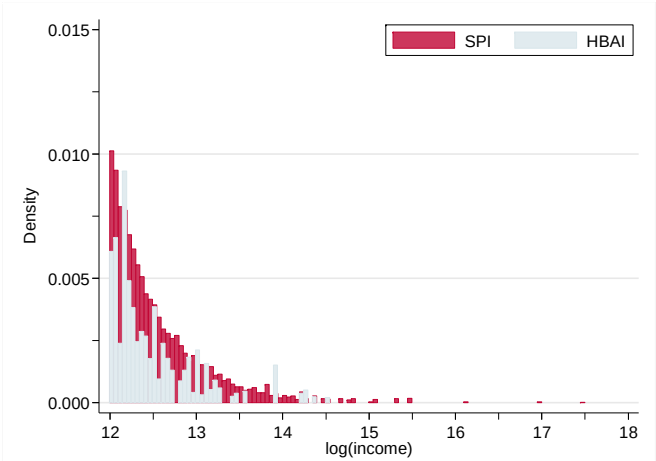

SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

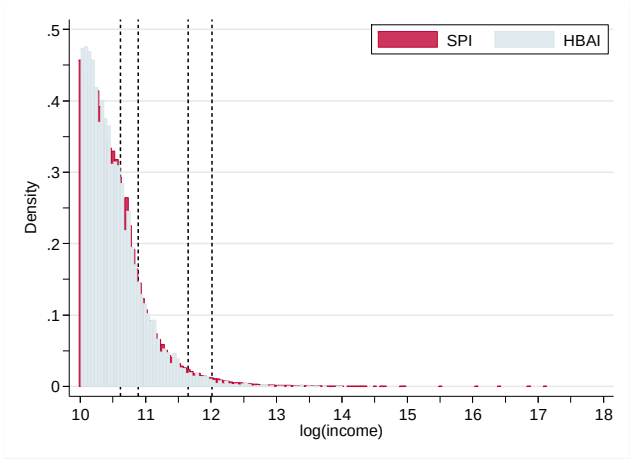
2003

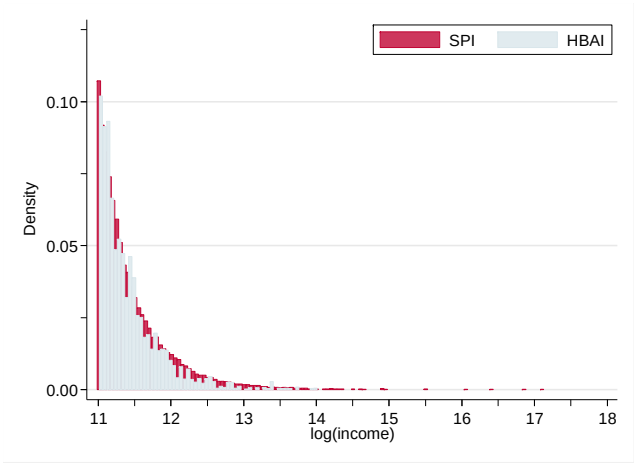

SPI
HBAI
Density
log(income)

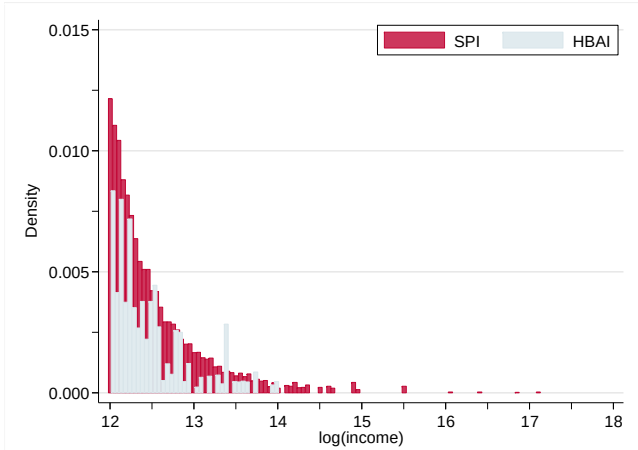

SPI
HBAI
Density
log(income)


SPI
HBAI
Density
log(income)

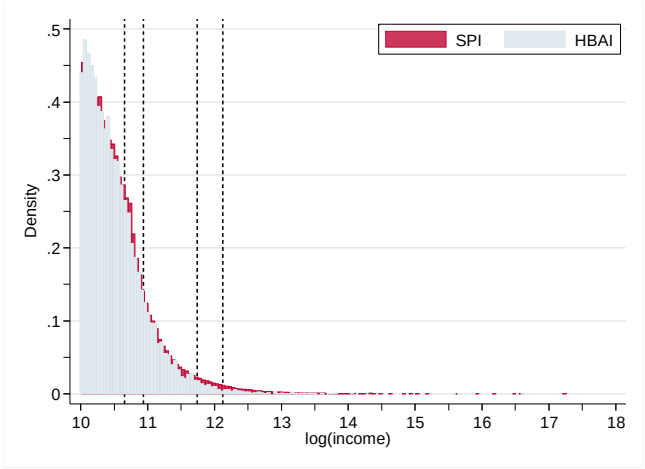
2004

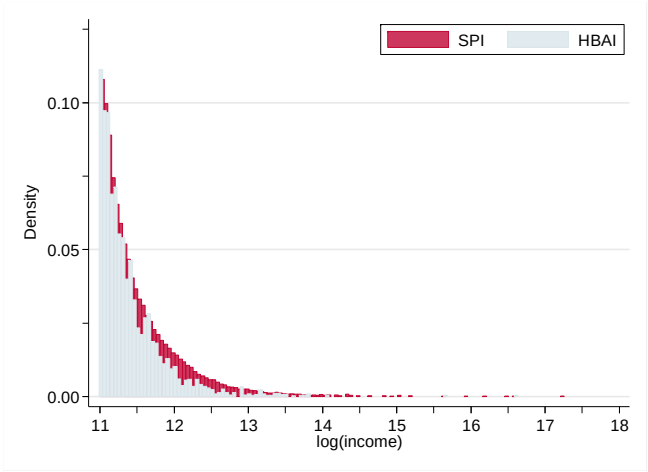

SPI
HBAI
Density
log(income)

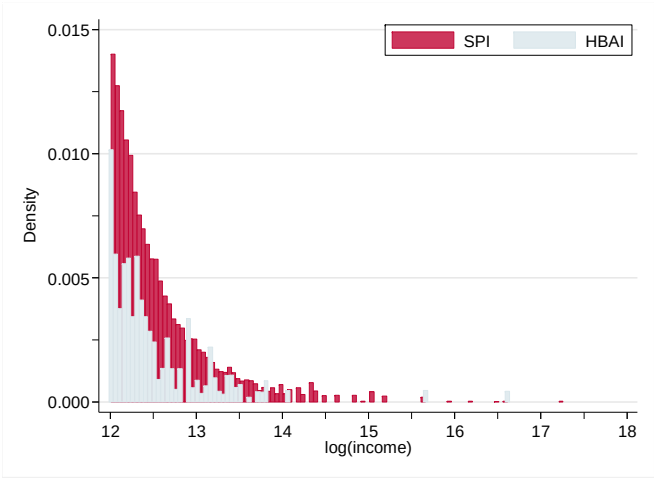

SPI
HBAI
Density
log(income)


SPI
HBAI
Density
log(income)

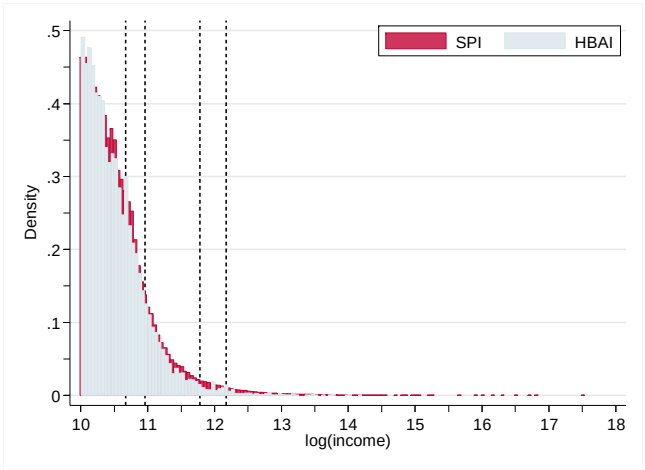
2005

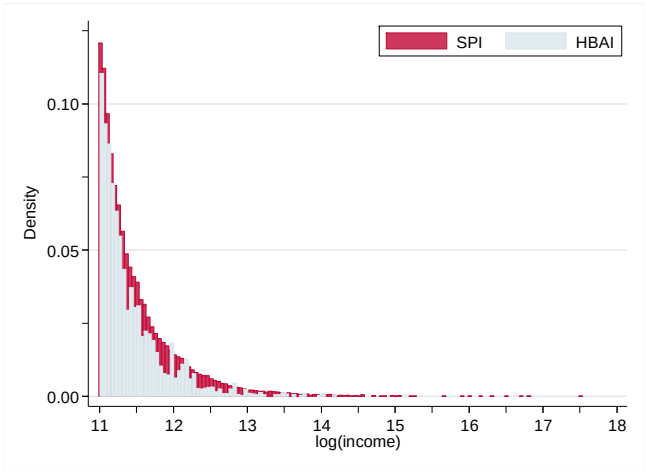

SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)

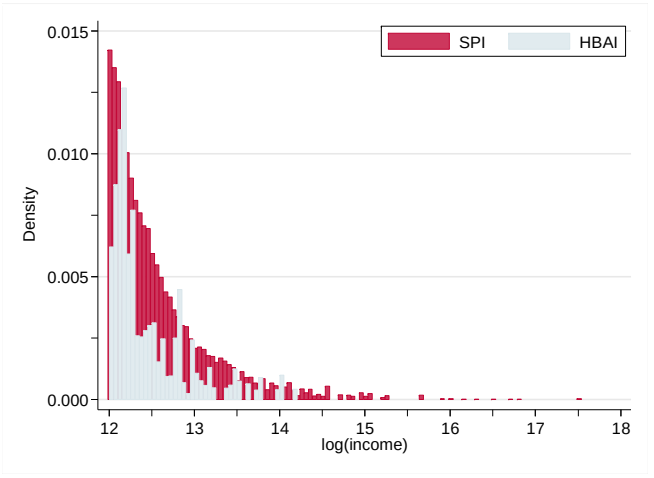

SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

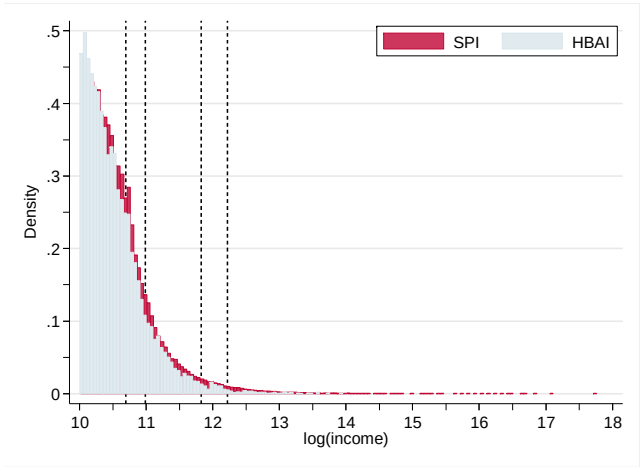
2006

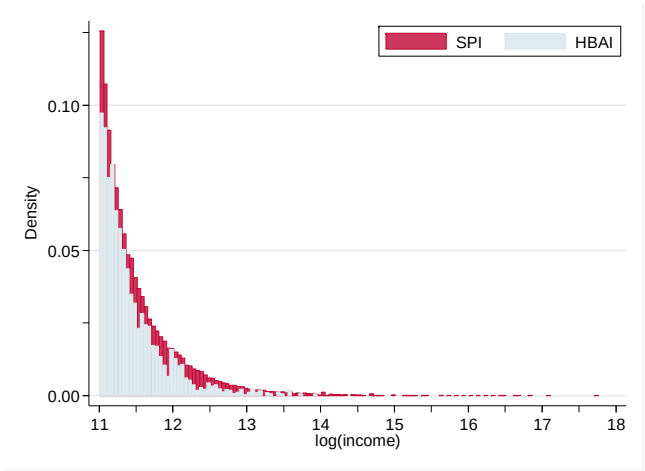

SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)

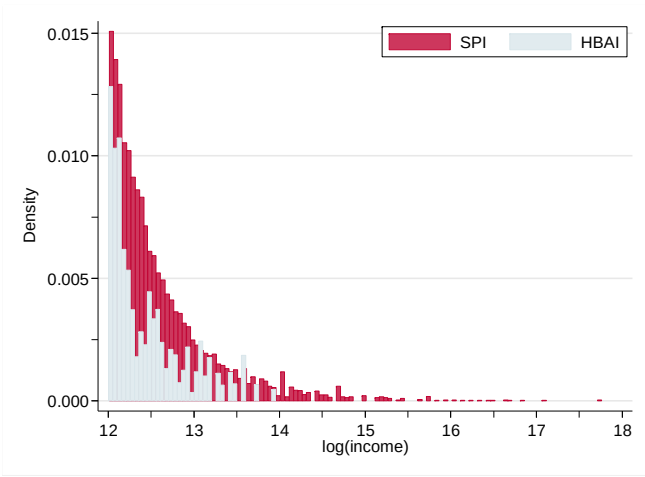

SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

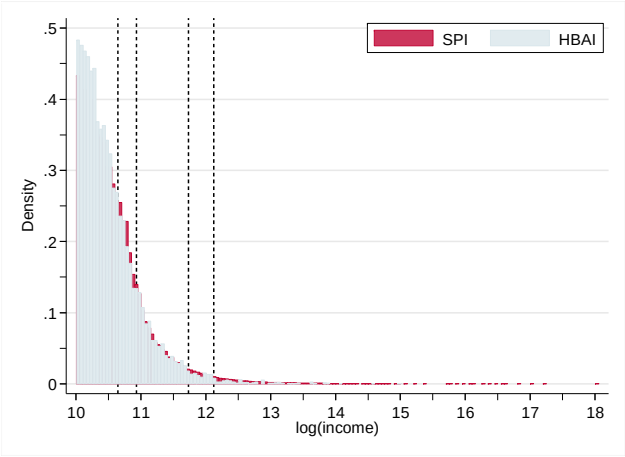
2007

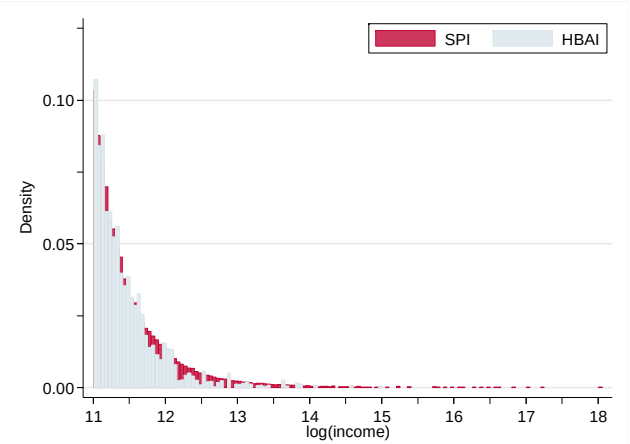

SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)

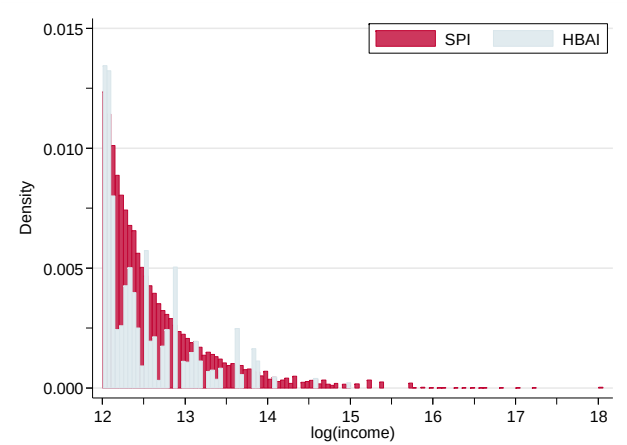

SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

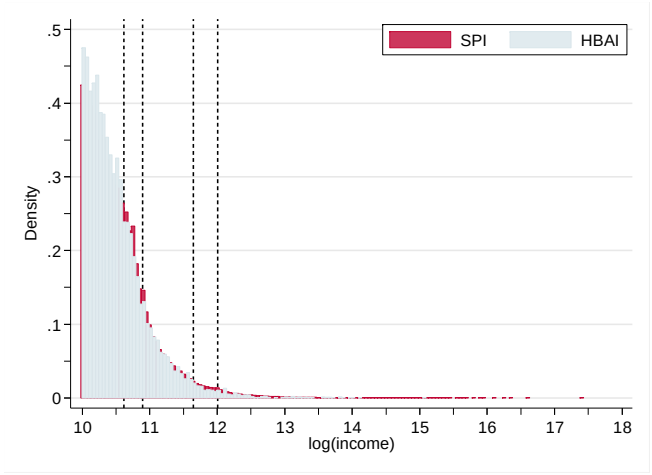
2009

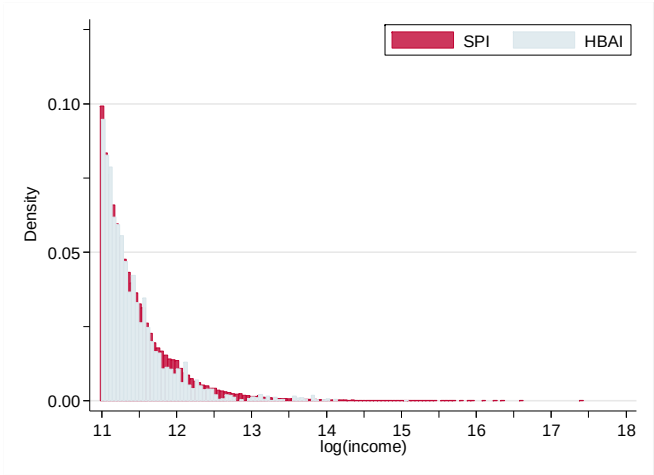

SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)

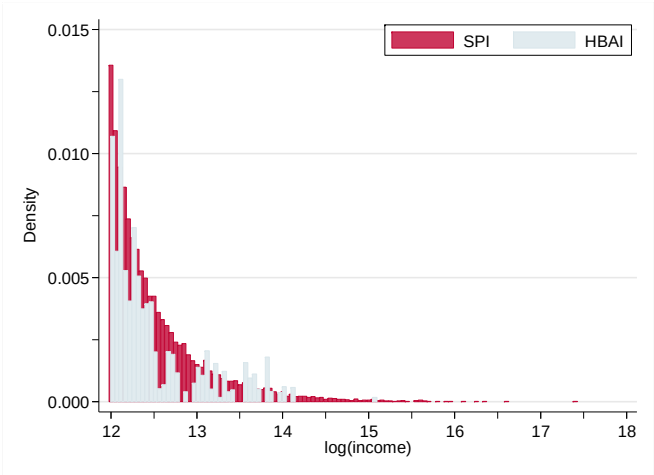

SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

2010


SPI
HBAI
Density
.5
.4
.3
.2
.1
0
10
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.10
0.05
0.00
11
12
13
14
15
16
17
18
log(income)


SPI
HBAI
Density
0.015
0.010
0.005
0.000
12
13
14
15
16
17
18
log(income)

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions (pareto02)

Atkinson (2016) explains that the Pareto shape parameter α may be estimated by OLS in three ways, depending on the information available:

1. the income range (e.g. above some specific income, x)
2. the fraction of income units with incomes greater than or equal to some value, the survivor function $S(x)$
3. The total income received by these income units, divided by the total population, $\Omega(x)$

With control totals for population and income available (as in this paper), Atkinson remarks that $\Omega(x)$ divided by the mean is the income share of those units with incomes of x or greater.

The first OLS estimation method is the 'Zipf' curve regression employed in the main text of this paper, in which $\log[S(x)]$ is regressed on $\log(x)$ using all income units with $x >$ threshold t. For estimated slope coefficient β_1, the estimate of α is $-\beta_1$.

The second OLS estimation method, which Atkinson (2016) attributes to Champernowne, is a regression of $\log[\Omega(x)]$ on $\log(x)$ using all income units with $x >$ threshold t. For estimated slope coefficient β_2, the estimate of α is $1-\beta_2$.

The third OLS estimation method, which Atkinson (2016) attributes to McGregor (1936) and Frechet (1945), is a regression of $\log[\Omega(x)]$ on $\log[S(x)]$ using all income units with $x >$ threshold t. For estimated slope coefficient β_3, the estimate of α is $1/(1-\beta_3)$.

Atkinson (2016) states that "the differences [in estimates of alpha] would not arise if the Pareto distribution provided a fully satisfactory representation of the data … [t]he differences between the results from the three methods provide therefore a simple diagnostic device".

In the remainder of this appendix, I present for each year, the three estimates derived for each of a large number of income thresholds t. At the end, I report the estimates in numerical rather than graphical form, together with the R^2 from the corresponding regression.

Overall, the results confirm Atkinson's conclusions, based on estimates from UK SPI data in grouped form, covering 1918/19 to 2012/13 (using a threshold of around the 95th percentile in each year), that the three estimates are not always in close agreement with each other.

At relatively low thresholds (but disregarding extremely low ones), the Zipf method provides the highest estimates of α and the McGregor/Frechet method the lowest. Estimates converge as the threshold is increased to between around the 95th and 99th percentile, and then diverge again thereafter. That is, there is a 'cross over' with the Zipf method providing the lowest estimates and the McGregor/Frechet method the highest estimates as the threshold increases.

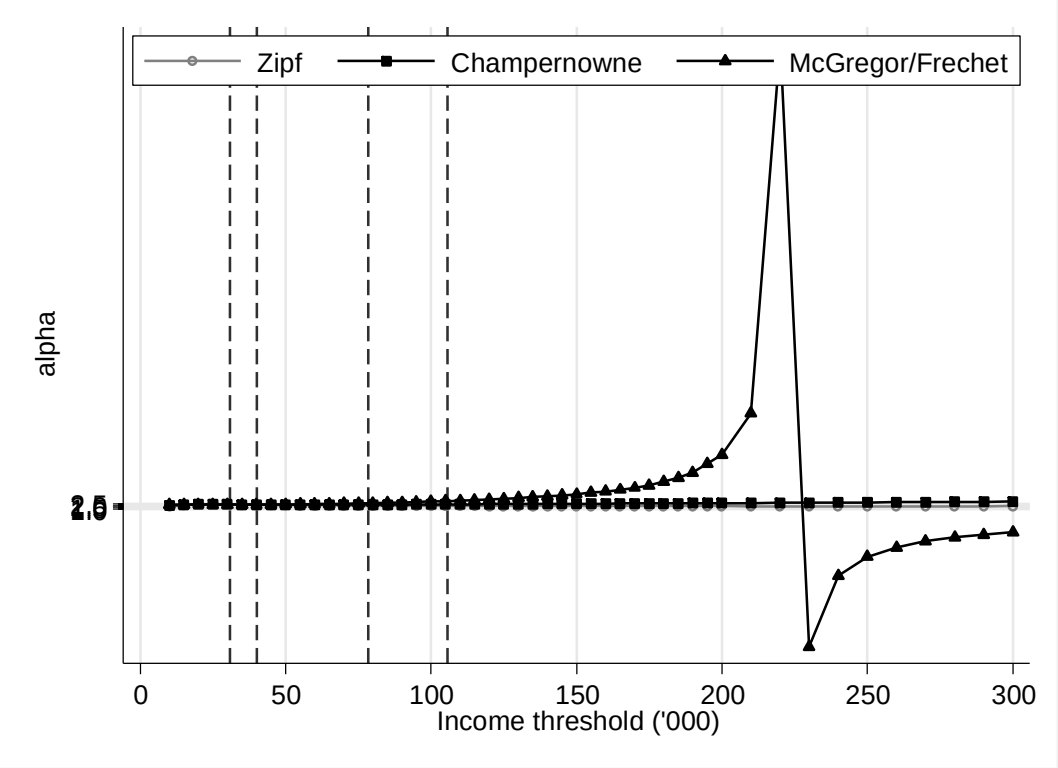
The Zipf estimates of the Pareto α are closer to the Maximum Likelihood estimates than the estimates from the other two OLS methods (see Figure 7 and Appendix F).

The Champernowne method consistently yields the smallest R^2. The Zipf method provides the largest R^2 at most thresholds, but the McGregor/Frechet method tends to provide the highest R^2 at relatively low thresholds. But there are some exceptions to these patterns across the different years.

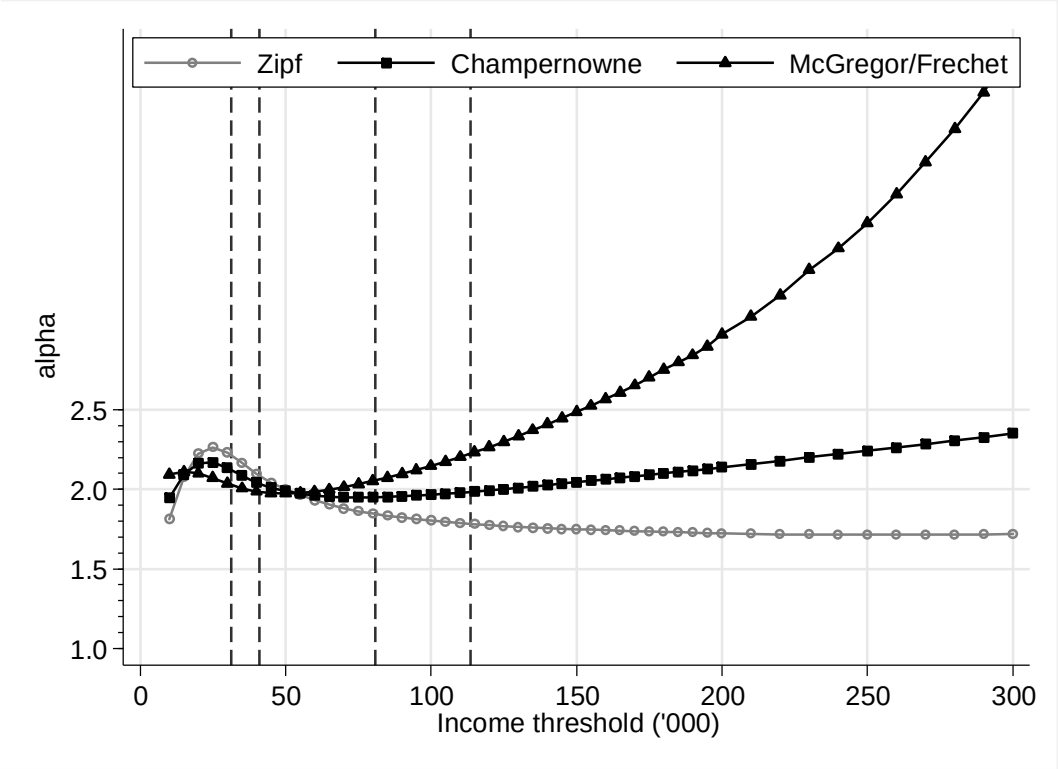
Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions (pareto02)

The vertical dashed lines show, from left to right, *p*90, *p*95, *p*99, *p*99.5 for each year.

1995

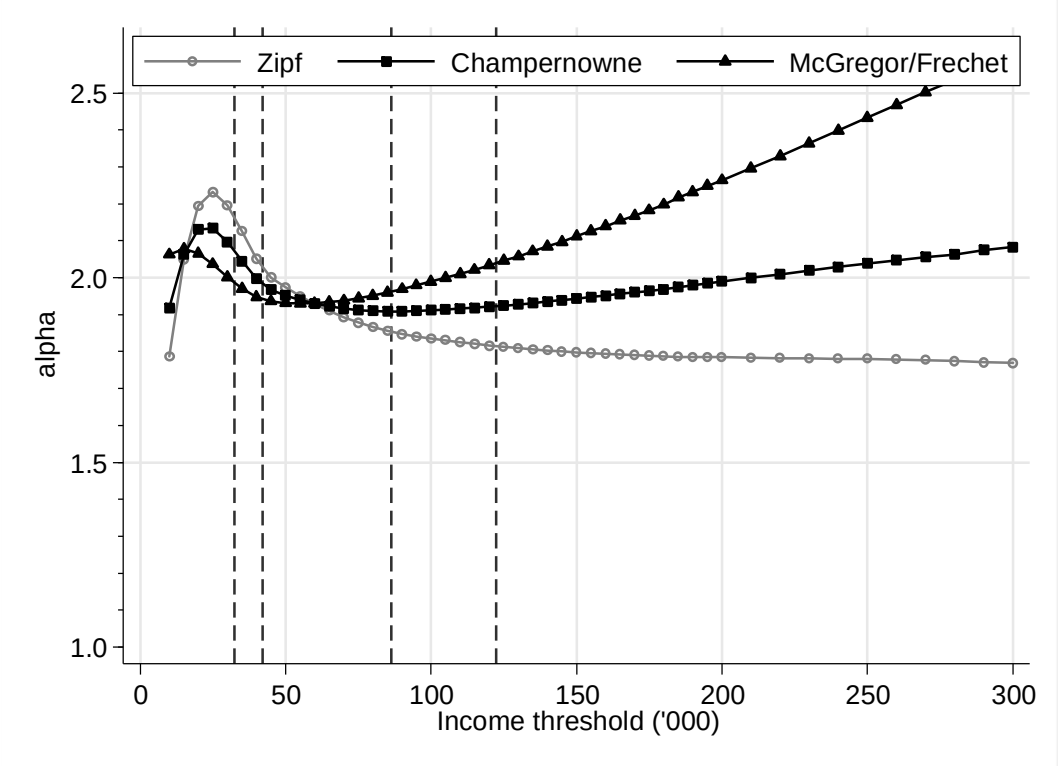


1996

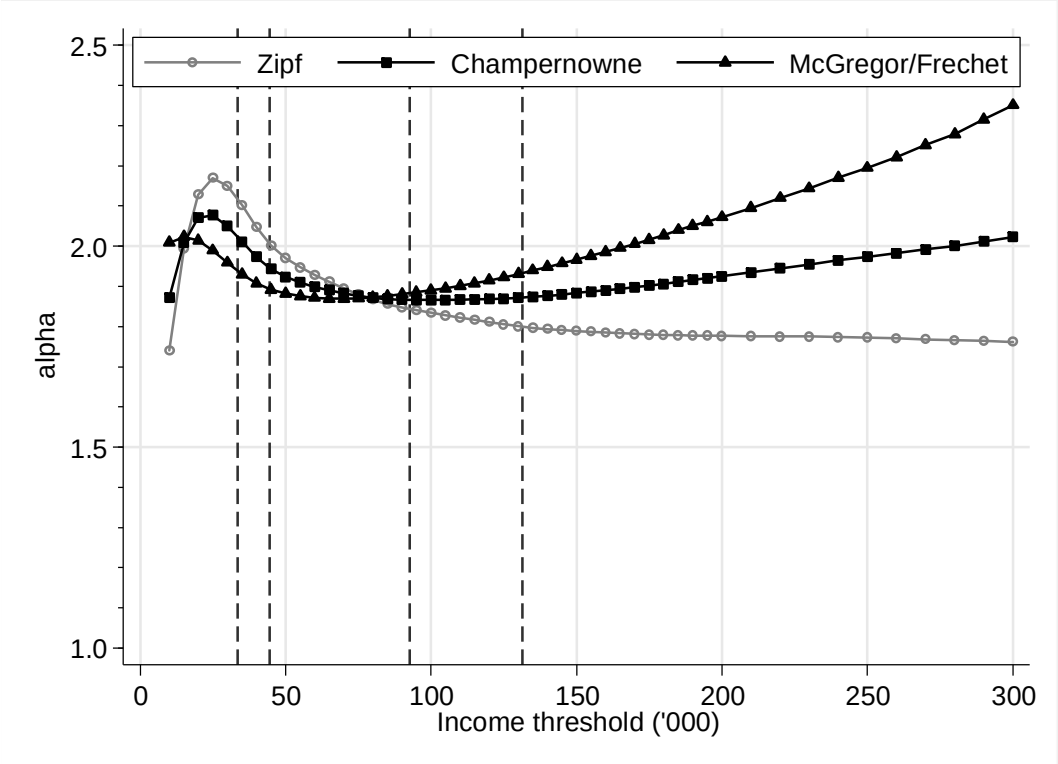


Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

1997

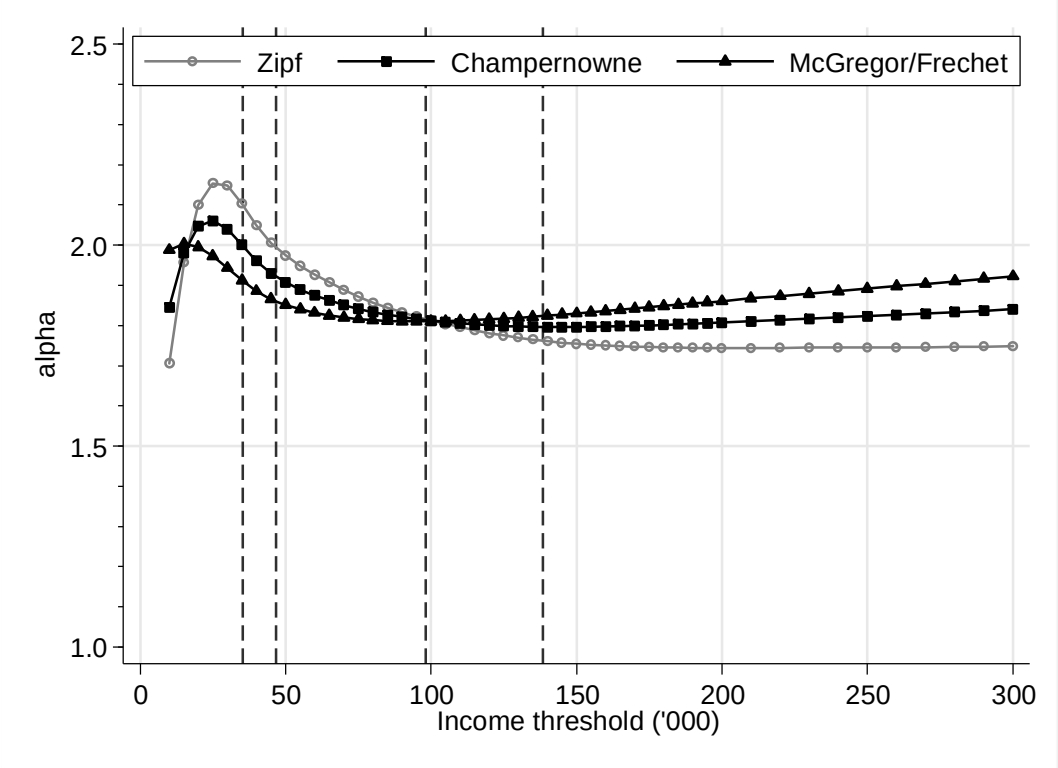


1998

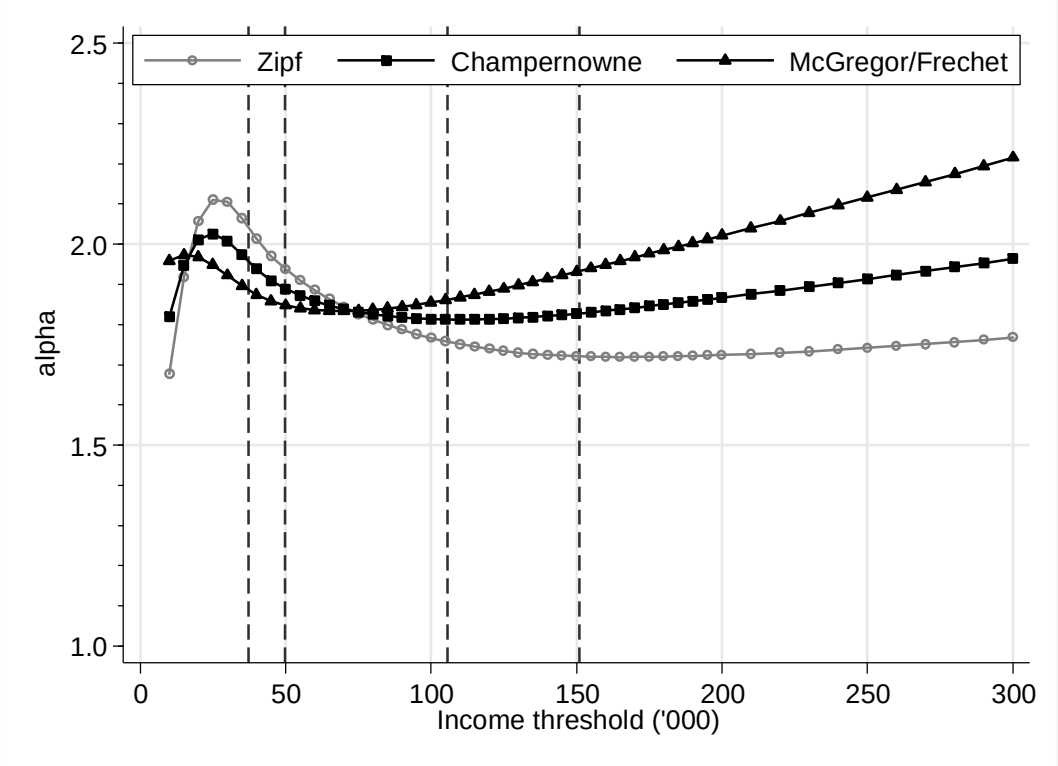


Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

1999

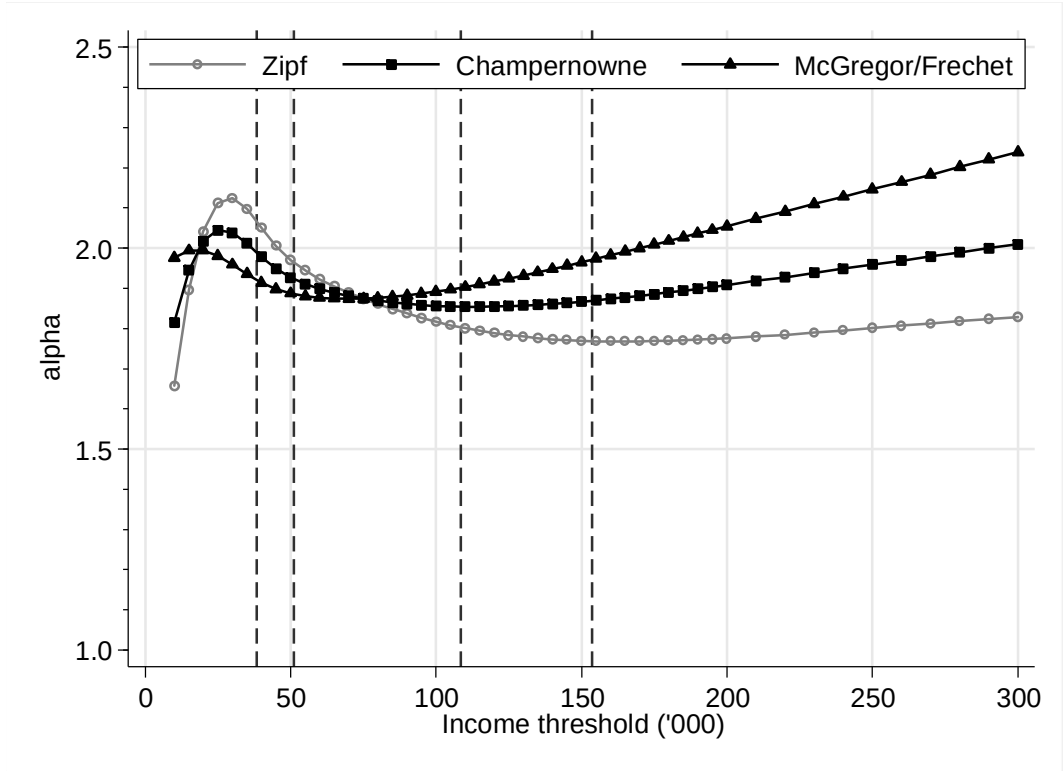


2000

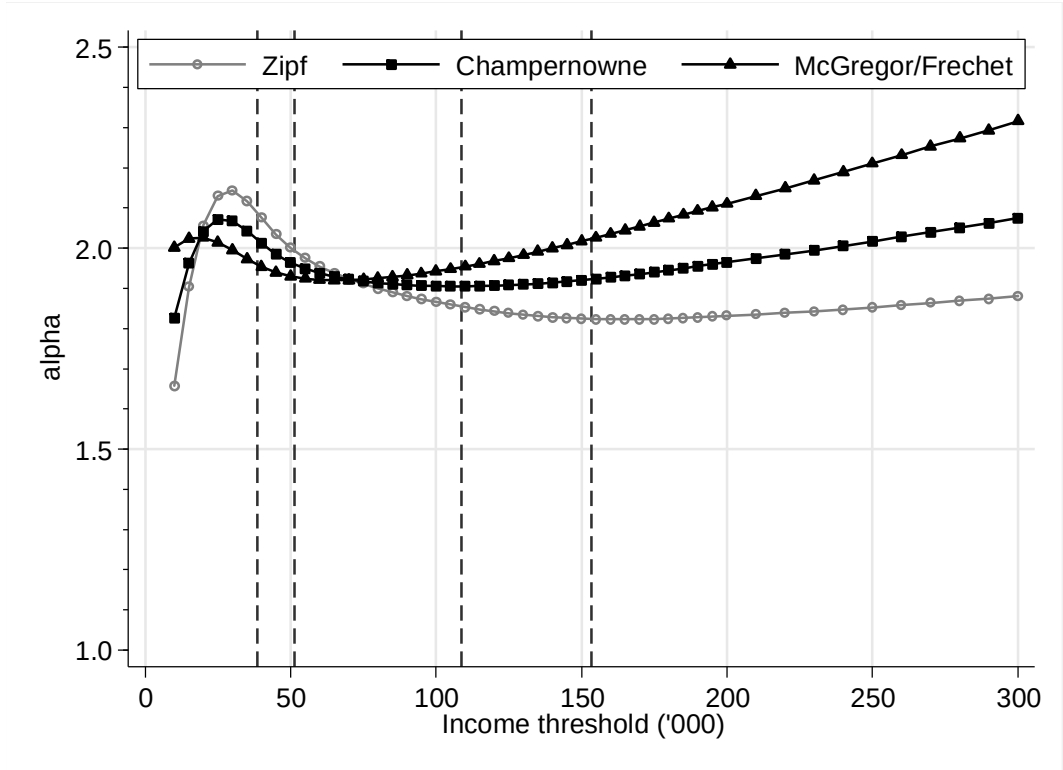


Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2001

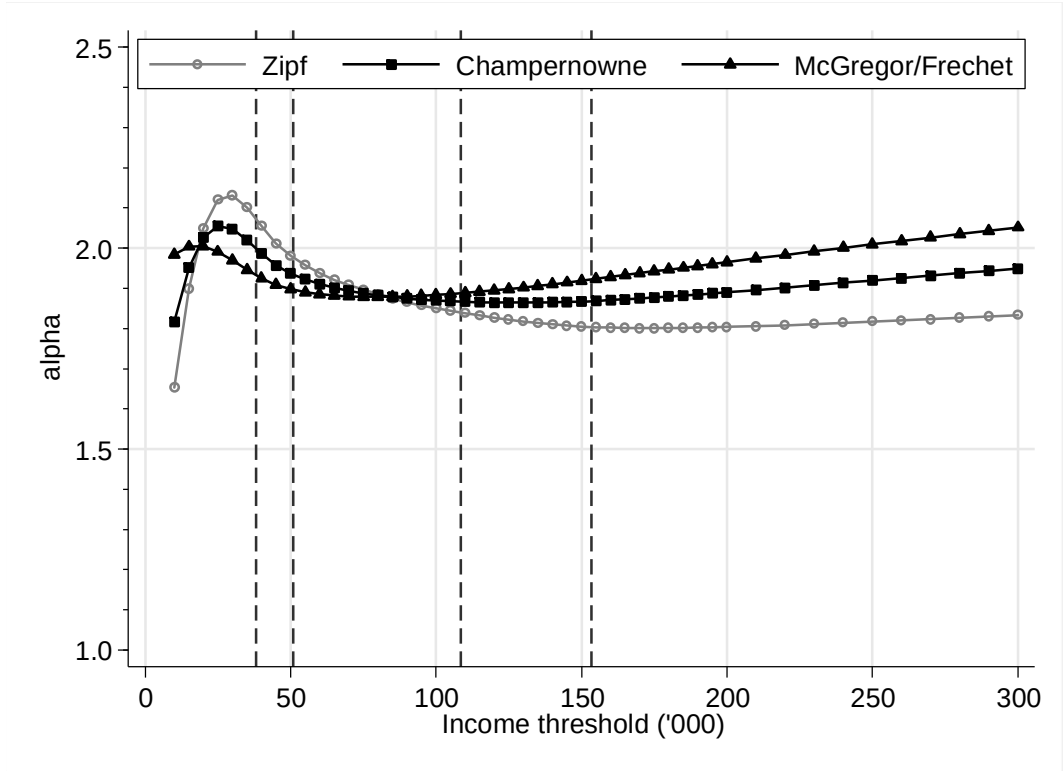


2002

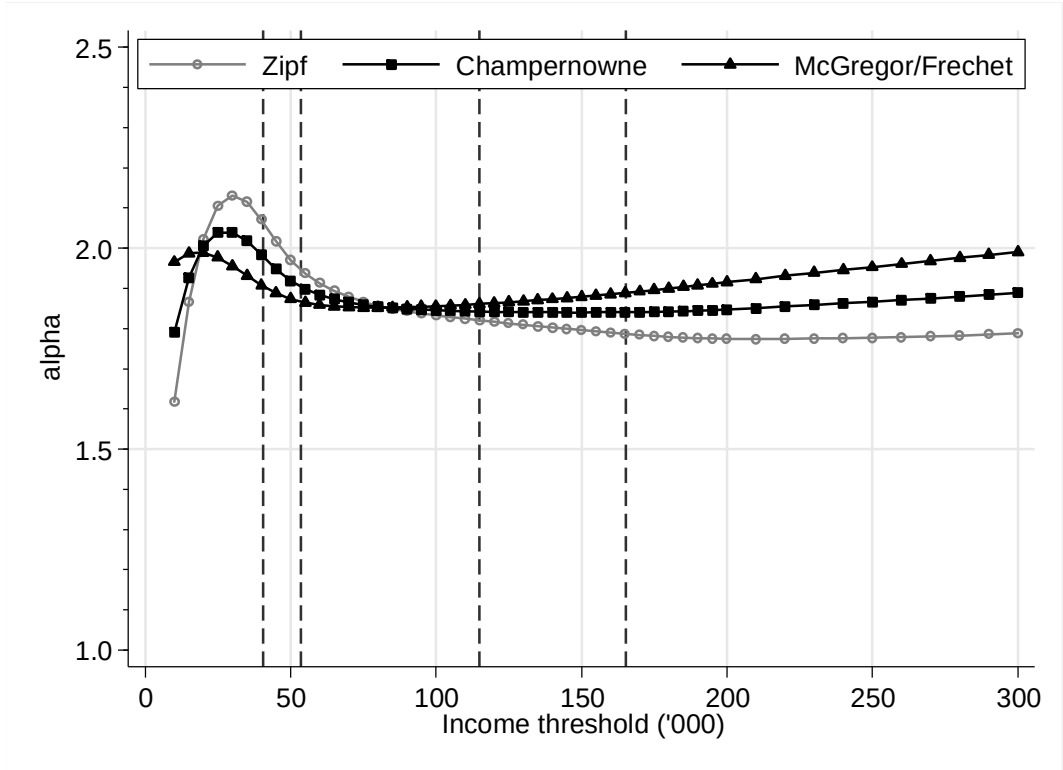


Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2003

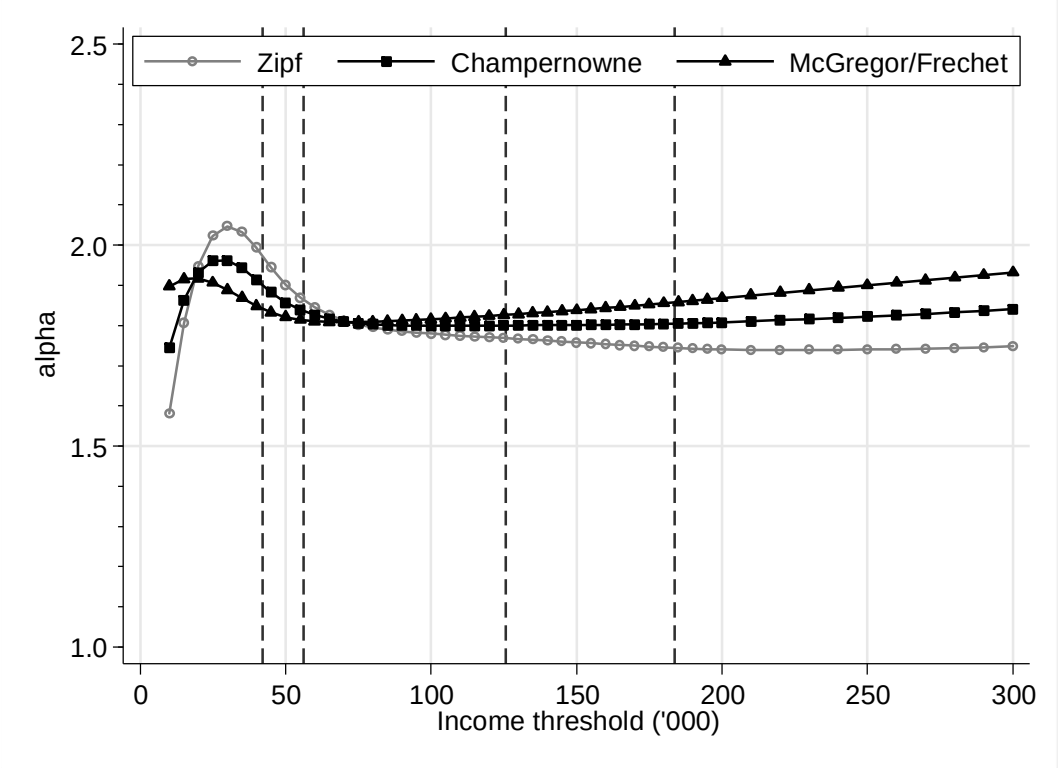


2004

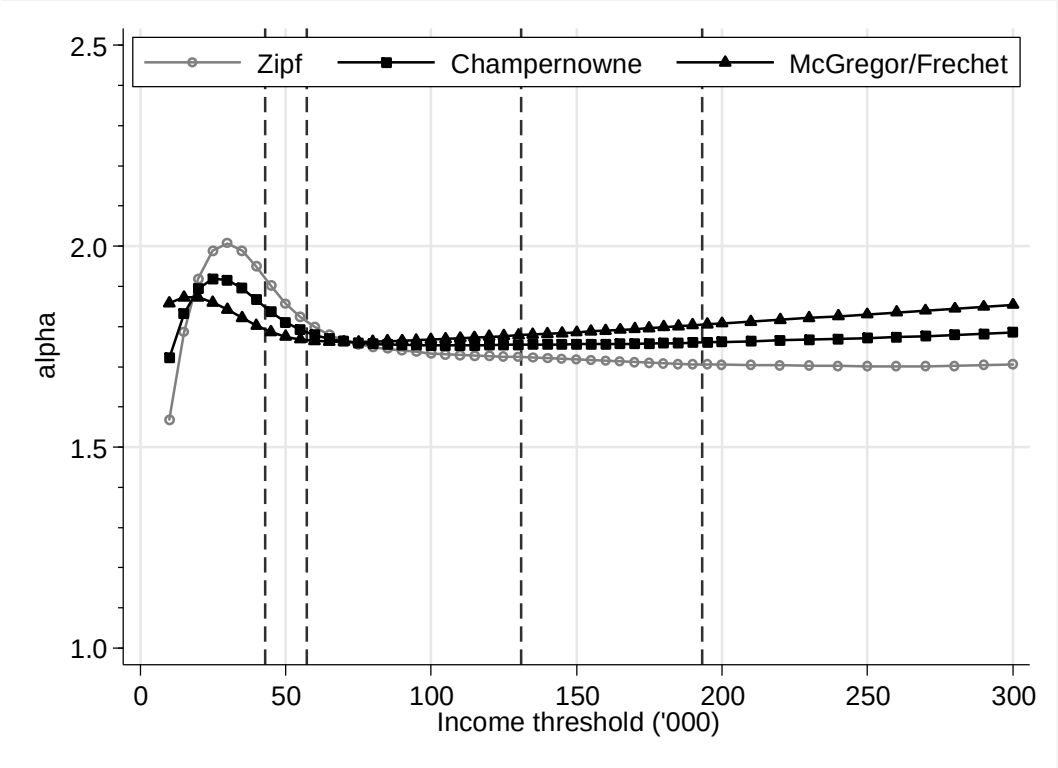


Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2005

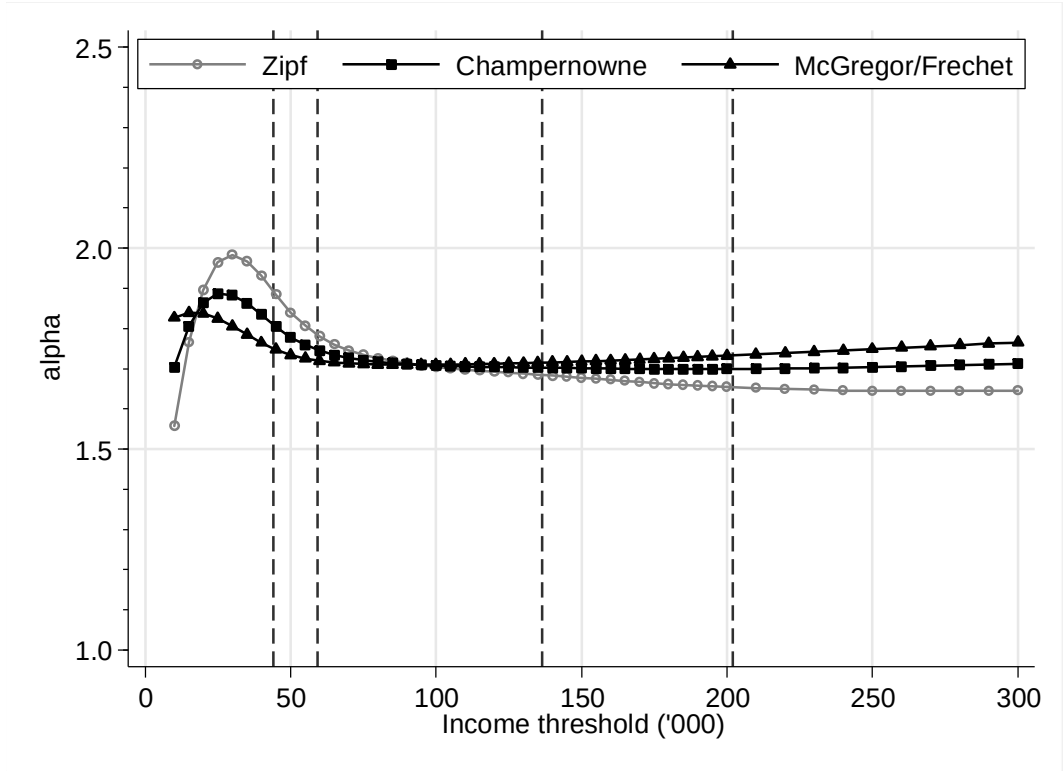


2006

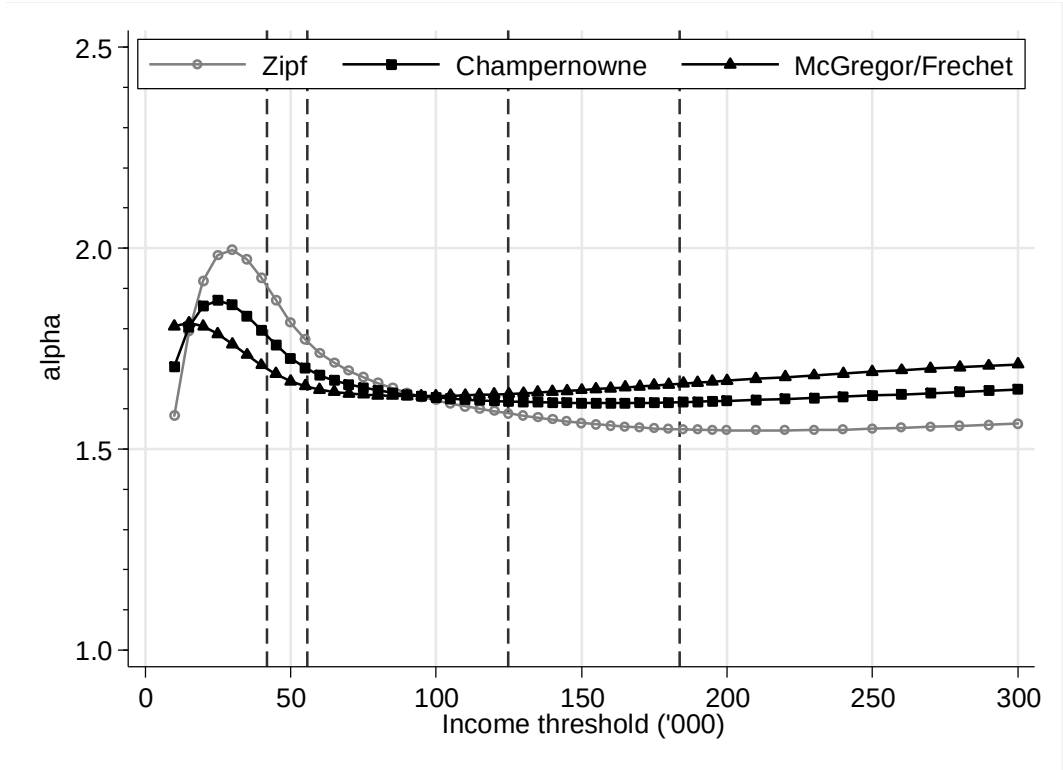


Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2007

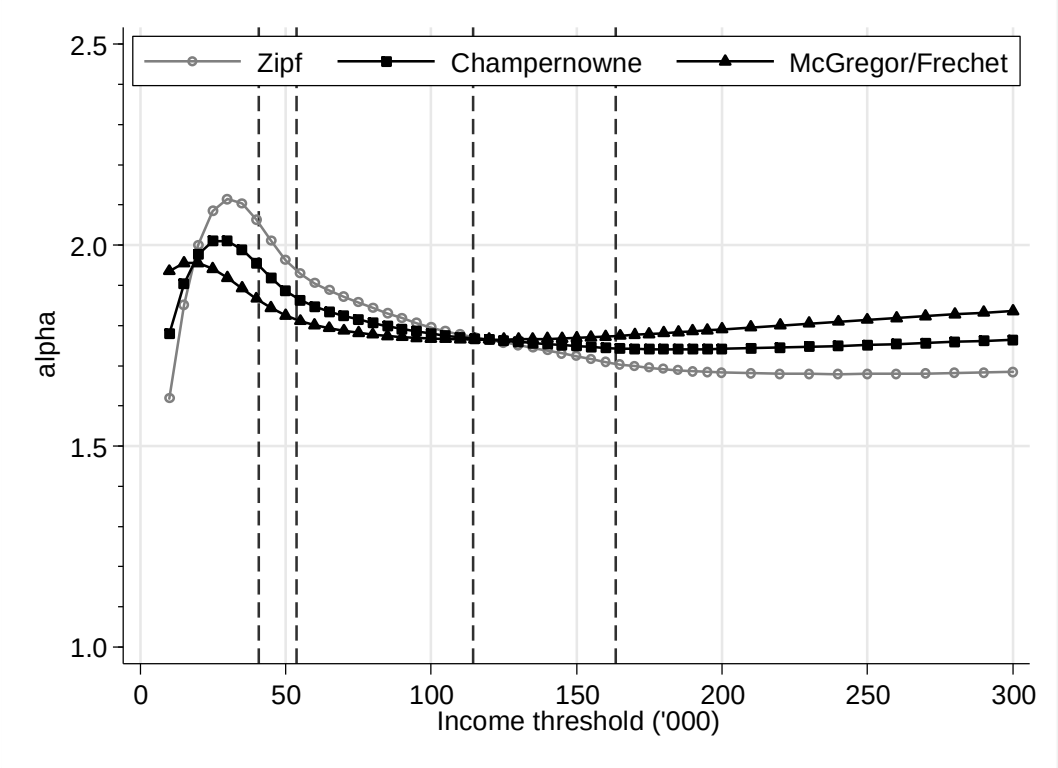


2009



Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions (pareto02)

2010



Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

Estimates of α are listed below for each model, together with the R^2 from the regression
Key:
Zipf: "ols", Champernowne "ols2", McGregor/Frechet (ols3"

```
list year threshold alpha_ols alpha_ols2 alpha_ols3 r2_ols r2_ols2 r2_ols3, noobs sepby(year)
```

year	thresh~d	alpha_~s	alpha_~2	alpha_o~3	r2_ols	r2_ols2	r2_ols3
1995	10000	1.842078	2.016769	2.246252	.954354	.9433437	.9974367
1995	15000	2.146887	2.206385	2.283974	.985137	.98283	.9973323
1995	20000	2.318627	2.307382	2.291517	.9945325	.9929944	.9963069
1995	25000	2.378236	2.33533	2.278956	.9945367	.9917693	.9946872
1995	30000	2.346169	2.309755	2.261108	.9929382	.9893767	.9923073
1995	35000	2.273248	2.266073	2.252713	.9929606	.988107	.9884446
1995	40000	2.204729	2.232077	2.261793	.993497	.985855	.9834747
1995	45000	2.156929	2.215601	2.286677	.9930517	.9816053	.9777972
1995	50000	2.124672	2.21011	2.319169	.9920871	.9767706	.9723192
1995	55000	2.10006	2.211349	2.360072	.9907016	.9714163	.9666806
1995	60000	2.0795	2.214756	2.403405	.9893155	.9663018	.9615814
1995	65000	2.059019	2.220057	2.455541	.9877642	.9607713	.9563669
1995	70000	2.040164	2.226691	2.512949	.9861513	.9553099	.951515
1995	75000	2.021583	2.233775	2.576258	.9845361	.949882	.9470292
1995	80000	2.000748	2.243257	2.659736	.9825583	.9435759	.9422311
1995	85000	1.985999	2.255581	2.746111	.9803591	.9380925	.9382416
1995	90000	1.972787	2.269228	2.84174	.9779752	.9328467	.9346814
1995	95000	1.96176	2.285009	2.949055	.9753132	.9278992	.9314993
1995	100000	1.952426	2.304247	3.078094	.972155	.9230276	.9285116
1995	105000	1.947538	2.324104	3.204714	.9690783	.9193286	.9262096
1995	110000	1.942958	2.339779	3.31681	.9664783	.916361	.9245554
1995	115000	1.93626	2.355845	3.45241	.9635407	.9129741	.9229738
1995	120000	1.92737	2.378127	3.660814	.9593113	.9085647	.9212145
1995	125000	1.923476	2.396996	3.834484	.955913	.9058152	.9201415
1995	130000	1.920428	2.42228	4.078705	.9514323	.9028688	.9190864
1995	135000	1.918802	2.443502	4.301288	.9476274	.9007592	.9184157
1995	140000	1.916484	2.462059	4.527122	.9440398	.8987644	.917929
1995	145000	1.913456	2.479303	4.770984	.9404566	.8967519	.9175695
1995	150000	1.909016	2.49701	5.070746	.9364451	.8944161	.9173107
1995	155000	1.904242	2.517485	5.465396	.9316779	.8918646	.9171728
1995	160000	1.900676	2.532568	5.801149	.9280174	.8899867	.9171491
1995	165000	1.894508	2.550088	6.297121	.92327	.8873368	.9172409
1995	170000	1.889225	2.564844	6.793129	.9190938	.885074	.9173973
1995	175000	1.882667	2.580251	7.443322	.9143768	.8824441	.9176563
1995	180000	1.874641	2.599339	8.453877	.9083003	.8791955	.9181103
1995	185000	1.869161	2.615272	9.477061	.9032506	.8767505	.9185395
1995	190000	1.862776	2.631124	10.85853	.8978189	.874043	.9190489
1995	195000	1.854774	2.649967	13.21169	.8910083	.8707019	.9197904
1995	200000	1.849046	2.663234	15.63433	.8859451	.8682213	.9203165
1995	210000	1.836531	2.693352	26.78219	.8738763	.8624935	.9215789
1995	220000	1.822622	2.72569	126.1193	.8596005	.8557858	.9230181
1995	230000	1.810687	2.77028	-35.91547	.8405885	.8480669	.9248375
1995	240000	1.807609	2.818608	-16.87738	.8217947	.8418055	.9260812
1995	250000	1.807158	2.86157	-11.748	.8045915	.8361614	.9266669
1995	260000	1.808551	2.901755	-9.299731	.7882563	.8308823	.9268131
1995	270000	1.811598	2.94893	-7.567878	.7682695	.8241833	.9263741
1995	280000	1.819195	2.998701	-6.493437	.7482621	.8177645	.9252896
1995	290000	1.827019	3.040745	-5.858939	.7310248	.8119905	.9238468
1995	300000	1.849434	3.121548	-5.110957	.7017434	.803059	.9205881
1995	400000	2.559833	4.256126	-4.251876	.5917667	.8219123	.8782439
1995	500000	3.454192	5.501071	-5.190177	.566201	.8744838	.8410628
1995	600000	4.337626	6.252917	-37.06926	.4602654	.7973911	.7744112
1996	10000	1.81429	1.945506	2.093523	.9616552	.9555989	.9982563
1996	15000	2.077037	2.093235	2.110409	.9875807	.9862447	.9979427
1996	20000	2.223107	2.164287	2.100091	.9948734	.9916192	.9971456
1996	25000	2.264165	2.169915	2.07158	.9941064	.9882629	.9963065
1996	30000	2.231411	2.134916	2.037096	.992304	.9857166	.9953011
1996	35000	2.165521	2.086303	2.006542	.9922144	.9859854	.9937001
1996	40000	2.096596	2.042281	1.98627	.9933559	.9870256	.9912836
1996	45000	2.040947	2.010458	1.97647	.9942189	.9865514	.9881023
1996	50000	2.000719	1.990164	1.974491	.9947044	.9849535	.9847485
1996	55000	1.963965	1.974224	1.978018	.9953182	.982697	.9809366
1996	60000	1.931504	1.962476	1.986291	.9959918	.9798691	.976895
1996	65000	1.904748	1.954949	1.998113	.9965536	.9766212	.9728811
1996	70000	1.880896	1.950614	2.014357	.9969999	.9727163	.9685473
1996	75000	1.862713	1.949577	2.032323	.9972423	.9688041	.9645226
1996	80000	1.847404	1.95055	2.0523	.9973999	.9648387	.9606302

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

1996	85000	1.834515	1.953111	2.073861	.9974834	.9609147	.9569112
1996	90000	1.823058	1.956935	2.097513	.9975243	.9569651	.9532771
1996	95000	1.813348	1.961687	2.122062	.9975206	.9531985	.9498914
1996	100000	1.804499	1.966872	2.147498	.9975116	.9495777	.9467303
1996	105000	1.796268	1.972444	2.174057	.9974996	.9460649	.9437523
1996	110000	1.788285	1.97891	2.203714	.9974713	.9424549	.9407709
1996	115000	1.781293	1.985623	2.233646	.9974248	.9390988	.938065
1996	120000	1.774792	1.992651	2.264848	.9973693	.9358652	.9355292
1996	125000	1.768552	2.00026	2.298571	.9973001	.9326446	.9330723
1996	130000	1.762821	2.008921	2.335943	.9971946	.9294202	.9306486
1996	135000	1.758688	2.017888	2.372544	.9970461	.9266339	.9285275
1996	140000	1.755244	2.026994	2.40953	.9968809	.9240832	.9265963
1996	145000	1.752088	2.03592	2.446649	.9967137	.9217175	.9248469
1996	150000	1.749015	2.0458	2.488326	.9965196	.9192923	.9230802
1996	155000	1.746433	2.054366	2.525625	.996347	.9172758	.921654
1996	160000	1.743509	2.063469	2.567209	.9961631	.9151479	.9202179
1996	165000	1.740634	2.072211	2.608878	.9959848	.9131435	.9189241
1996	170000	1.737674	2.081475	2.654549	.9957916	.9111049	.9176594
1996	175000	1.735071	2.091299	2.703362	.9955744	.9091396	.9164504
1996	180000	1.732879	2.101052	2.752567	.9953493	.907349	.9153638
1996	185000	1.730712	2.109697	2.798681	.9951473	.905743	.9144503
1996	190000	1.728369	2.117583	2.843636	.9949645	.9042124	.9136478
1996	195000	1.72555	2.126752	2.898403	.9947494	.9024435	.9127756
1996	200000	1.722844	2.139488	2.973583	.9944322	.9004123	.9117789
1996	210000	1.72051	2.158303	3.085213	.9939312	.8979182	.9105033
1996	220000	1.717272	2.178483	3.219058	.9933642	.8951585	.9092902
1996	230000	1.716057	2.202332	3.379089	.9926716	.8927521	.9081592
1996	240000	1.715524	2.221139	3.514563	.9921007	.8910481	.907384
1996	250000	1.714673	2.241076	3.67276	.9914578	.889212	.906633
1996	260000	1.713956	2.262305	3.856486	.9907393	.8873472	.90591
1996	270000	1.714166	2.283944	4.055539	.9899917	.8857377	.9052448
1996	280000	1.715087	2.305349	4.266506	.9892387	.8843589	.9046262
1996	290000	1.716447	2.326716	4.494789	.9884673	.8831026	.9040179
1996	300000	1.719188	2.352382	4.78819	.9875513	.8818973	.9032896
1996	400000	1.76708	2.573726	9.102162	.9804314	.880906	.8975928
1996	500000	1.765861	2.708644	28.53136	.9730909	.8742464	.8935123
1996	600000	1.714929	2.759379	-35.63996	.9696519	.8574803	.8880096
1997	10000	1.786295	1.918212	2.06288	.959937	.9542783	.9987873
1997	15000	2.050162	2.063387	2.077465	.9872045	.9860109	.9986351
1997	20000	2.194104	2.131001	2.065727	.9948101	.9913324	.9981908
1997	25000	2.232445	2.13401	2.036737	.9940737	.9879766	.9979829
1997	30000	2.195671	2.096457	2.001798	.9926053	.9863396	.9979782
1997	35000	2.12549	2.045128	1.969956	.9933661	.9885681	.9978544
1997	40000	2.051772	1.998189	1.948169	.9957748	.9923205	.9973354
1997	45000	2.000439	1.967511	1.936513	.9970066	.9936016	.9964134
1997	50000	1.973794	1.952292	1.931746	.9971637	.9931952	.9954668
1997	55000	1.949437	1.93976	1.930026	.9973688	.9925539	.9943662
1997	60000	1.929731	1.93068	1.930738	.9975187	.9916795	.9932338
1997	65000	1.910344	1.922725	1.933494	.9977687	.9906583	.9919956
1997	70000	1.892629	1.916516	1.938299	.9980006	.9893672	.9906477
1997	75000	1.878702	1.912654	1.944316	.9981321	.9879341	.9893379
1997	80000	1.866297	1.910118	1.951733	.9982238	.9863183	.987973
1997	85000	1.855658	1.908918	1.960342	.998262	.9845616	.9865811
1997	90000	1.847324	1.908947	1.96933	.998242	.9828181	.9852636
1997	95000	1.840432	1.910289	1.979779	.9981544	.9809021	.983858
1997	100000	1.835379	1.91208	1.989371	.9980498	.9792089	.982643
1997	105000	1.830468	1.914192	1.999705	.9979364	.9774411	.9814065
1997	110000	1.825722	1.916397	2.010266	.9978274	.9756891	.9802179
1997	115000	1.820888	1.918698	2.021377	.9977215	.9738909	.9790372
1997	120000	1.816059	1.921495	2.033876	.997597	.9719415	.9777921
1997	125000	1.812257	1.924684	2.046252	.9974515	.9700973	.9766314
1997	130000	1.809004	1.927797	2.057967	.9973096	.9684052	.975591
1997	135000	1.805931	1.931694	2.071572	.9971316	.966535	.9744589
1997	140000	1.803128	1.935322	2.084403	.9969636	.9648082	.973443
1997	145000	1.800518	1.938991	2.0973	.9967923	.9631234	.9724765
1997	150000	1.798117	1.943423	2.112086	.996586	.9612961	.9714453
1997	155000	1.796032	1.947538	2.125854	.9963895	.9596322	.9705278
1997	160000	1.79409	1.951646	2.139632	.9961905	.9580173	.9696603
1997	165000	1.792339	1.956443	2.155206	.9959576	.9562766	.9687378
1997	170000	1.790955	1.9605	2.168451	.9957565	.9548362	.9679927
1997	175000	1.789128	1.964561	2.182649	.995548	.9532964	.9672377
1997	180000	1.78709	1.968853	2.198107	.9953232	.9516575	.9664671
1997	185000	1.785888	1.97499	2.217706	.9950176	.949805	.9655818
1997	190000	1.785239	1.979578	2.232192	.9947836	.9484715	.964947
1997	195000	1.784831	1.984975	2.248857	.9945102	.947015	.9642572
1997	200000	1.784488	1.989846	2.264091	.9942579	.9457183	.9636588

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

1997	210000	1.783424	1.999832	2.296747	.9937134	.9430019	.9624785
1997	220000	1.782407	2.009364	2.328928	.9931694	.9404511	.9614388
1997	230000	1.781539	2.019437	2.363606	.9925753	.9378604	.9604451
1997	240000	1.780894	2.029444	2.398724	.9919652	.9353839	.9595506
1997	250000	1.780024	2.038939	2.433724	.9913522	.9329911	.9587585
1997	260000	1.779006	2.047717	2.467619	.9907556	.9307427	.9580774
1997	270000	1.777123	2.055949	2.50275	.990146	.9283893	.9574549
1997	280000	1.774373	2.062717	2.535715	.9896004	.9261285	.9569437
1997	290000	1.770786	2.074821	2.593598	.9886432	.9226339	.9563161
1997	300000	1.769647	2.083373	2.632323	.9879605	.9204715	.9559202
1997	400000	1.798681	2.208663	3.156063	.9791327	.9031504	.953341
1997	500000	1.873444	2.355122	3.75624	.9724935	.8999645	.9523372
1997	600000	1.952106	2.481714	4.288064	.969758	.9036796	.9510152
1998	10000	1.740285	1.871952	2.008976	.9612613	.9546931	.9979084
1998	15000	1.994609	2.008951	2.023253	.9886332	.986528	.9975042
1998	20000	2.12852	2.071123	2.014035	.9962683	.9918702	.9966143
1998	25000	2.169295	2.077244	1.989493	.9962393	.9888123	.9956346
1998	30000	2.149392	2.050374	1.959128	.9950801	.9860157	.9944821
1998	35000	2.101925	2.011497	1.930047	.9949651	.9851372	.9928152
1998	40000	2.047515	1.9736	1.907589	.9960595	.9854215	.9904645
1998	45000	2.001765	1.943699	1.891756	.9970641	.9845603	.98728
1998	50000	1.970188	1.923701	1.881744	.9975156	.9822623	.983613
1998	55000	1.947057	1.909743	1.875504	.9977293	.9792486	.9796715
1998	60000	1.928408	1.899233	1.871721	.9978603	.9757962	.9754751
1998	65000	1.911614	1.890583	1.869762	.9980199	.9720808	.9710529
1998	70000	1.894674	1.882698	1.869358	.9983069	.9681183	.9663208
1998	75000	1.879618	1.876493	1.870467	.9985217	.963412	.9609518
1998	80000	1.867038	1.872016	1.872733	.998693	.9585774	.9555731
1998	85000	1.856203	1.868894	1.876077	.9988139	.9533549	.949884
1998	90000	1.847486	1.867146	1.88021	.9988736	.9479766	.9441162
1998	95000	1.840276	1.866438	1.885039	.998892	.9424205	.9382128
1998	100000	1.834451	1.86632	1.889849	.9988978	.9373405	.9328451
1998	105000	1.827986	1.866215	1.895361	.998936	.9319494	.9271754
1998	110000	1.822038	1.866696	1.901615	.9989583	.926259	.9212191
1998	115000	1.816535	1.867414	1.908032	.9989849	.9207605	.9154899
1998	120000	1.810913	1.868339	1.91509	.9990238	.915049	.9095674
1998	125000	1.805568	1.869635	1.922743	.9990599	.9092029	.9035291
1998	130000	1.800794	1.87146	1.931012	.9990782	.9032157	.8973567
1998	135000	1.796715	1.874018	1.940175	.9990707	.8969518	.8908939
1998	140000	1.79388	1.876761	1.948567	.999044	.8914643	.8852156
1998	145000	1.791444	1.880032	1.957721	.9990038	.8857344	.8792703
1998	150000	1.789213	1.883284	1.966743	.9989639	.8803006	.8736377
1998	155000	1.787018	1.886573	1.975925	.9989249	.8749656	.8681177
1998	160000	1.785021	1.890361	1.986048	.9988759	.8693267	.8622695
1998	165000	1.78349	1.8942	1.995883	.9988223	.8640693	.8567977
1998	170000	1.781989	1.897994	2.005707	.9987698	.858999	.8515267
1998	175000	1.78053	1.902024	2.016114	.9987134	.8538242	.8461452
1998	180000	1.779065	1.906285	2.027168	.9986535	.8485323	.8406432
1998	185000	1.778154	1.911787	2.040425	.9985729	.8425224	.8343227
1998	190000	1.778045	1.916302	2.050622	.9985064	.8381172	.8296281
1998	195000	1.777605	1.920648	2.060965	.9984403	.8337567	.8250136
1998	200000	1.7772	1.925356	2.072194	.998368	.8291866	.8201677
1998	210000	1.776274	1.934688	2.094996	.9982226	.8203855	.8108408
1998	220000	1.775533	1.944837	2.119996	.9980616	.8114258	.8013058
1998	230000	1.774985	1.954499	2.144122	.9979058	.8034028	.7927346
1998	240000	1.773962	1.964562	2.170536	.9977385	.7951902	.7840034
1998	250000	1.772835	1.973494	2.194901	.9975876	.7881172	.7765042
1998	260000	1.770936	1.982383	2.221038	.9974385	.780962	.7690136
1998	270000	1.76847	1.992194	2.251362	.9972762	.7732392	.7609839
1998	280000	1.766142	2.000813	2.27909	.9971353	.7666835	.7542007
1998	290000	1.76386	2.012285	2.315669	.9969389	.7588642	.7460339
1998	300000	1.761857	2.023201	2.351351	.9967488	.7518818	.7387279
1998	400000	1.75315	2.144211	2.798252	.994455	.6997433	.6821928
1998	500000	1.750763	2.26615	3.425282	.9918205	.6710307	.6485234
1998	600000	1.74519	2.39821	4.541643	.9885294	.6521078	.6249622
1999	10000	1.705801	1.845357	1.987897	.9585576	.9521103	.9987963
1999	15000	1.956411	1.979742	2.002954	.9864486	.9853048	.998741
1999	20000	2.099462	2.046924	1.995654	.9954987	.9926549	.9983452
1999	25000	2.153924	2.060394	1.973018	.9962698	.9906601	.9981551
1999	30000	2.147776	2.039991	1.943159	.9950001	.9881202	.9982793
1999	35000	2.103029	2.001144	1.912263	.9948021	.9886618	.9985873
1999	40000	2.049397	1.96135	1.886013	.9957091	.9910497	.9988685
1999	45000	2.005408	1.929613	1.865652	.9963911	.9928622	.999046
1999	50000	1.974258	1.907298	1.851312	.9966911	.9938634	.9991381
1999	55000	1.948253	1.88928	1.840245	.9969206	.9946316	.9991748

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

1999	60000	1.92695	1.874849	1.831653	.9970417	.9951355	.9991675
1999	65000	1.907622	1.862568	1.825187	.997247	.9956623	.9991254
1999	70000	1.888512	1.851082	1.819884	.9975097	.9961863	.9990425
1999	75000	1.871931	1.841612	1.816123	.9977659	.9966033	.9989327
1999	80000	1.85688	1.83339	1.813352	.9979829	.9968917	.9987915
1999	85000	1.844064	1.826717	1.811568	.9981546	.9970574	.9986346
1999	90000	1.832731	1.82112	1.810539	.9983013	.9971389	.9984641
1999	95000	1.822428	1.816318	1.81013	.9984449	.99717	.9982851
1999	100000	1.812464	1.811958	1.81027	.9986009	.9971645	.9980913
1999	105000	1.803759	1.808414	1.810893	.998722	.9970866	.9978925
1999	110000	1.795909	1.805434	1.811879	.9988407	.9969819	.9976981
1999	115000	1.788298	1.802763	1.813266	.9989625	.9968419	.9974945
1999	120000	1.781217	1.800516	1.815032	.9990724	.9966592	.9972855
1999	125000	1.775132	1.798822	1.817019	.999158	.9964465	.9970848
1999	130000	1.769608	1.797535	1.81932	.9992242	.9961954	.9968796
1999	135000	1.764881	1.796705	1.821824	.999267	.9959199	.9966781
1999	140000	1.760787	1.796214	1.824451	.9992953	.9956337	.9964835
1999	145000	1.757201	1.796077	1.827327	.9993068	.9953246	.9962855
1999	150000	1.754475	1.796373	1.830288	.9992964	.9950135	.9960932
1999	155000	1.752376	1.796901	1.833156	.9992763	.9947197	.9959152
1999	160000	1.750611	1.797651	1.836165	.9992484	.99442	.9957354
1999	165000	1.749055	1.798483	1.839164	.9992181	.9941263	.9955622
1999	170000	1.747733	1.799371	1.842074	.9991863	.9938466	.9953993
1999	175000	1.746566	1.800371	1.845075	.999151	.9935638	.995236
1999	180000	1.74542	1.801439	1.848204	.9991135	.9932718	.9950704
1999	185000	1.744919	1.803014	1.851727	.9990637	.9929758	.994892
1999	190000	1.744648	1.804426	1.854727	.9990192	.9927272	.99474
1999	195000	1.744264	1.805699	1.857578	.9989769	.9924858	.9945976
1999	200000	1.743785	1.807024	1.860631	.9989315	.9922251	.9944479
1999	210000	1.743611	1.81034	1.867322	.9988264	.9916969	.9941279
1999	220000	1.744204	1.813704	1.873393	.9987286	.9912605	.9938433
1999	230000	1.744746	1.816979	1.879363	.9986289	.9908315	.993567
1999	240000	1.745032	1.820126	1.88536	.9985243	.9903882	.9932941
1999	250000	1.745152	1.823245	1.891501	.9984137	.9899272	.9930202
1999	260000	1.745484	1.826557	1.897846	.9982964	.989467	.992743
1999	270000	1.745981	1.829671	1.903646	.9981869	.9890618	.9924936
1999	280000	1.746533	1.833158	1.910171	.9980597	.9886079	.9922168
1999	290000	1.747269	1.836639	1.916514	.9979337	.9881838	.9919511
1999	300000	1.748643	1.840368	1.922696	.9978144	.9878236	.9916936
1999	400000	1.763353	1.876436	1.981238	.9965674	.9847143	.9892269
1999	500000	1.777488	1.90833	2.03118	.9952106	.9820508	.9866046
1999	600000	1.784775	1.933446	2.074334	.9935476	.9786553	.983604
2000	10000	1.67751	1.818917	1.95906	.9571629	.9499953	.9981178
2000	15000	1.918771	1.946296	1.972987	.9860769	.984103	.9979017
2000	20000	2.056582	2.011008	1.967734	.9952909	.9917002	.997224
2000	25000	2.110395	2.025672	1.948395	.9961178	.9896054	.9965207
2000	30000	2.105224	2.008204	1.922745	.9948288	.9865414	.9958338
2000	35000	2.064232	1.973941	1.896413	.9944384	.9857336	.9949111
2000	40000	2.014381	1.938616	1.87458	.9950499	.9861133	.9935286
2000	45000	1.970136	1.909316	1.858377	.9957205	.9858723	.9915951
2000	50000	1.937803	1.888695	1.847753	.9960144	.9845966	.989366
2000	55000	1.910318	1.872374	1.840739	.9963264	.9829484	.9868989
2000	60000	1.885899	1.858908	1.836303	.9966505	.9808804	.9841365
2000	65000	1.863899	1.847724	1.83402	.997001	.9784414	.9811149
2000	70000	1.84404	1.838476	1.833556	.9973837	.9757084	.9779096
2000	75000	1.826751	1.831194	1.834653	.9976681	.9724362	.9743902
2000	80000	1.812029	1.825671	1.836951	.9979024	.96896	.9708218
2000	85000	1.799503	1.821527	1.840066	.9981065	.9654585	.9673375
2000	90000	1.787206	1.817942	1.844183	.9983441	.9616466	.9636164
2000	95000	1.776474	1.815416	1.849083	.9985344	.9576508	.9598218
2000	100000	1.766818	1.813736	1.854805	.9986919	.9534052	.955873
2000	105000	1.758724	1.812934	1.860953	.9988019	.9491559	.951987
2000	110000	1.75178	1.812744	1.86737	.9988844	.9449642	.9481999
2000	115000	1.745467	1.812955	1.874128	.9989573	.9407589	.9444373
2000	120000	1.739294	1.813569	1.881736	.9990264	.9362348	.9404259
2000	125000	1.734424	1.814735	1.889308	.9990578	.9319166	.9366185
2000	130000	1.730096	1.816457	1.897638	.9990669	.9273435	.9326003
2000	135000	1.726979	1.818689	1.905899	.9990441	.9229724	.9287537
2000	140000	1.724407	1.821365	1.914664	.9990053	.9184787	.9247962
2000	145000	1.722622	1.824192	1.922994	.998956	.9143328	.9211341
2000	150000	1.72129	1.82732	1.931587	.9988978	.9101703	.9174458
2000	155000	1.720168	1.83045	1.940042	.9988381	.9061626	.913897
2000	160000	1.719452	1.834041	1.94917	.9987684	.9019524	.9101496
2000	165000	1.719128	1.837711	1.958108	.9986972	.8979338	.9065557
2000	170000	1.719283	1.841868	1.967692	.9986199	.8937491	.9027814
2000	175000	1.719865	1.84602	1.976821	.9985469	.8898657	.8992463

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2000	180000	1.720664	1.850149	1.985696	.9984774	.8861722	.8958638
2000	185000	1.721442	1.854	1.993996	.9984123	.8827732	.8927475
2000	190000	1.722214	1.857994	2.002716	.9983429	.879254	.8895243
2000	195000	1.723285	1.862447	2.012239	.9982699	.8755004	.8860633
2000	200000	1.724537	1.866933	2.021675	.9982004	.8718628	.8826873
2000	210000	1.726888	1.875334	2.039532	.9980683	.8651461	.8764455
2000	220000	1.729869	1.884258	2.058087	.9979434	.8584602	.870158
2000	230000	1.733462	1.893978	2.078074	.9978213	.8515589	.8635989
2000	240000	1.737748	1.903674	2.097217	.9977352	.8452837	.857502
2000	250000	1.742063	1.913313	2.11638	.9976563	.8392211	.8515707
2000	260000	1.746864	1.923241	2.135771	.9976012	.8333587	.8457372
2000	270000	1.751473	1.932723	2.154442	.9975559	.8279086	.8402742
2000	280000	1.756664	1.943066	2.17474	.9975245	.8222225	.8345023
2000	290000	1.761846	1.953257	2.194823	.9975067	.8168235	.8289643
2000	300000	1.768009	1.96432	2.215895	.9975388	.8114694	.8233083
2000	400000	1.805437	2.052252	2.414879	.9972819	.7676522	.7778927
2000	500000	1.830101	2.1337	2.646952	.99672	.7312257	.7401201
2000	600000	1.841039	2.225133	3.015376	.9955093	.6934617	.7026932
2001	10000	1.657559	1.815531	1.975865	.9550027	.9459922	.9981713
2001	15000	1.896344	1.945222	1.994246	.9842471	.9819884	.9981575
2001	20000	2.041023	2.017665	1.994601	.9945227	.9919522	.9975795
2001	25000	2.112527	2.044888	1.980567	.9966709	.9916348	.9968757
2001	30000	2.124597	2.03843	1.959095	.9958082	.9887911	.9961836
2001	35000	2.096822	2.012005	1.935774	.995109	.9873074	.9953422
2001	40000	2.05075	1.978423	1.914403	.9955282	.987402	.9940786
2001	45000	2.00586	1.948748	1.898553	.9964109	.9875493	.9923431
2001	50000	1.970776	1.926504	1.887695	.9969473	.9865752	.9901531
2001	55000	1.944695	1.910773	1.880996	.9972483	.9849361	.9877797
2001	60000	1.923036	1.898495	1.87683	.9974554	.9828042	.9851406
2001	65000	1.905063	1.888998	1.874642	.9975979	.9803479	.9823494
2001	70000	1.888932	1.881152	1.87396	.997743	.9776321	.9793787
2001	75000	1.874136	1.874578	1.874553	.9978957	.9746568	.9762205
2001	80000	1.860594	1.869134	1.876249	.9980547	.9714689	.9729219
2001	85000	1.848471	1.864798	1.878881	.9981975	.9680887	.9695172
2001	90000	1.837113	1.861216	1.882378	.9983494	.9645123	.9659799
2001	95000	1.826691	1.858381	1.886585	.9985012	.9608176	.9623891
2001	100000	1.816918	1.856204	1.891613	.9986459	.9568717	.9586199
2001	105000	1.808382	1.85485	1.89723	.998757	.9528185	.9548129
2001	110000	1.800737	1.854149	1.903431	.9988458	.9486338	.9509324
2001	115000	1.794301	1.854264	1.91023	.9988899	.9442922	.9469479
2001	120000	1.788961	1.854822	1.916953	.9989144	.9401833	.9432041
2001	125000	1.78372	1.855811	1.924614	.9989308	.9356893	.9391343
2001	130000	1.779757	1.857233	1.931958	.9989196	.9315371	.935385
2001	135000	1.776104	1.859105	1.940057	.9988939	.9271068	.931395
2001	140000	1.773301	1.861339	1.948119	.9988506	.9228374	.9275474
2001	145000	1.771087	1.863876	1.956292	.9987953	.918633	.9237539
2001	150000	1.769262	1.866847	1.965095	.9987269	.9142294	.9197733
2001	155000	1.768269	1.870273	1.974052	.998646	.9098884	.9158177
2001	160000	1.76795	1.873935	1.982832	.9985616	.9057523	.9120172
2001	165000	1.767969	1.877653	1.991413	.9984775	.9017996	.9083675
2001	170000	1.768194	1.881599	2.000347	.9983894	.8977692	.9046342
2001	175000	1.768714	1.885556	2.009013	.9983042	.893944	.901069
2001	180000	1.769729	1.890048	2.018383	.9982154	.8899128	.8972734
2001	185000	1.7709	1.894358	2.027184	.9981342	.8862038	.8937587
2001	190000	1.772323	1.898999	2.036515	.9980507	.8823497	.890085
2001	195000	1.773921	1.903532	2.04541	.9979758	.8787558	.88663
2001	200000	1.775608	1.908025	2.05414	.9979051	.8752924	.8832818
2001	210000	1.780153	1.918416	2.073618	.9977705	.8678344	.8759602
2001	220000	1.784479	1.927673	2.09076	.9976665	.8615006	.8696665
2001	230000	1.789915	1.938472	2.110378	.997572	.8545357	.8626387
2001	240000	1.795647	1.948722	2.128304	.9975258	.848465	.8563648
2001	250000	1.801494	1.959264	2.146945	.9974843	.8423361	.8499866
2001	260000	1.806998	1.969213	2.164681	.9974532	.8366842	.8440576
2001	270000	1.812335	1.979071	2.182559	.9974225	.8311452	.8382227
2001	280000	1.8182	1.98994	2.202439	.9973972	.8251958	.8319042
2001	290000	1.823843	2.000094	2.220877	.9973955	.819905	.8261985
2001	300000	1.828797	2.009585	2.238762	.9973786	.814882	.8208053
2001	400000	1.863458	2.102053	2.448324	.9966146	.7653935	.7683964
2001	500000	1.877702	2.181331	2.690163	.9953657	.7260909	.7274758
2001	600000	1.909678	2.301015	3.098217	.9941338	.6902412	.6861027
2002	10000	1.657691	1.826017	2.002043	.9530511	.942882	.9981501
2002	15000	1.903986	1.96269	2.023541	.9833587	.9808553	.998239
2002	20000	2.055593	2.041197	2.026366	.9946879	.9924644	.9977124
2002	25000	2.129704	2.071943	2.014728	.9972291	.9929688	.9970157
2002	30000	2.14338	2.067764	1.994998	.9965879	.9905502	.9962503

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2002	35000	2.117373	2.043047	1.972996	.9959847	.9890811	.9952588
2002	40000	2.076128	2.012807	1.953795	.9962787	.9887911	.9938723
2002	45000	2.033881	1.984953	1.939449	.9970626	.9886032	.9919951
2002	50000	2.000891	1.964265	1.930042	.9975811	.9874686	.989695
2002	55000	1.975266	1.949169	1.924466	.9979423	.9857457	.9871545
2002	60000	1.954487	1.937757	1.921467	.9981928	.9835221	.9843476
2002	65000	1.937767	1.92929	1.920405	.9983543	.980947	.9813721
2002	70000	1.923508	1.922741	1.920798	.9984664	.9780341	.9781697
2002	75000	1.911393	1.917755	1.922283	.9985593	.9750184	.9749426
2002	80000	1.899789	1.913515	1.924808	.998666	.9717111	.9714603
2002	85000	1.889649	1.910411	1.928254	.9987392	.9681457	.9677901
2002	90000	1.88064	1.908212	1.932493	.998792	.964372	.9639655
2002	95000	1.873153	1.906894	1.937102	.9988202	.960659	.9602488
2002	100000	1.866174	1.906052	1.942261	.9988456	.956813	.9564311
2002	105000	1.859428	1.905548	1.947977	.9988761	.9528202	.9524968
2002	110000	1.853103	1.905535	1.954382	.9988967	.948593	.9483619
2002	115000	1.847806	1.906053	1.960935	.9988981	.9444682	.9443504
2002	120000	1.84296	1.906978	1.96797	.9988889	.9402175	.940235
2002	125000	1.838641	1.908305	1.975402	.9988672	.9358912	.9360602
2002	130000	1.834513	1.90979	1.98308	.9988465	.9315664	.9319057
2002	135000	1.831059	1.911857	1.991377	.9988031	.9270567	.9275723
2002	140000	1.828254	1.91426	1.999781	.9987472	.9226304	.9233168
2002	145000	1.826042	1.917004	2.008359	.9986792	.9182458	.9190929
2002	150000	1.824442	1.920271	2.017476	.9985962	.9137258	.9147168
2002	155000	1.823502	1.923654	2.026163	.9985101	.9095361	.9106379
2002	160000	1.822879	1.927489	2.035572	.9984134	.9051101	.9063107
2002	165000	1.822783	1.931515	2.044853	.9983154	.9008605	.9021245
2002	170000	1.822938	1.935595	2.054022	.9982174	.8967535	.8980607
2002	175000	1.823558	1.940234	2.063968	.998112	.8924136	.8937289
2002	180000	1.824714	1.945132	2.073898	.9980106	.8882015	.8894734
2002	185000	1.826345	1.950212	2.083697	.9979173	.8841586	.8853347
2002	190000	1.828304	1.955336	2.093218	.9978341	.8803279	.8813648
2002	195000	1.830174	1.960096	2.102036	.9977588	.8768374	.8777295
2002	200000	1.831855	1.964711	2.110807	.9976802	.8734	.8741574
2002	210000	1.835147	1.974511	2.129961	.9975013	.8660442	.8665184
2002	220000	1.838862	1.984547	2.149292	.9973311	.8589196	.8590301
2002	230000	1.842588	1.994675	2.169032	.9971583	.851903	.8516114
2002	240000	1.847064	2.005641	2.189963	.9969936	.8448061	.8439874
2002	250000	1.852308	2.017048	2.21103	.996859	.8380331	.8365495
2002	260000	1.857817	2.028337	2.231538	.996752	.8317438	.8295154
2002	270000	1.863502	2.039986	2.252847	.9966494	.825465	.8224192
2002	280000	1.869265	2.051321	2.273336	.9965746	.8197119	.8157958
2002	290000	1.874432	2.062074	2.293396	.9964901	.8142415	.8094966
2002	300000	1.880563	2.074546	2.316587	.9964114	.8082113	.8024421
2002	400000	1.922727	2.181785	2.545354	.9953341	.759859	.7448723
2002	500000	1.949023	2.289373	2.842427	.9935468	.7195978	.6953136
2002	600000	1.971643	2.406095	3.246034	.9913122	.6883991	.6537082
2003	10000	1.653749	1.817157	1.984579	.9531948	.9444011	.9989878
2003	15000	1.899351	1.95135	2.003969	.9834496	.9820028	.9992208
2003	20000	2.048523	2.026594	2.004618	.9946073	.9932318	.9989993
2003	25000	2.120529	2.05458	1.991047	.9970115	.993676	.9988265
2003	30000	2.131197	2.047491	1.969356	.9962446	.9916161	.998858
2003	35000	2.101205	2.019927	1.945725	.9957571	.9912843	.9990184
2003	40000	2.055959	1.986979	1.924753	.996337	.9927136	.9991605
2003	45000	2.011092	1.95716	1.908616	.9973602	.9944853	.9991941
2003	50000	1.981158	1.937354	1.897826	.9977636	.9950832	.99913
2003	55000	1.95748	1.92245	1.890542	.9980805	.9953787	.9990112
2003	60000	1.937706	1.910583	1.885424	.9983228	.9954232	.9988453
2003	65000	1.92121	1.901187	1.882015	.9985029	.9952646	.9986436
2003	70000	1.908046	1.894057	1.879919	.9986026	.9949206	.9984193
2003	75000	1.896192	1.888063	1.878801	.9987022	.9944975	.9981753
2003	80000	1.885188	1.882899	1.878498	.9988105	.9940054	.9979129
2003	85000	1.875045	1.878497	1.878898	.998931	.9934675	.9976423
2003	90000	1.865293	1.874647	1.880003	.9990464	.9928064	.9973398
2003	95000	1.857558	1.871991	1.88161	.9991084	.9920678	.9970382
2003	100000	1.850802	1.869968	1.883583	.9991572	.9912953	.9967393
2003	105000	1.844631	1.86836	1.88586	.9992037	.9904966	.9964424
2003	110000	1.8385	1.866926	1.888473	.999264	.9896626	.9961418
2003	115000	1.832539	1.865731	1.891432	.99933	.9887816	.9958363
2003	120000	1.826883	1.864864	1.894783	.9993922	.9878353	.9955225
2003	125000	1.822085	1.864476	1.898312	.9994349	.9868751	.9952179
2003	130000	1.817597	1.864354	1.902113	.9994738	.9858753	.9949125
2003	135000	1.813705	1.864541	1.906004	.9995018	.98488	.9946194
2003	140000	1.810066	1.865046	1.910308	.9995221	.9838073	.994314
2003	145000	1.807205	1.865984	1.914757	.9995242	.9827272	.9940144
2003	150000	1.805067	1.867283	1.919252	.9995129	.9816624	.9937247

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2003	155000	1.803469	1.868812	1.923707	.9994938	.9806279	.9934475
2003	160000	1.802272	1.870595	1.928299	.9994687	.9795812	.9931706
2003	165000	1.801554	1.87262	1.932916	.9994386	.9785506	.9928994
2003	170000	1.801101	1.87477	1.937542	.9994066	.977531	.9926336
2003	175000	1.801062	1.877171	1.942267	.9993721	.9765108	.9923672
2003	180000	1.801347	1.879658	1.946853	.9993384	.9755374	.9921125
2003	185000	1.801724	1.882096	1.951266	.9993057	.9746055	.9918702
2003	190000	1.802135	1.884522	1.95563	.9992731	.9736854	.9916332
2003	195000	1.802851	1.887365	1.960521	.9992376	.9726695	.9913707
2003	200000	1.80369	1.89	1.964882	.9992075	.9717759	.9911385
2003	210000	1.805769	1.895784	1.974236	.9991454	.9698755	.9906456
2003	220000	1.80817	1.901467	1.983072	.9990925	.9681072	.9901846
2003	230000	1.811065	1.907512	1.992125	.9990461	.9663244	.9897148
2003	240000	1.814353	1.913609	2.000867	.9990135	.9646395	.9892619
2003	250000	1.817429	1.919391	2.009202	.9989823	.9630215	.9888291
2003	260000	1.820246	1.925002	2.017472	.9989464	.9613925	.9884001
2003	270000	1.823261	1.931133	2.02658	.998906	.9595898	.987929
2003	280000	1.826946	1.937424	2.035272	.9988909	.9579444	.9874789
2003	290000	1.830152	1.943168	2.043371	.9988721	.9563865	.9870578
2003	300000	1.833406	1.948957	2.051502	.9988562	.9548254	.9866338
2003	400000	1.852842	1.999594	2.133301	.998389	.938194	.9825084
2003	500000	1.855294	2.039351	2.215513	.9976448	.9208573	.9788067
2003	600000	1.859149	2.084692	2.312694	.9966343	.9021813	.9751292
2004	10000	1.617271	1.79072	1.965596	.9503185	.9402251	.9987645
2004	15000	1.86588	1.926653	1.987391	.9802468	.9785361	.9990662
2004	20000	2.021236	2.005171	1.98931	.9926	.9913002	.9988189
2004	25000	2.104834	2.039219	1.976996	.9958545	.9924461	.9985849
2004	30000	2.13037	2.039145	1.955673	.9951966	.9899083	.9985766
2004	35000	2.115152	2.018045	1.931761	.9939796	.9882346	.9987848
2004	40000	2.070584	1.983627	1.907891	.994106	.9894588	.9990845
2004	45000	2.017248	1.947941	1.888176	.9955654	.9925365	.999314
2004	50000	1.970386	1.918314	1.873582	.9969495	.9951233	.9994089
2004	55000	1.937104	1.898107	1.864583	.9978127	.9966248	.9993975
2004	60000	1.912879	1.883897	1.858878	.99832	.997444	.9993274
2004	65000	1.894132	1.873389	1.855328	.9986655	.9979249	.9992235
2004	70000	1.878638	1.865177	1.853253	.9989383	.9982249	.9990968
2004	75000	1.86592	1.858824	1.85227	.9991317	.9983642	.9989513
2004	80000	1.856186	1.854278	1.852109	.9992303	.9983535	.9987934
2004	85000	1.849038	1.851211	1.852499	.9992658	.9982557	.9986352
2004	90000	1.842815	1.848767	1.853272	.9992915	.9981309	.9984755
2004	95000	1.837429	1.846886	1.854391	.9992995	.9979711	.9983085
2004	100000	1.832817	1.845499	1.855779	.9992935	.9977867	.998138
2004	105000	1.828567	1.844366	1.85735	.9992875	.9975944	.9979693
2004	110000	1.824395	1.843353	1.859101	.9992871	.9973956	.9978006
2004	115000	1.820305	1.842486	1.86108	.9992879	.997182	.9976273
2004	120000	1.816452	1.841813	1.863245	.9992869	.9969562	.9974528
2004	125000	1.812926	1.841361	1.86556	.9992814	.9967203	.997279
2004	130000	1.809494	1.840957	1.867909	.9992824	.9964882	.9971136
2004	135000	1.80567	1.840489	1.870528	.9992961	.9962388	.9969413
2004	140000	1.802201	1.84028	1.873351	.9993007	.9959738	.9967678
2004	145000	1.798999	1.840199	1.876207	.9993045	.9957101	.9966025
2004	150000	1.795901	1.840183	1.879123	.9993106	.9954455	.9964435
2004	155000	1.792771	1.840248	1.882262	.9993183	.9951652	.9962823
2004	160000	1.789766	1.840358	1.885405	.9993288	.9948885	.9961305
2004	165000	1.786681	1.840562	1.888853	.9993404	.9945897	.9959744
2004	170000	1.784005	1.84093	1.89226	.9993454	.9942999	.9958295
2004	175000	1.781366	1.841449	1.895975	.9993472	.9939899	.9956814
2004	180000	1.779173	1.842262	1.899874	.999337	.9936762	.9955353
2004	185000	1.777472	1.84334	1.90385	.9993163	.9933702	.9953947
2004	190000	1.77625	1.84444	1.907404	.9992931	.9931064	.9952748
2004	195000	1.775023	1.845732	1.911389	.9992654	.9928168	.9951469
2004	200000	1.774324	1.847341	1.915505	.99923	.9925411	.9950207
2004	210000	1.773701	1.850619	1.923105	.9991586	.992066	.9948003
2004	220000	1.774187	1.854645	1.931158	.9990803	.9916322	.9945818
2004	230000	1.775231	1.858761	1.938846	.9990057	.9912608	.9943832
2004	240000	1.776332	1.862634	1.945991	.9989354	.9909317	.9942059
2004	250000	1.777205	1.86632	1.95303	.9988623	.9906008	.9940386
2004	260000	1.778625	1.87072	1.961044	.998781	.9902632	.9938569
2004	270000	1.780679	1.875242	1.968631	.9987124	.9900069	.993691
2004	280000	1.782778	1.879695	1.976037	.9986458	.989774	.9935333
2004	290000	1.785486	1.884517	1.983561	.9985899	.9895971	.9933764
2004	300000	1.78823	1.889069	1.990449	.9985452	.9894682	.993234
2004	400000	1.808667	1.929168	2.056321	.9979389	.9881229	.991939
2004	500000	1.823232	1.963493	2.117628	.9971635	.9869143	.9907565
2004	600000	1.834717	1.994995	2.177589	.9961665	.9855632	.9894667

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2005	10000	1.580641	1.744421	1.897573	.9540405	.9447801	.99886
2005	15000	1.806155	1.863122	1.915884	.9818283	.9802902	.9991382
2005	20000	1.947238	1.93171	1.917475	.9930002	.9918398	.9989129
2005	25000	2.023149	1.961304	1.906776	.9958696	.9927613	.998711
2005	30000	2.046969	1.961666	1.888883	.9953004	.9905097	.9987338
2005	35000	2.032735	1.943131	1.868685	.9941705	.9890677	.9989526
2005	40000	1.994165	1.914278	1.849078	.9941658	.9900657	.9992434
2005	45000	1.944777	1.882876	1.832763	.9955995	.9930599	.9994673
2005	50000	1.900828	1.85671	1.821076	.9970117	.9956049	.9995586
2005	55000	1.869271	1.838858	1.814253	.9979158	.9970803	.9995501
2005	60000	1.845303	1.826056	1.810402	.9985528	.9979789	.9994895
2005	65000	1.826399	1.816592	1.808519	.9990273	.9985278	.9994007
2005	70000	1.811763	1.809771	1.808007	.9993287	.9987804	.9992929
2005	75000	1.802068	1.805643	1.808404	.9994584	.9988042	.9991838
2005	80000	1.794921	1.802935	1.80933	.9995236	.9987412	.9990754
2005	85000	1.789508	1.801202	1.810633	.9995483	.9986266	.9989657
2005	90000	1.785647	1.800306	1.812206	.9995405	.9984781	.9988546
2005	95000	1.782634	1.79983	1.813858	.9995229	.9983242	.998749
2005	100000	1.779874	1.799524	1.81563	.9995034	.9981641	.998644
2005	105000	1.777283	1.799345	1.817511	.9994829	.9979982	.9985397
2005	110000	1.774732	1.799278	1.819586	.9994606	.997819	.9984317
2005	115000	1.772534	1.799361	1.821652	.9994363	.9976431	.9983297
2005	120000	1.770481	1.799598	1.823902	.9994073	.9974543	.998224
2005	125000	1.768716	1.79996	1.826149	.999376	.9972683	.998123
2005	130000	1.766828	1.800235	1.828358	.9993499	.9970849	.9980279
2005	135000	1.764732	1.800483	1.830718	.9993263	.9968897	.9979312
2005	140000	1.762576	1.800729	1.833148	.9993053	.9966899	.9978369
2005	145000	1.760404	1.800972	1.835607	.999287	.996489	.9977469
2005	150000	1.75814	1.801225	1.838191	.9992709	.9962794	.9976581
2005	155000	1.755905	1.801568	1.840948	.9992529	.9960584	.9975697
2005	160000	1.753768	1.801932	1.843677	.9992363	.9958413	.997488
2005	165000	1.751681	1.802312	1.846417	.9992211	.9956248	.9974117
2005	170000	1.749476	1.802718	1.849344	.9992074	.9953949	.9973366
2005	175000	1.747414	1.803292	1.852497	.9991876	.9951526	.9972624
2005	180000	1.745618	1.803923	1.855533	.999166	.9949229	.997197
2005	185000	1.744003	1.804675	1.85866	.9991398	.9946917	.9971354
2005	190000	1.742631	1.805572	1.861869	.9991083	.9944616	.9970778
2005	195000	1.741422	1.806499	1.864996	.9990758	.9942412	.9970266
2005	200000	1.74036	1.807588	1.868332	.9990379	.9940138	.9969771
2005	210000	1.73907	1.810182	1.87508	.9989501	.9935867	.9968911
2005	220000	1.738919	1.813156	1.881513	.9988579	.9932299	.9968234
2005	230000	1.739171	1.816332	1.888003	.9987617	.9928944	.9967672
2005	240000	1.73953	1.819395	1.894187	.9986669	.9925843	.9967238
2005	250000	1.740062	1.822551	1.900428	.9985689	.9922848	.9966899
2005	260000	1.740989	1.825917	1.906735	.9984691	.9920101	.9966646
2005	270000	1.74187	1.829083	1.91269	.998372	.9917548	.9966487
2005	280000	1.743148	1.832738	1.919314	.9982637	.9914956	.9966395
2005	290000	1.745494	1.836938	1.925962	.99817	.9913216	.9966378
2005	300000	1.748029	1.841022	1.932156	.9980882	.9911929	.9966407
2005	400000	1.774158	1.877676	1.984173	.9974788	.9907382	.9967678
2005	500000	1.799078	1.911734	2.031975	.9968173	.990583	.9969912
2005	600000	1.823157	1.940135	2.067088	.9964166	.9909351	.9970683
2006	10000	1.568518	1.721735	1.85812	.9555042	.947487	.9988326
2006	15000	1.785646	1.832111	1.872874	.982778	.9814849	.9989827
2006	20000	1.918628	1.893817	1.872284	.9932016	.9916604	.9987064
2006	25000	1.988682	1.918632	1.860211	.9956138	.991774	.9985063
2006	30000	2.006964	1.915677	1.842064	.994785	.9892372	.9985707
2006	35000	1.988991	1.895279	1.821744	.9936304	.9879145	.9988497
2006	40000	1.95008	1.866626	1.802348	.9935943	.9890049	.9992056
2006	45000	1.90213	1.836623	1.786583	.9950079	.9921154	.9994909
2006	50000	1.856352	1.8101	1.774812	.9966739	.9951505	.9996431
2006	55000	1.824282	1.792411	1.768013	.9976879	.9968519	.9996788
2006	60000	1.799336	1.77948	1.764136	.9984629	.9979739	.9996562
2006	65000	1.779319	1.769757	1.762182	.9990408	.9986789	.9996007
2006	70000	1.765638	1.763529	1.761601	.9993256	.9989489	.9995322
2006	75000	1.756063	1.759523	1.761835	.9994704	.9990215	.9994618
2006	80000	1.749174	1.756958	1.76258	.9995335	.9989883	.9993899
2006	85000	1.74441	1.755473	1.763616	.9995468	.9989011	.9993202
2006	90000	1.740606	1.754472	1.764784	.9995484	.9988021	.9992551
2006	95000	1.736863	1.753585	1.766125	.9995541	.9986968	.9991905
2006	100000	1.733285	1.752897	1.76771	.9995568	.9985759	.9991238
2006	105000	1.730784	1.752734	1.769399	.9995391	.9984435	.9990594
2006	110000	1.728784	1.75283	1.771166	.9995143	.9983069	.9989966
2006	115000	1.727475	1.753239	1.772955	.9994809	.9981719	.9989361
2006	120000	1.726458	1.753774	1.774745	.9994447	.9980388	.9988776
2006	125000	1.725548	1.75431	1.776457	.9994094	.9979112	.9988233

Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2006	130000	1.724393	1.754779	1.778255	.9993746	.9977741	.998768
2006	135000	1.72308	1.755167	1.780046	.9993424	.9976353	.9987153
2006	140000	1.72157	1.755471	1.781855	.999314	.9974934	.9986648
2006	145000	1.719913	1.755753	1.783758	.9992868	.9973436	.9986147
2006	150000	1.718168	1.756038	1.785755	.9992603	.9971867	.9985657
2006	155000	1.71652	1.756342	1.787715	.9992347	.9970332	.998521
2006	160000	1.714755	1.756663	1.789823	.999209	.9968686	.9984767
2006	165000	1.713144	1.757039	1.791915	.9991821	.9967067	.9984363
2006	170000	1.711544	1.757414	1.794011	.9991564	.9965447	.9983994
2006	175000	1.709865	1.757779	1.79617	.9991323	.9963774	.9983651
2006	180000	1.70831	1.758316	1.798562	.9991004	.9961966	.9983313
2006	185000	1.707177	1.759034	1.800942	.9990619	.9960244	.9983014
2006	190000	1.706347	1.759917	1.803381	.9990178	.9958556	.9982741
2006	195000	1.705812	1.760833	1.805626	.9989743	.9957064	.9982515
2006	200000	1.705217	1.761685	1.807812	.9989322	.9955585	.9982318
2006	210000	1.704212	1.763533	1.812323	.9988416	.9952593	.9981982
2006	220000	1.703389	1.765477	1.816888	.9987458	.9949622	.9981735
2006	230000	1.702591	1.767366	1.821354	.9986493	.9946705	.998158
2006	240000	1.701737	1.769274	1.825945	.9985477	.9943683	.9981512
2006	250000	1.701231	1.771286	1.830443	.9984431	.994085	.998153
2006	260000	1.700996	1.773563	1.83524	.9983271	.9937962	.9981638
2006	270000	1.701493	1.776147	1.839968	.9982097	.993549	.9981819
2006	280000	1.702466	1.778977	1.844743	.9980901	.993327	.9982066
2006	290000	1.704067	1.78207	1.84944	.9979763	.9931503	.9982362
2006	300000	1.706225	1.785419	1.854107	.99787	.9930162	.9982699
2006	400000	1.739745	1.820045	1.890366	.9974889	.993406	.9985712
2006	500000	1.770721	1.848648	1.915262	.9974412	.9943225	.9986972
2006	600000	1.786843	1.867244	1.934207	.9968955	.9941368	.998696
2007	10000	1.558274	1.703271	1.827221	.9561598	.9490001	.998671
2007	15000	1.765959	1.805814	1.839246	.982832	.9815411	.9986979
2007	20000	1.895226	1.8636	1.837324	.9930906	.9910861	.9983473
2007	25000	1.964653	1.886647	1.824475	.9955482	.9908408	.9980994
2007	30000	1.983413	1.882915	1.805678	.9947178	.9879902	.9981527
2007	35000	1.967547	1.862846	1.784795	.9934853	.986361	.9984605
2007	40000	1.931583	1.835221	1.764951	.9933168	.987244	.9988815
2007	45000	1.884622	1.805057	1.747762	.9946327	.9904488	.9992673
2007	50000	1.839364	1.777915	1.733965	.9962006	.9936824	.999532
2007	55000	1.807318	1.759469	1.725327	.9972169	.9956704	.9996595
2007	60000	1.780805	1.744986	1.719396	.998084	.9971848	.9997131
2007	65000	1.759843	1.733978	1.715419	.9986936	.998171	.999718
2007	70000	1.745428	1.72657	1.712953	.9989933	.9986449	.9996966
2007	75000	1.73477	1.721275	1.711433	.999167	.9989001	.9996631
2007	80000	1.726055	1.71712	1.710485	.9992805	.9990465	.999621
2007	85000	1.719477	1.714107	1.709979	.9993334	.9990997	.9995752
2007	90000	1.713921	1.711698	1.709786	.9993737	.9991236	.9995286
2007	95000	1.708612	1.709534	1.709843	.9994094	.9991277	.9994769
2007	100000	1.704456	1.70796	1.710094	.9994211	.9990999	.9994259
2007	105000	1.70075	1.706659	1.7105	.9994258	.9990582	.999374
2007	110000	1.697436	1.705598	1.711041	.9994232	.9990034	.9993209
2007	115000	1.694656	1.704826	1.7117	.9994088	.998934	.9992664
2007	120000	1.692428	1.704301	1.712393	.9993885	.998861	.9992151
2007	125000	1.690075	1.703749	1.713138	.9993742	.9987892	.9991646
2007	130000	1.687655	1.703208	1.713957	.9993632	.9987152	.9991138
2007	135000	1.685266	1.70272	1.714853	.9993526	.9986368	.9990627
2007	140000	1.682859	1.702239	1.715782	.9993472	.9985598	.9990136
2007	145000	1.680299	1.701733	1.716791	.9993477	.9984807	.9989645
2007	150000	1.677759	1.701271	1.71787	.9993498	.9983982	.9989161
2007	155000	1.675184	1.700781	1.718938	.9993618	.998322	.998872
2007	160000	1.672265	1.700232	1.720173	.9993829	.9982381	.9988255
2007	165000	1.669431	1.69974	1.721459	.999406	.9981521	.9987817
2007	170000	1.66675	1.699361	1.722839	.999425	.9980586	.9987392
2007	175000	1.664227	1.699067	1.724263	.9994422	.9979621	.9986995
2007	180000	1.661824	1.698881	1.7258	.999455	.9978575	.9986609
2007	185000	1.659779	1.698828	1.727309	.9994612	.9977548	.9986265
2007	190000	1.657897	1.698948	1.729013	.9994588	.9976401	.9985914
2007	195000	1.656488	1.699185	1.730563	.9994504	.9975379	.9985622
2007	200000	1.655089	1.699413	1.732099	.9994437	.9974365	.9985358
2007	210000	1.65227	1.699897	1.735264	.9994321	.9972281	.9984887
2007	220000	1.649856	1.700592	1.738524	.9994128	.9970197	.99845
2007	230000	1.6476	1.701398	1.741893	.9993905	.9968078	.9984196
2007	240000	1.645749	1.702412	1.745346	.9993603	.9965997	.9983976
2007	250000	1.644759	1.703914	1.749018	.9993148	.9964069	.9983828
2007	260000	1.644507	1.705617	1.752465	.9992661	.9962513	.9983751
2007	270000	1.644486	1.707277	1.755646	.9992193	.9961171	.9983723
2007	280000	1.644493	1.709018	1.758973	.9991687	.9959782	.9983739
2007	290000	1.644832	1.710855	1.762205	.9991184	.99586	.9983795

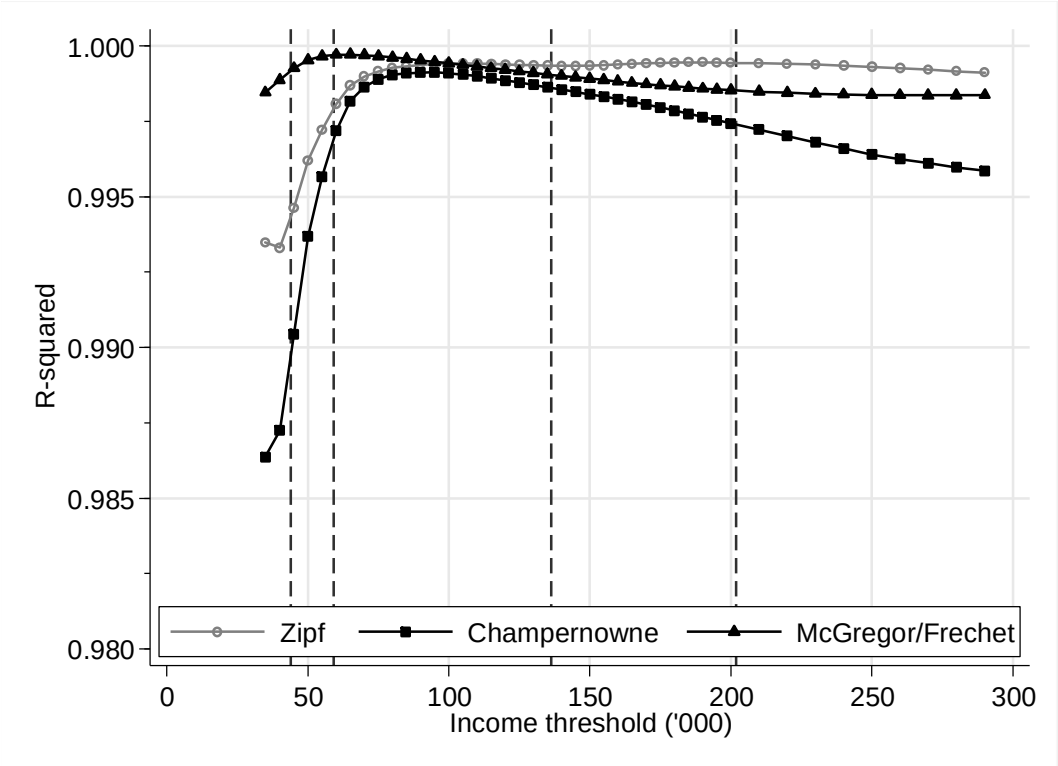
Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

2007	300000	1.645247	1.71267	1.765337	.9990685	.9957504	.9983883
2007	400000	1.657386	1.733264	1.794496	.998632	.9952739	.9985836
2007	500000	1.680731	1.75555	1.816262	.9986544	.9960516	.9987454
2007	600000	1.699266	1.772893	1.832208	.9986586	.9966416	.9987863
2009	10000	1.583394	1.705281	1.805765	.9586348	.953208	.9978807
2009	15000	1.792605	1.804323	1.813314	.9834834	.9816579	.9974799
2009	20000	1.918914	1.855431	1.805353	.9920251	.9873409	.9967942
2009	25000	1.982551	1.870325	1.786209	.9932718	.9841952	.9963781
2009	30000	1.995914	1.858986	1.761207	.9914494	.9791973	.9964378
2009	35000	1.972231	1.831203	1.734584	.9894965	.9765468	.9969166
2009	40000	1.926832	1.796089	1.709302	.9890692	.977671	.9976158
2009	45000	1.871061	1.759685	1.687267	.9905384	.9820347	.9983074
2009	50000	1.814724	1.726072	1.669134	.992863	.9873195	.9988307
2009	55000	1.773044	1.70222	1.657026	.9944208	.9907554	.9991081
2009	60000	1.739037	1.683606	1.648317	.9956062	.9932148	.9992368
2009	65000	1.714681	1.670582	1.642504	.996218	.9945374	.9992653
2009	70000	1.695145	1.66057	1.638493	.9966586	.9954252	.9992378
2009	75000	1.678808	1.652564	1.635702	.9969922	.9960333	.9991738
2009	80000	1.664552	1.645946	1.633854	.9973003	.9964994	.9990865
2009	85000	1.651654	1.640274	1.632698	.9975783	.996842	.9989796
2009	90000	1.640204	1.635511	1.632128	.997801	.997048	.9988581
2009	95000	1.630585	1.631742	1.632046	.9979783	.997152	.9987369
2009	100000	1.62148	1.628403	1.632365	.9981349	.9971829	.9986049
2009	105000	1.613941	1.625828	1.632965	.9982516	.997149	.9984818
2009	110000	1.60655	1.623486	1.633882	.9983655	.9970722	.9983522
2009	115000	1.600338	1.621693	1.63497	.9984452	.9969523	.9982322
2009	120000	1.594451	1.620121	1.636239	.9985275	.9968145	.9981166
2009	125000	1.588988	1.618783	1.637647	.9986071	.9966607	.998008
2009	130000	1.583715	1.6176	1.63922	.9986895	.9964904	.9979039
2009	135000	1.578854	1.616618	1.640884	.9987674	.9963094	.9978091
2009	140000	1.573938	1.61574	1.642794	.9988481	.9961014	.9977154
2009	145000	1.569428	1.615069	1.64481	.9989183	.9958792	.9976307
2009	150000	1.565436	1.614654	1.646937	.9989673	.9956404	.9975545
2009	155000	1.561956	1.614442	1.649079	.9990012	.9953997	.9974888
2009	160000	1.558778	1.614377	1.651289	.999026	.9951533	.9974311
2009	165000	1.555848	1.614527	1.653725	.9990354	.994886	.9973779
2009	170000	1.553591	1.614887	1.656059	.9990265	.9946368	.9973356
2009	175000	1.551597	1.615381	1.65846	.9990092	.9943871	.9972998
2009	180000	1.550038	1.616038	1.660846	.9989809	.9941499	.9972711
2009	185000	1.548822	1.616893	1.663345	.9989419	.9939155	.9972474
2009	190000	1.54804	1.61786	1.665731	.9988977	.9937071	.99723
2009	195000	1.547493	1.618863	1.668009	.9988525	.9935175	.9972176
2009	200000	1.547096	1.619946	1.670332	.9988041	.9933324	.9972088
2009	210000	1.546696	1.622157	1.674773	.9987068	.9929991	.9972023
2009	220000	1.546654	1.624497	1.6792	.9986052	.9926893	.9972078
2009	230000	1.547243	1.627096	1.683632	.9985015	.9924234	.9972241
2009	240000	1.548465	1.630039	1.688218	.9983962	.9921964	.9972504
2009	250000	1.550515	1.633208	1.692562	.9983091	.992059	.997282
2009	260000	1.552748	1.636228	1.696477	.9982376	.9919724	.9973145
2009	270000	1.555158	1.639258	1.700263	.9981738	.9919146	.9973489
2009	280000	1.557724	1.642336	1.70401	.9981145	.9918773	.9973854
2009	290000	1.560537	1.645497	1.707705	.9980646	.9918711	.9974234
2009	300000	1.563298	1.648458	1.711057	.9980261	.991889	.9974589
2009	400000	1.591725	1.676544	1.740569	.9978229	.9925508	.997787
2009	500000	1.61507	1.698856	1.762684	.9976977	.9933051	.9980029
2009	600000	1.633319	1.716863	1.780448	.997496	.9938346	.9981256
2010	10000	1.618442	1.779075	1.935572	.9549968	.9458587	.9985656
2010	15000	1.85077	1.903897	1.9551	.9816251	.9798554	.9987221
2010	20000	2.000128	1.977188	1.955444	.9923394	.990446	.9983345
2010	25000	2.085539	2.009975	1.941309	.9955933	.9909754	.9979869
2010	30000	2.114611	2.010117	1.918849	.9950831	.987987	.9979178
2010	35000	2.103141	1.988964	1.892727	.9937049	.9856182	.9981256
2010	40000	2.063532	1.955553	1.866943	.9934753	.9861947	.9984903
2010	45000	2.011343	1.918544	1.843764	.99471	.9892078	.9988508
2010	50000	1.962686	1.885712	1.824519	.996067	.9921531	.9991065
2010	55000	1.92884	1.863037	1.811229	.9967775	.9938045	.9992462
2010	60000	1.905112	1.846635	1.800994	.996979	.9945349	.9993271
2010	65000	1.887889	1.834665	1.793403	.997005	.9948956	.9993734
2010	70000	1.87202	1.824047	1.787051	.9970412	.9952289	.9993989
2010	75000	1.857582	1.814807	1.781942	.9971052	.9955508	.9994047
2010	80000	1.843746	1.806329	1.777654	.9971954	.9958677	.999392
2010	85000	1.830664	1.798657	1.774166	.9973099	.9961709	.9993623
2010	90000	1.818257	1.791711	1.771412	.9974554	.9964663	.9993176
2010	95000	1.806428	1.78535	1.769232	.9976057	.9967228	.9992579
2010	100000	1.795597	1.779745	1.767619	.9977418	.996918	.9991864

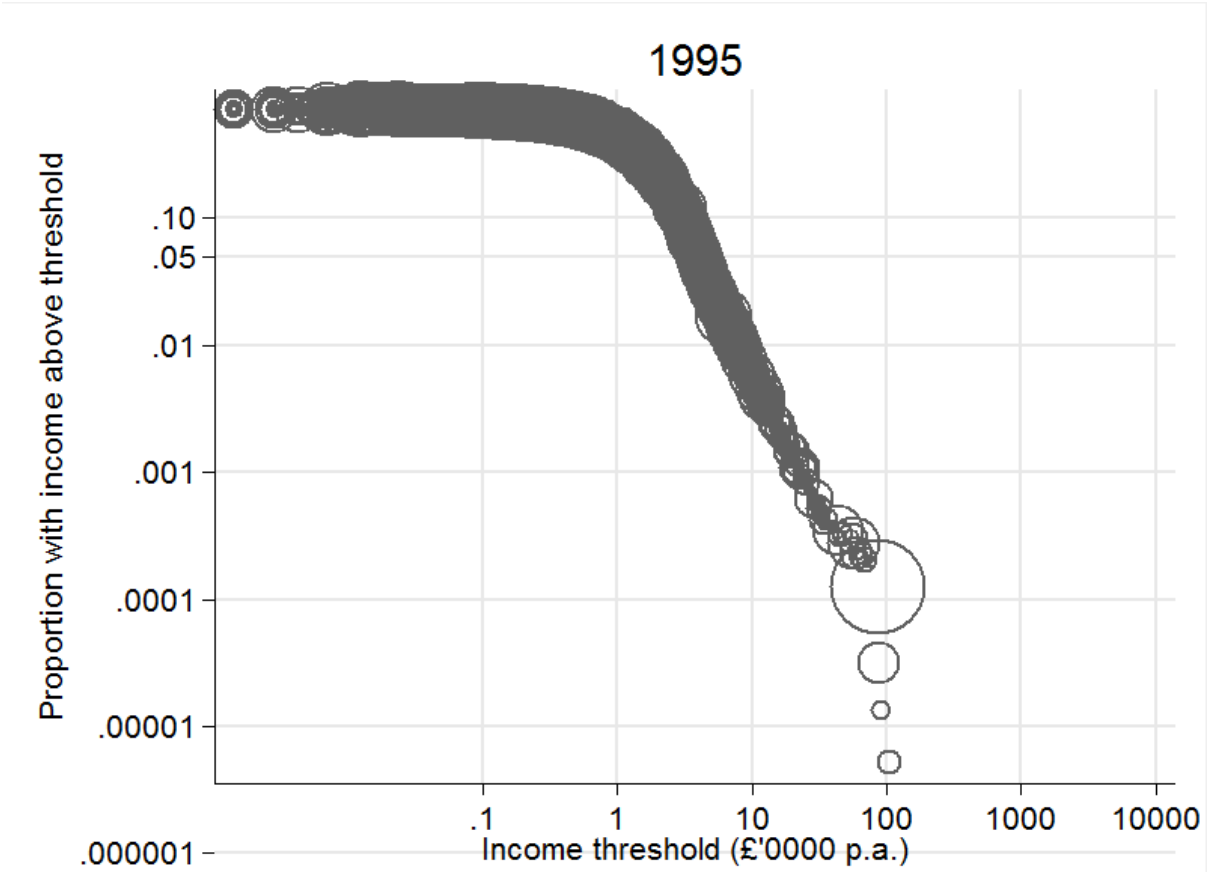
Estimates of Pareto I alpha: Zipf, Champernowne, McGregor/Frechet OLS regressions
(pareto02)

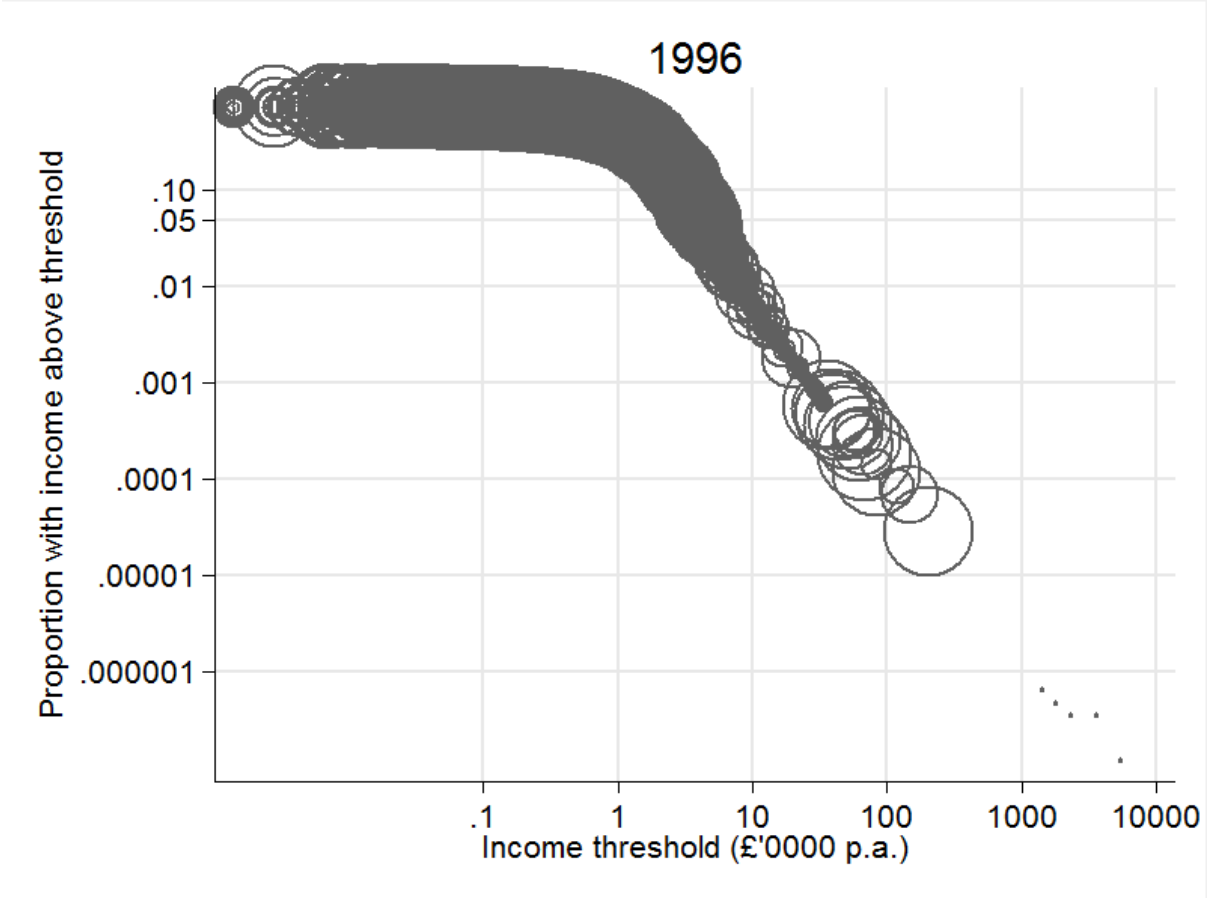
2010	105000	1.785494	1.774707	1.766454	.9978511	.9970427	.9991021
2010	110000	1.777386	1.770769	1.765715	.9978704	.9970337	.9990103
2010	115000	1.770676	1.767631	1.765324	.9978651	.9969791	.9989204
2010	120000	1.764302	1.764776	1.765182	.9978679	.9969169	.9988288
2010	125000	1.758188	1.762148	1.765256	.9978948	.9968666	.9987398
2010	130000	1.751557	1.759403	1.765539	.9979716	.996846	.9986476
2010	135000	1.744557	1.756617	1.766058	.9980857	.9968357	.9985508
2010	140000	1.737657	1.753983	1.766797	.9982211	.996823	.9984543
2010	145000	1.730541	1.751375	1.767785	.9983888	.9968108	.9983557
2010	150000	1.723528	1.74891	1.768985	.9985806	.9967961	.9982601
2010	155000	1.716486	1.746557	1.770453	.9987901	.9967651	.9981645
2010	160000	1.70936	1.744353	1.772312	.9989986	.9966893	.9980647
2010	165000	1.703208	1.742756	1.774536	.9991359	.9965293	.9979648
2010	170000	1.698755	1.741831	1.776621	.9992118	.9963534	.9978834
2010	175000	1.694807	1.741233	1.778925	.9992626	.9961485	.9978038
2010	180000	1.691576	1.740925	1.781189	.9992931	.9959438	.9977337
2010	185000	1.688759	1.740829	1.783522	.999311	.995733	.9976684
2010	190000	1.686447	1.74102	1.78599	.9993111	.995513	.9976059
2010	195000	1.684625	1.741405	1.788415	.9993007	.9953014	.9975497
2010	200000	1.683296	1.741987	1.790798	.9992806	.9951009	.9974986
2010	210000	1.681251	1.743444	1.795631	.9992298	.994709	.9974057
2010	220000	1.679924	1.745085	1.800215	.9991726	.9943532	.9973289
2010	230000	1.679366	1.747228	1.805135	.9991029	.9940022	.9972569
2010	240000	1.67932	1.749335	1.809528	.999037	.9937068	.9971996
2010	250000	1.679477	1.751694	1.814277	.9989636	.9933967	.9971448
2010	260000	1.679827	1.753909	1.818565	.9988954	.9931262	.9971009
2010	270000	1.680567	1.756584	1.823462	.9988171	.992836	.997057
2010	280000	1.68188	1.759251	1.827787	.9987528	.9926198	.9970224
2010	290000	1.683206	1.761917	1.832117	.9986866	.9924039	.9969912
2010	300000	1.684676	1.764572	1.836291	.9986241	.9922087	.9969641
2010	400000	1.708503	1.794199	1.875638	.9982446	.9911646	.9967797
2010	500000	1.734657	1.821174	1.906798	.9982275	.991017	.9966096
2010	600000	1.758726	1.844952	1.932922	.9983437	.9911214	.9964098

The relationship between R^2 and threshold in 2007 is shown below:

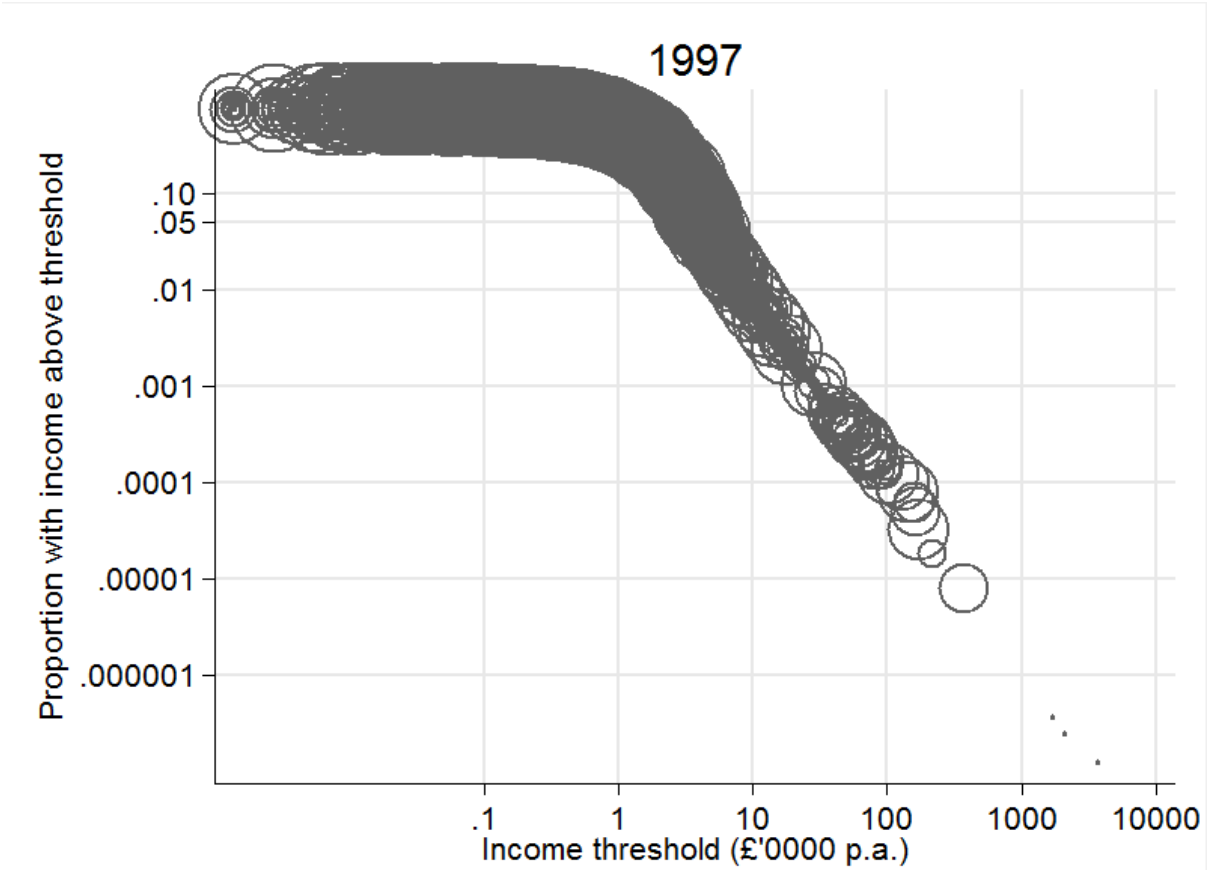


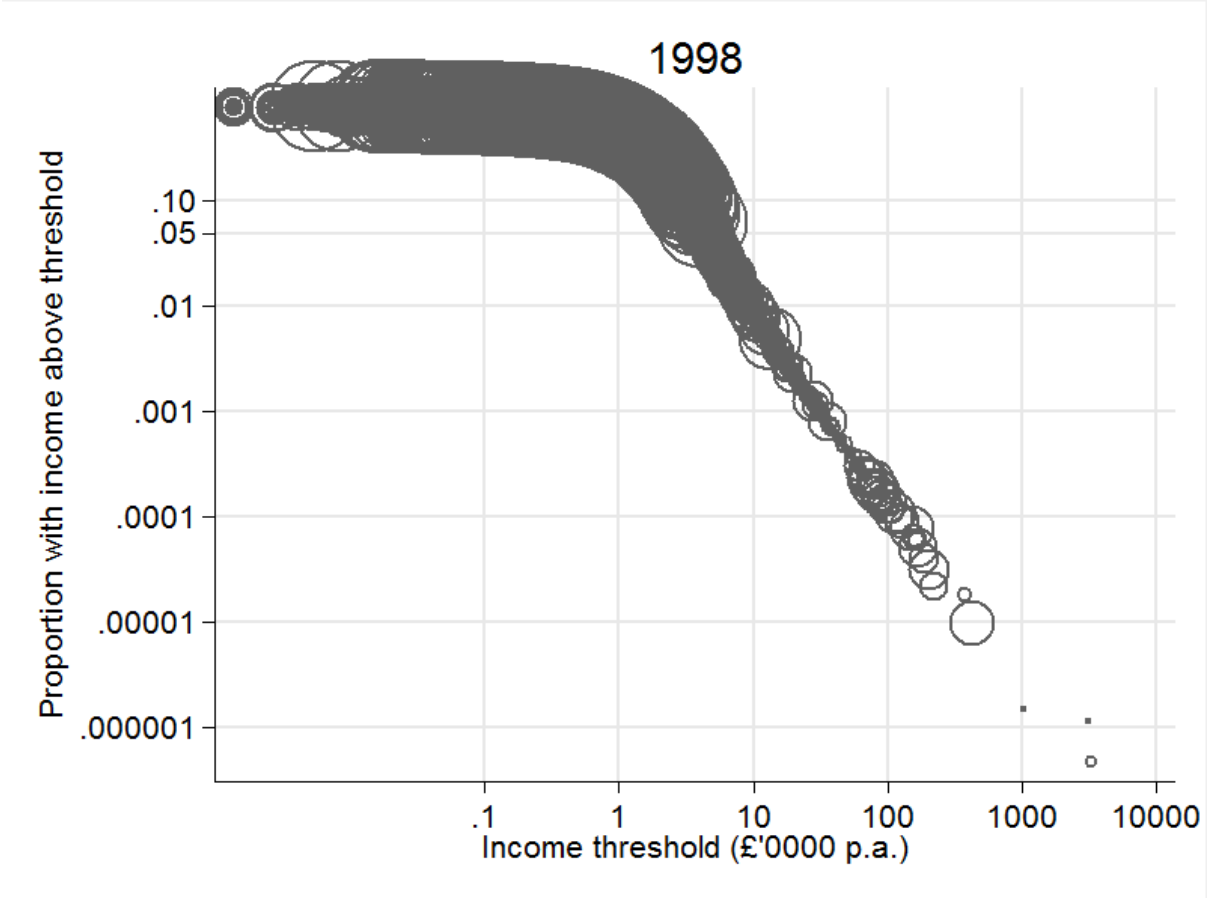
Zipf plots, by year: size of marker proportional to survey weight (pareto11)



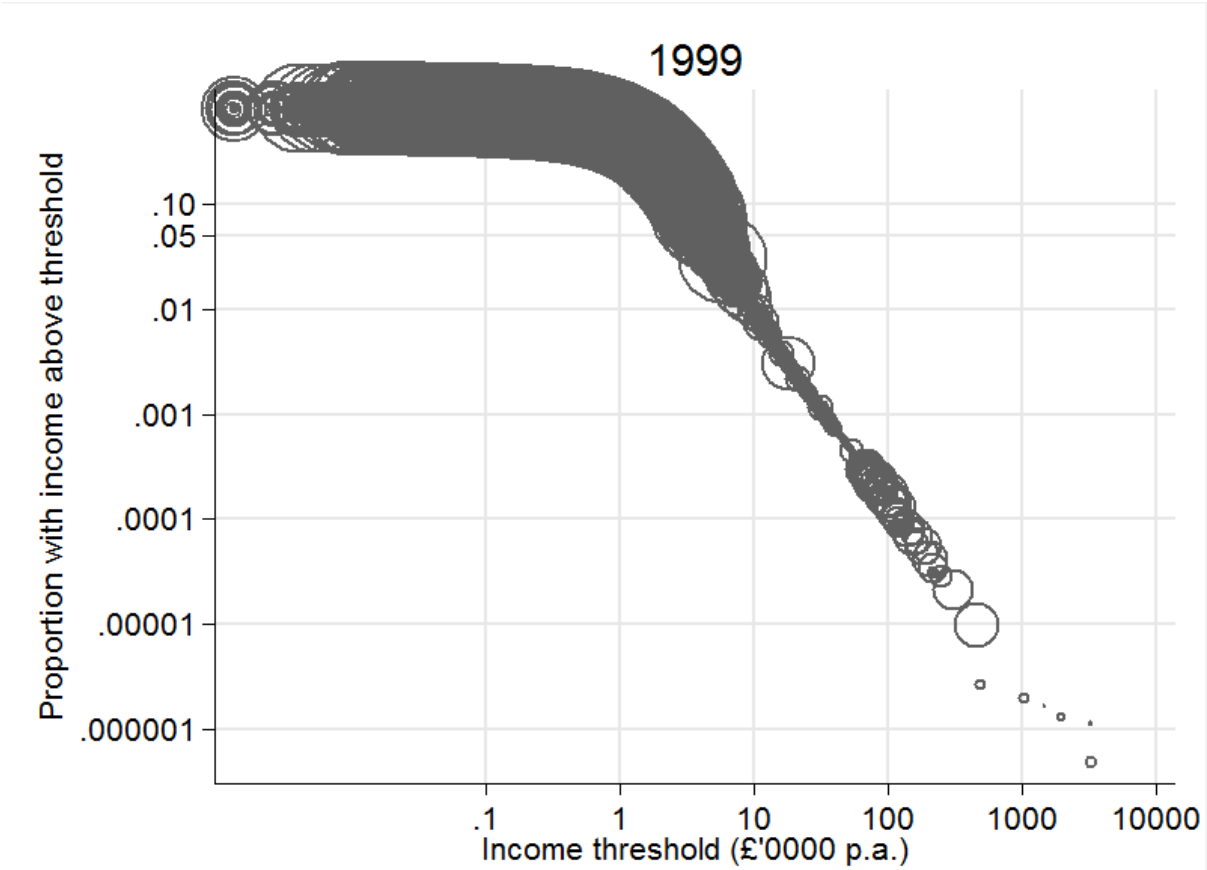


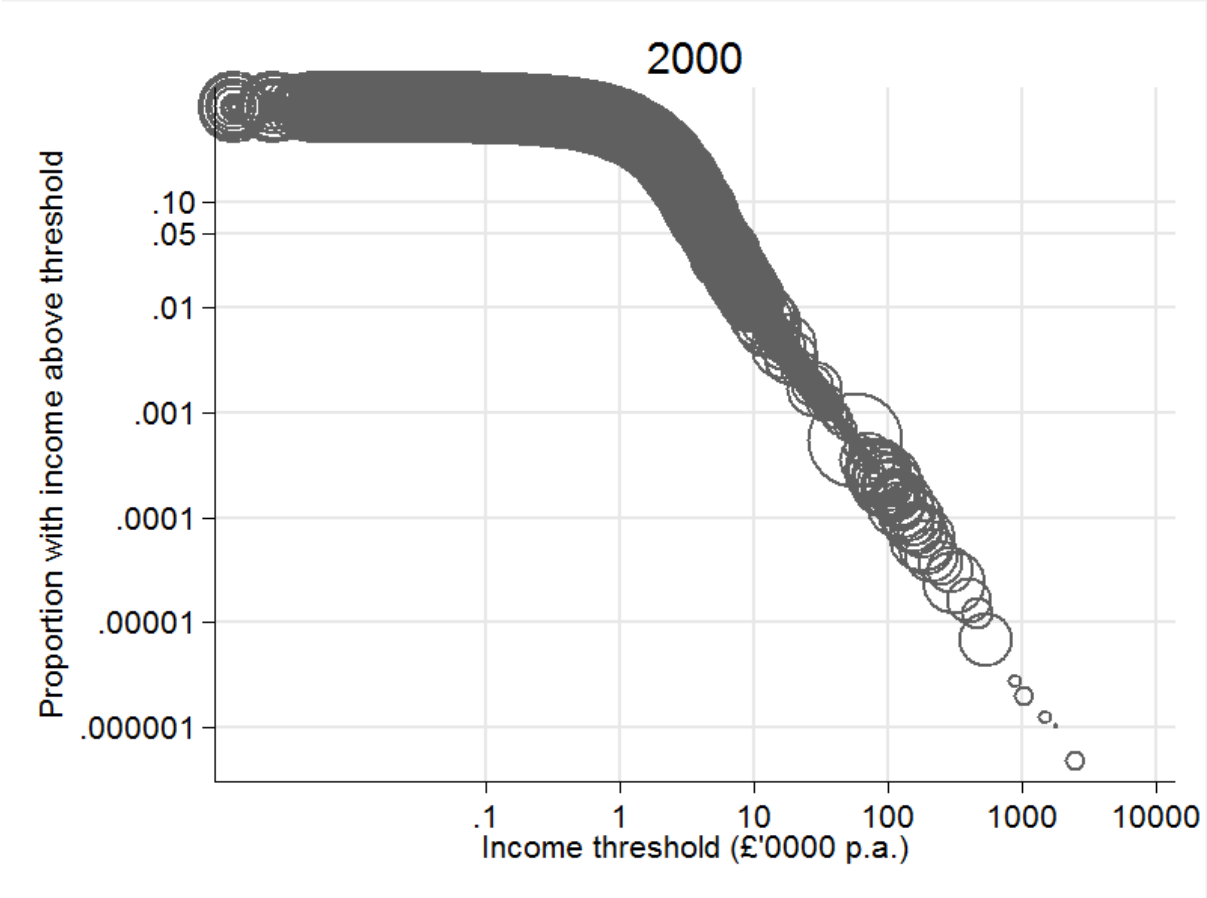
Zipf plots, by year: size of marker proportional to survey weight (pareto11)



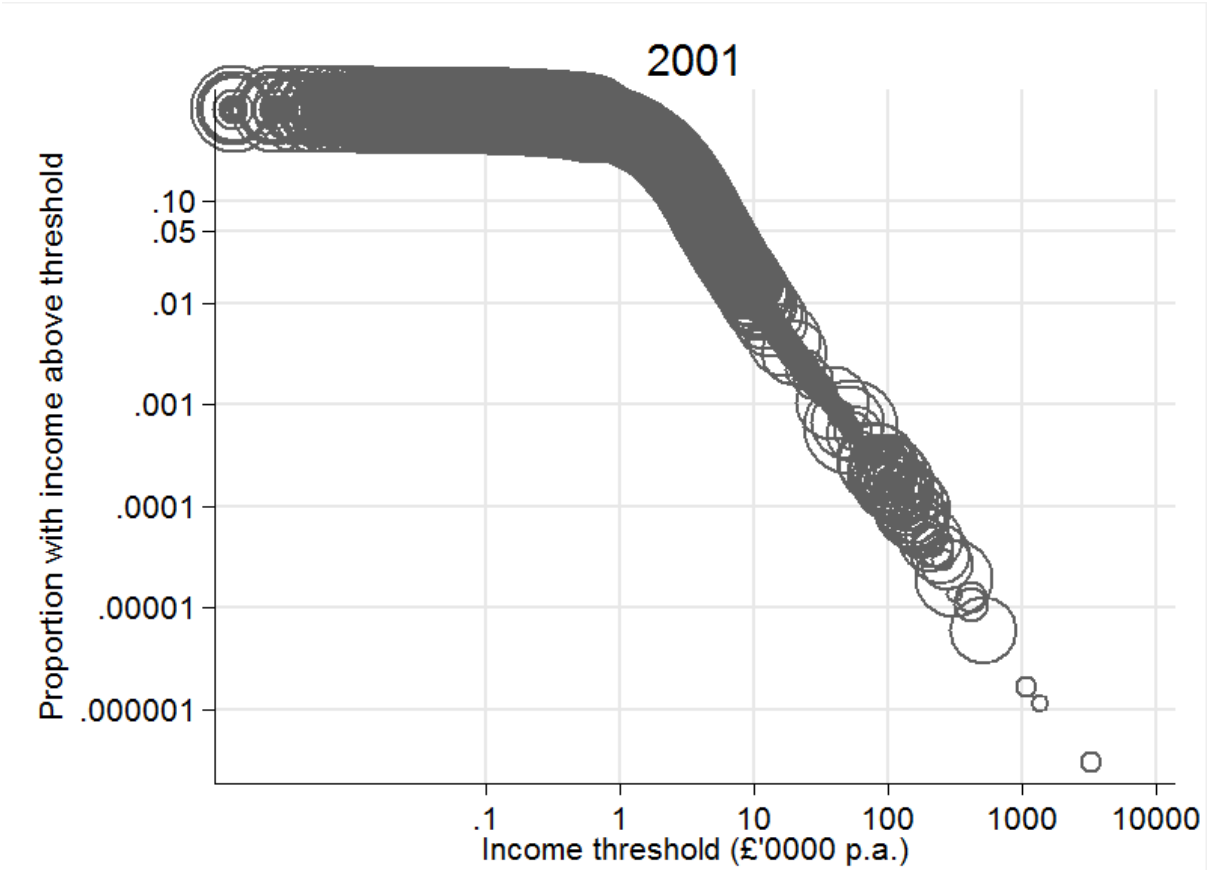


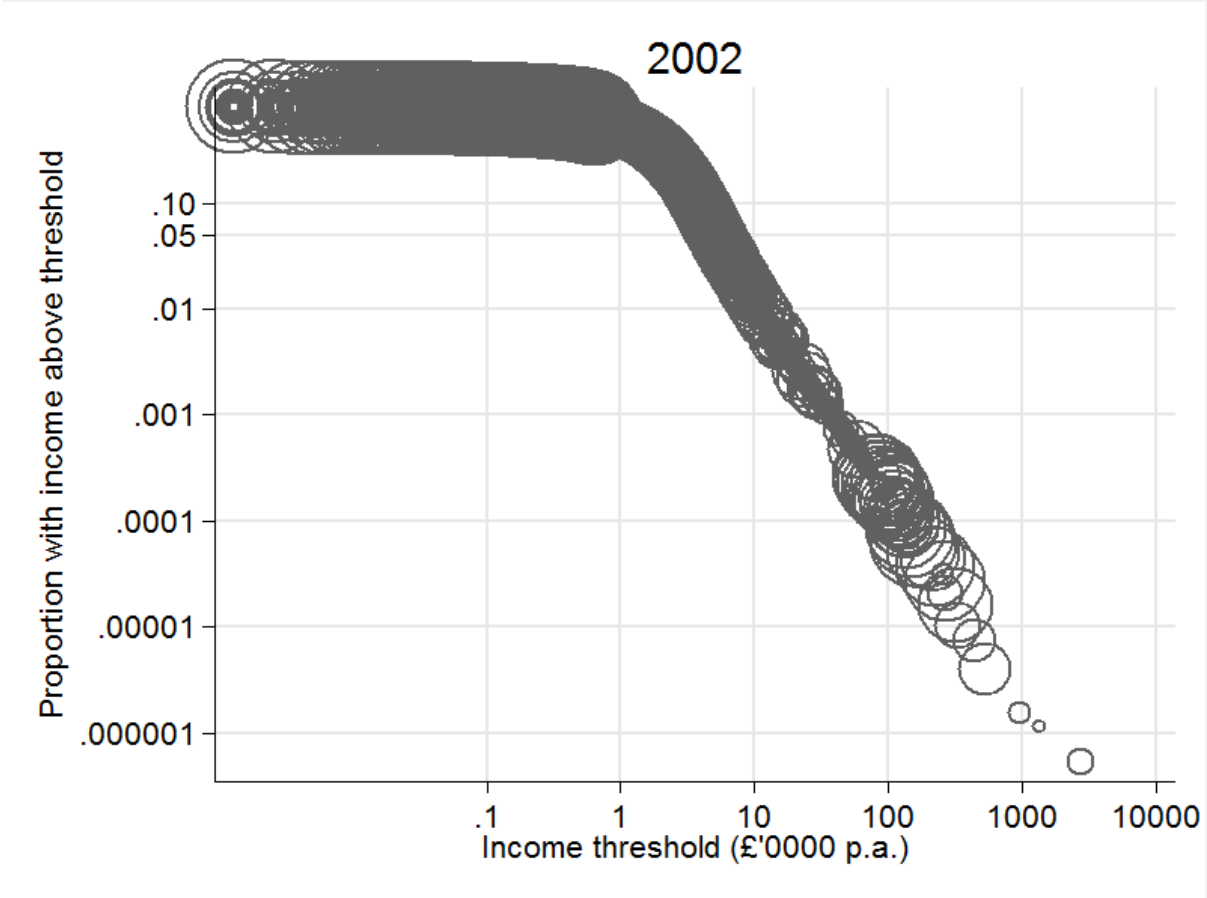
Zipf plots, by year: size of marker proportional to survey weight (pareto11)



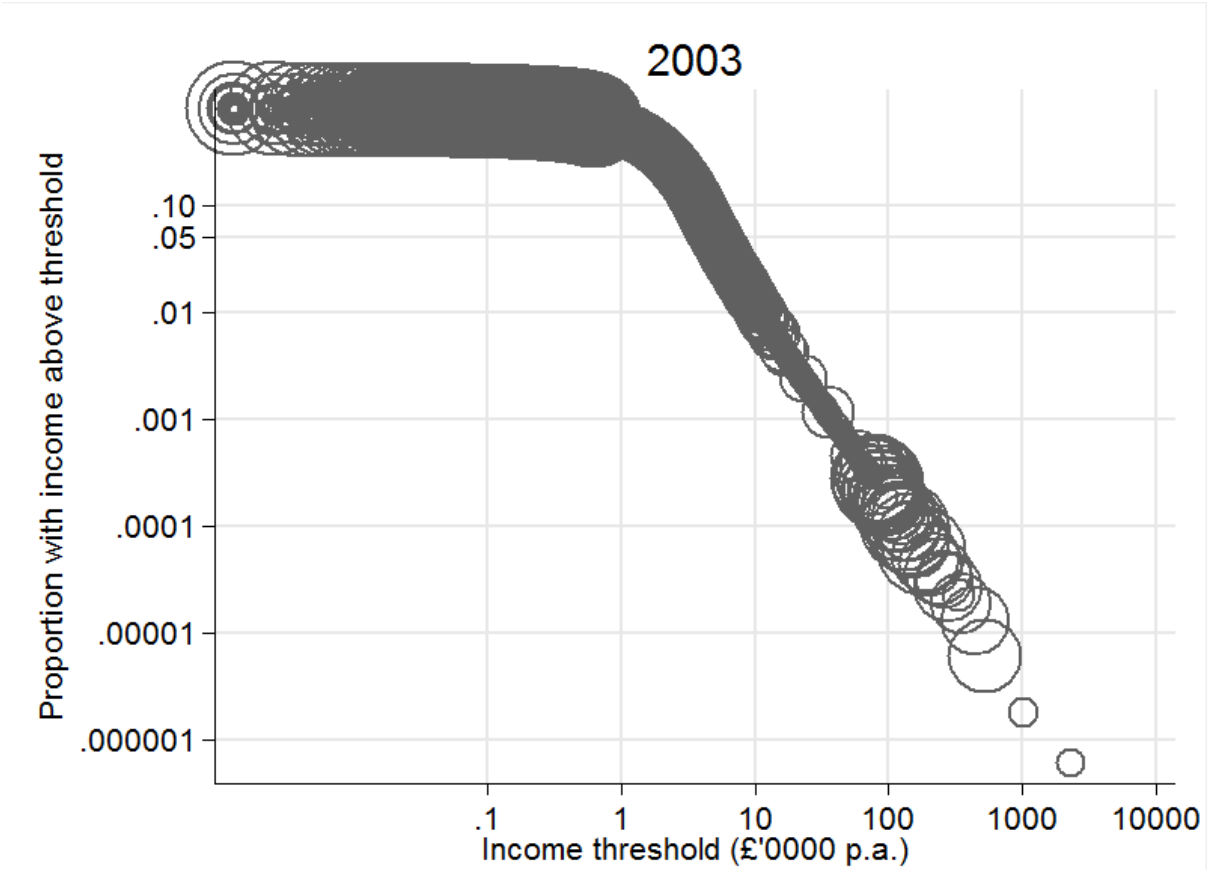


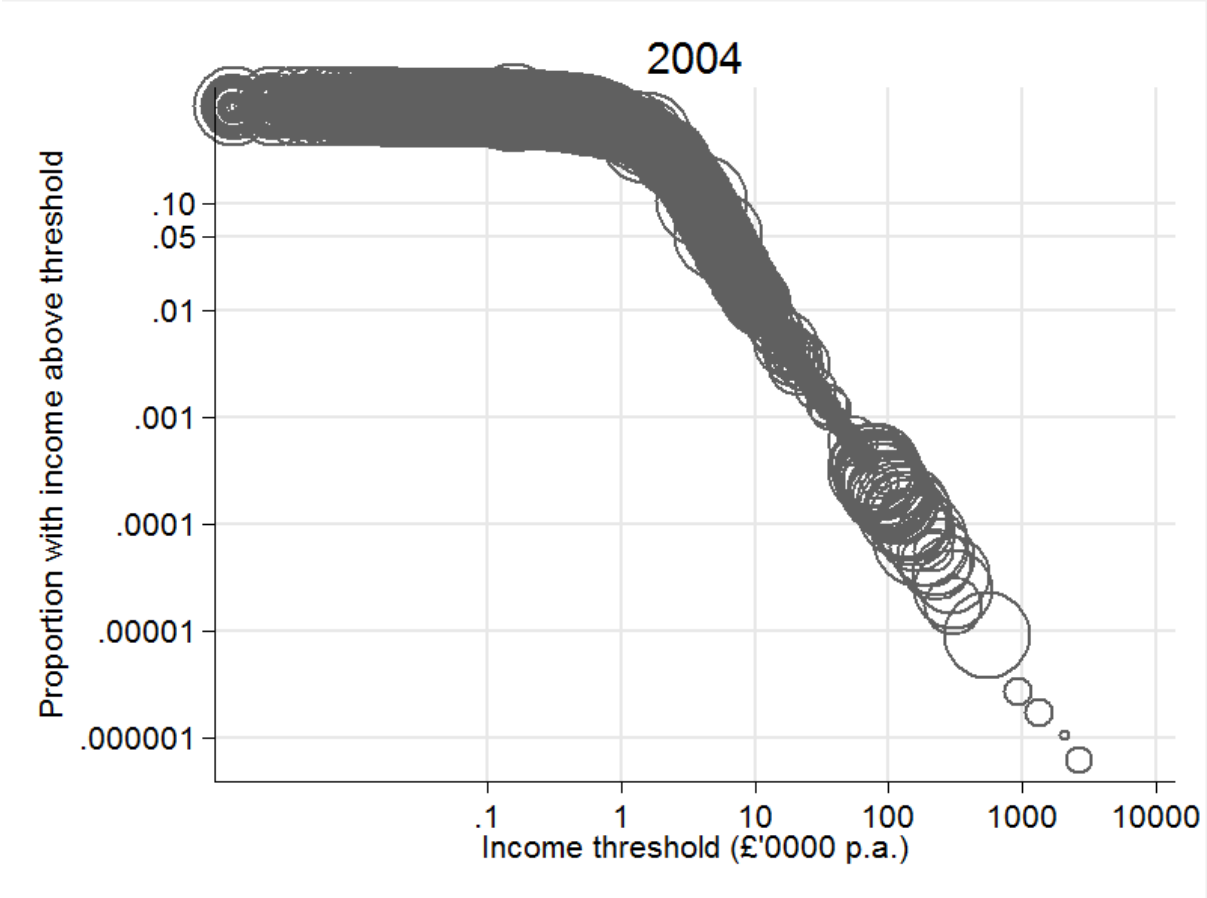
Zipf plots, by year: size of marker proportional to survey weight (pareto11)



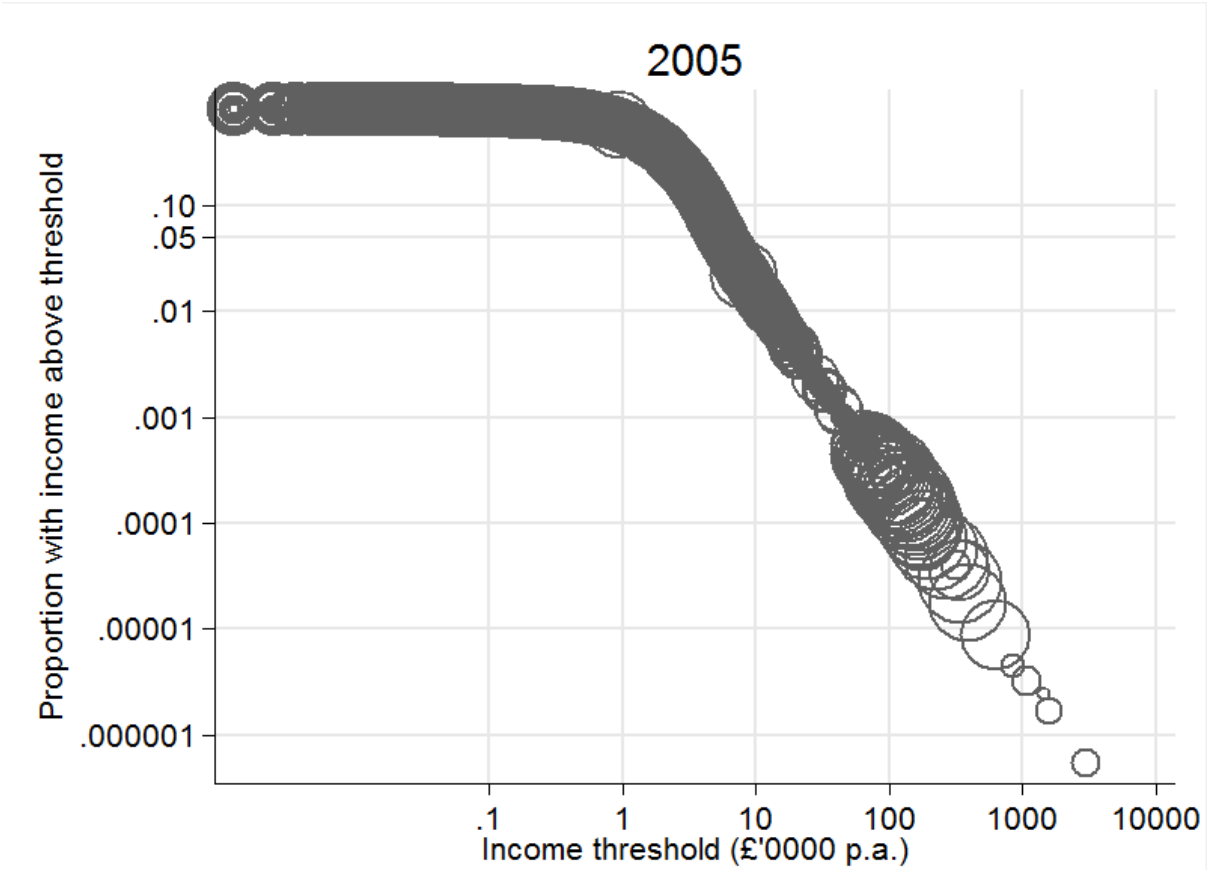


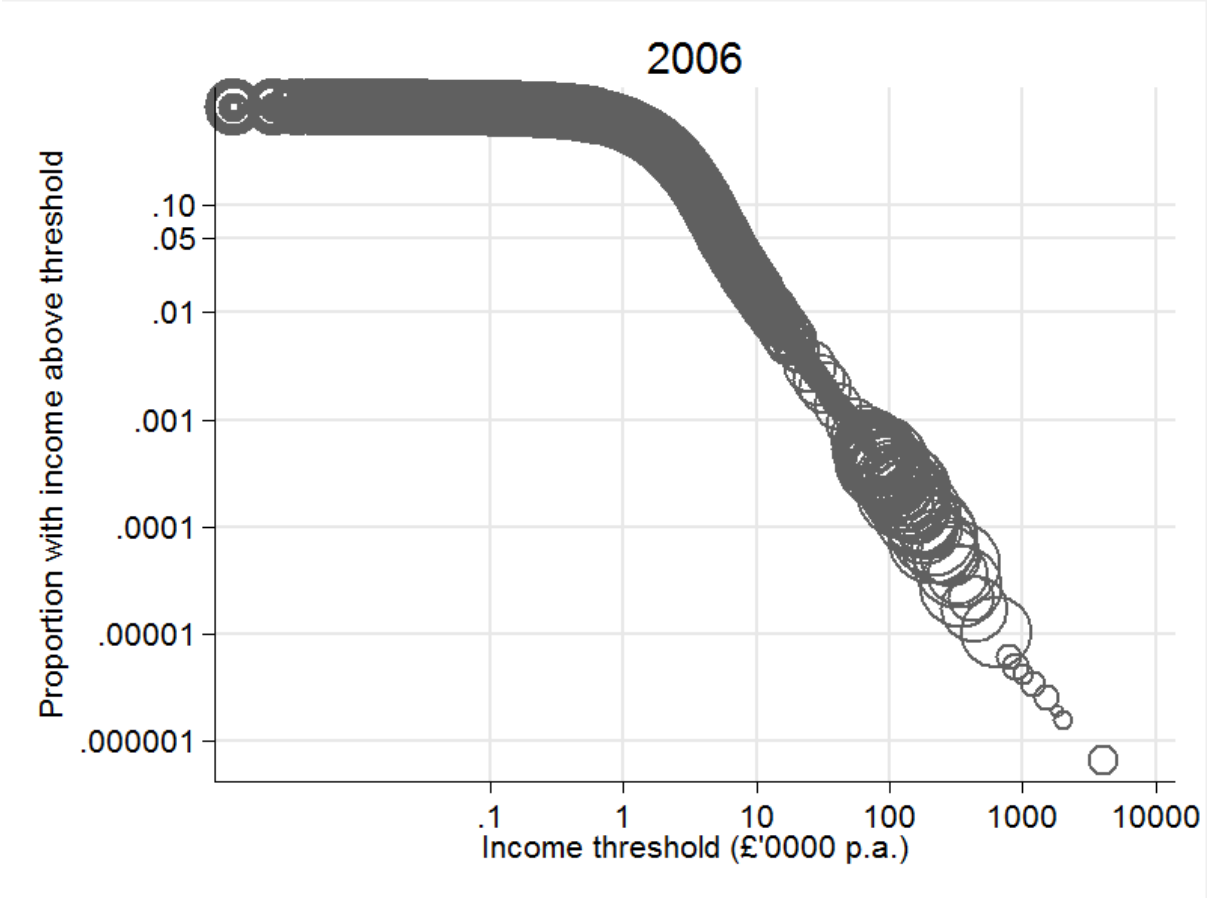
Zipf plots, by year: size of marker proportional to survey weight (pareto11)



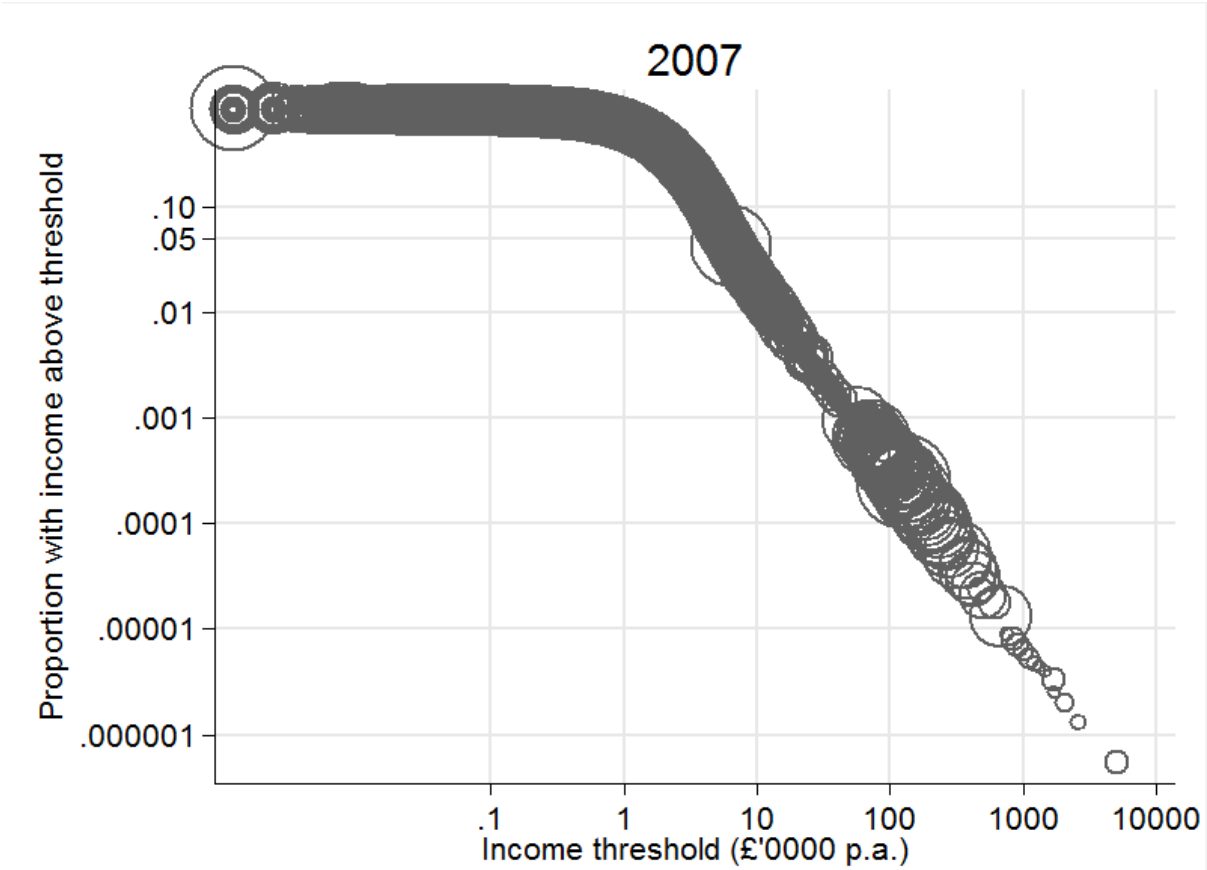


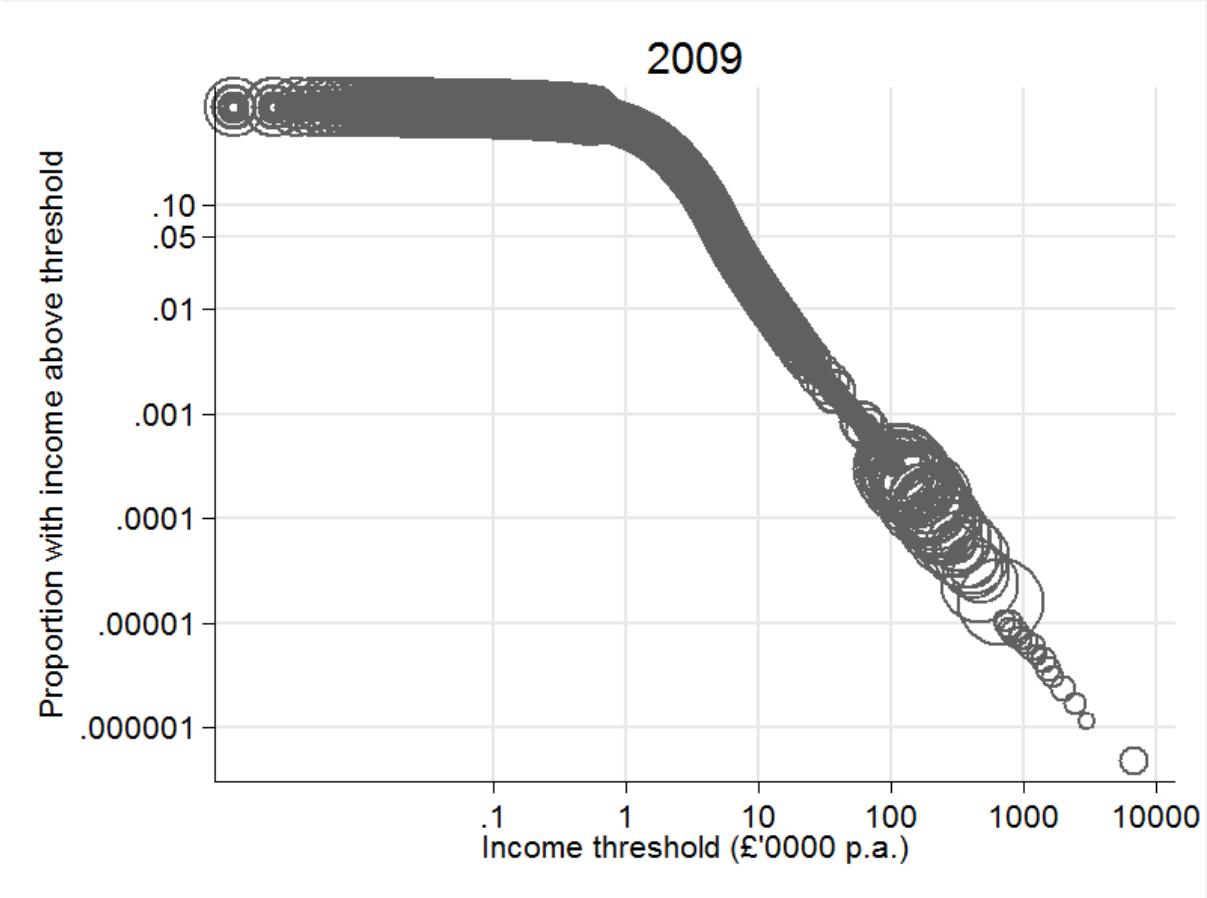
Zipf plots, by year: size of marker proportional to survey weight (pareto11)



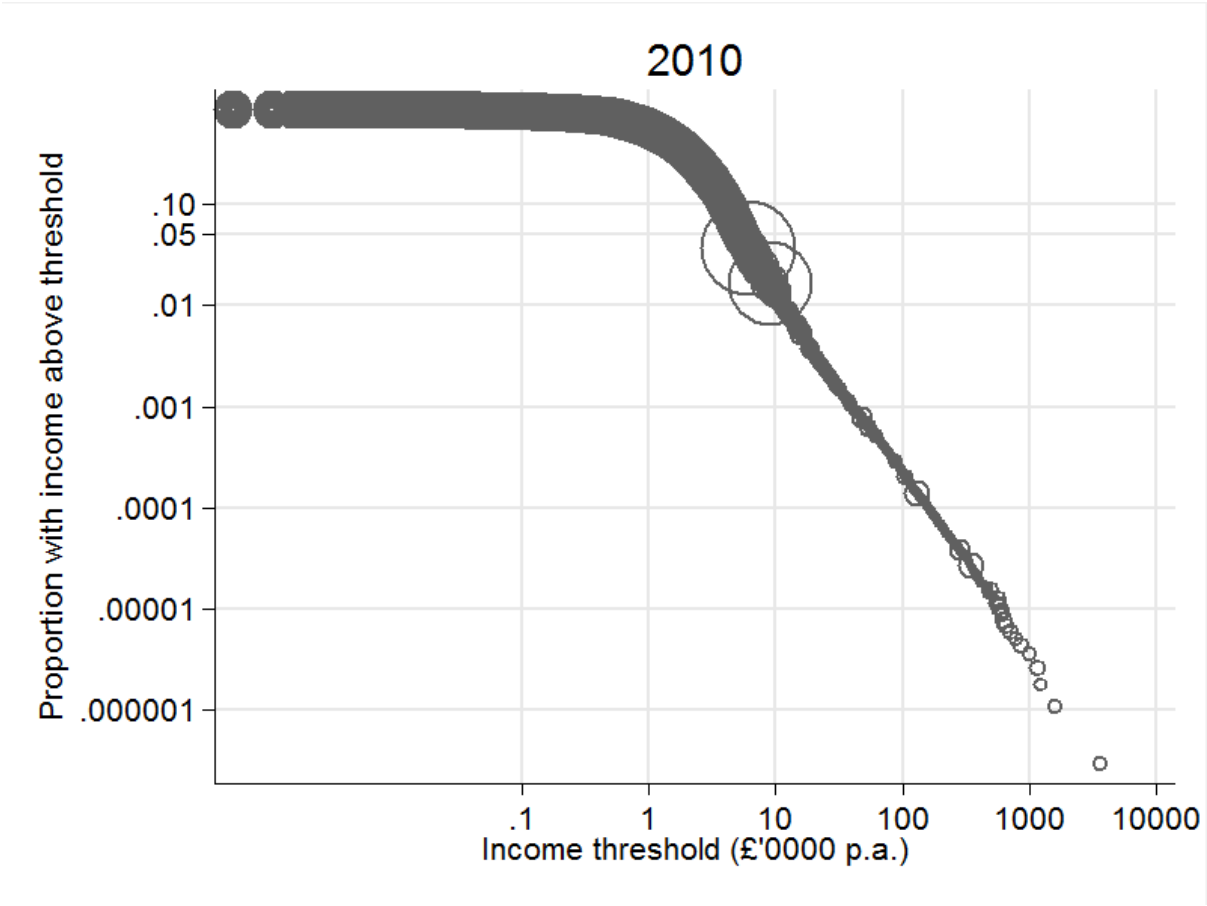


Zipf plots, by year: size of marker proportional to survey weight (pareto11)



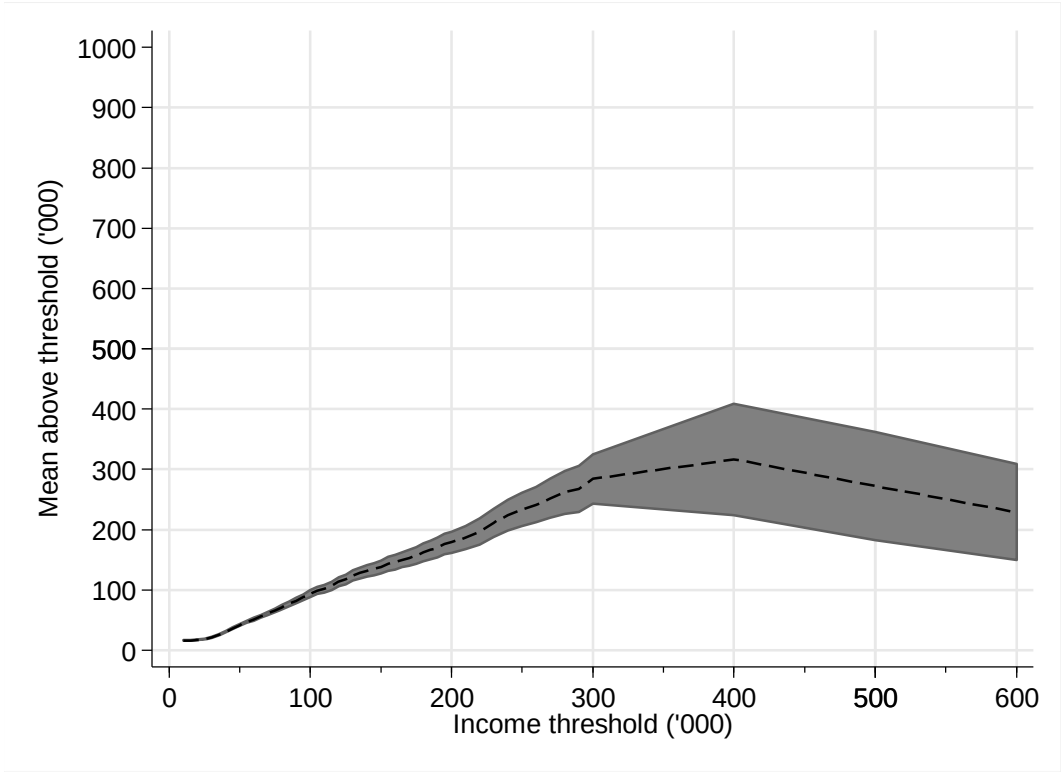


Zipf plots, by year: size of marker proportional to survey weight (pareto11)

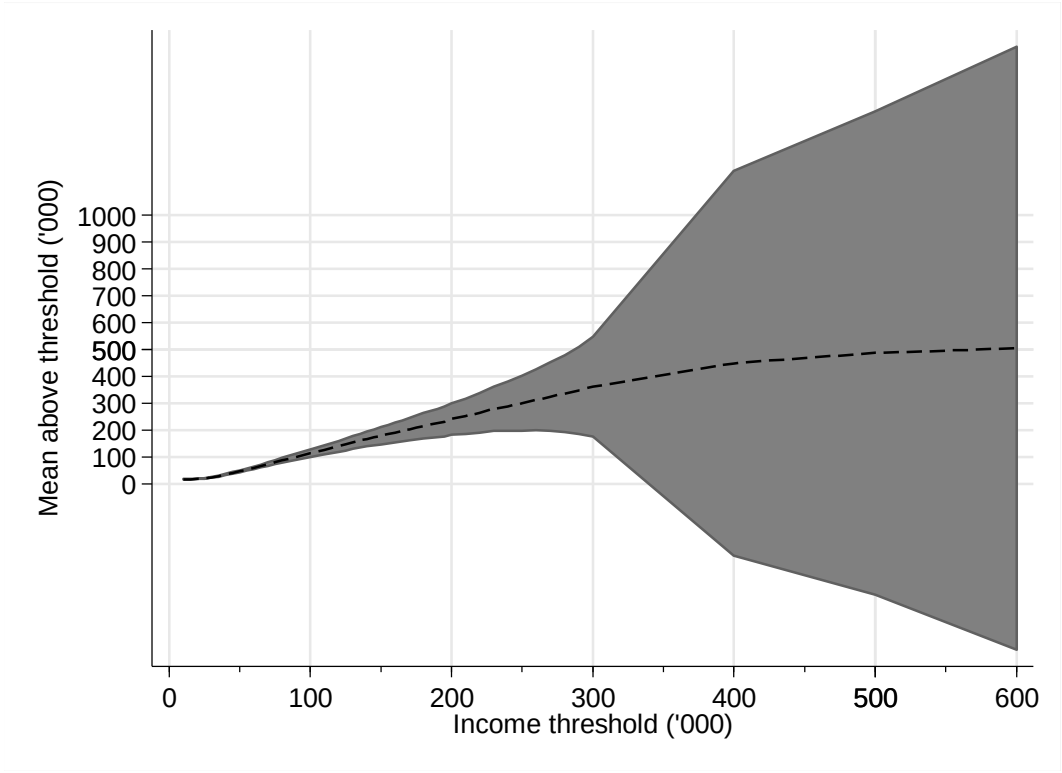


Mean excess plots, by year (pareto03)

1995

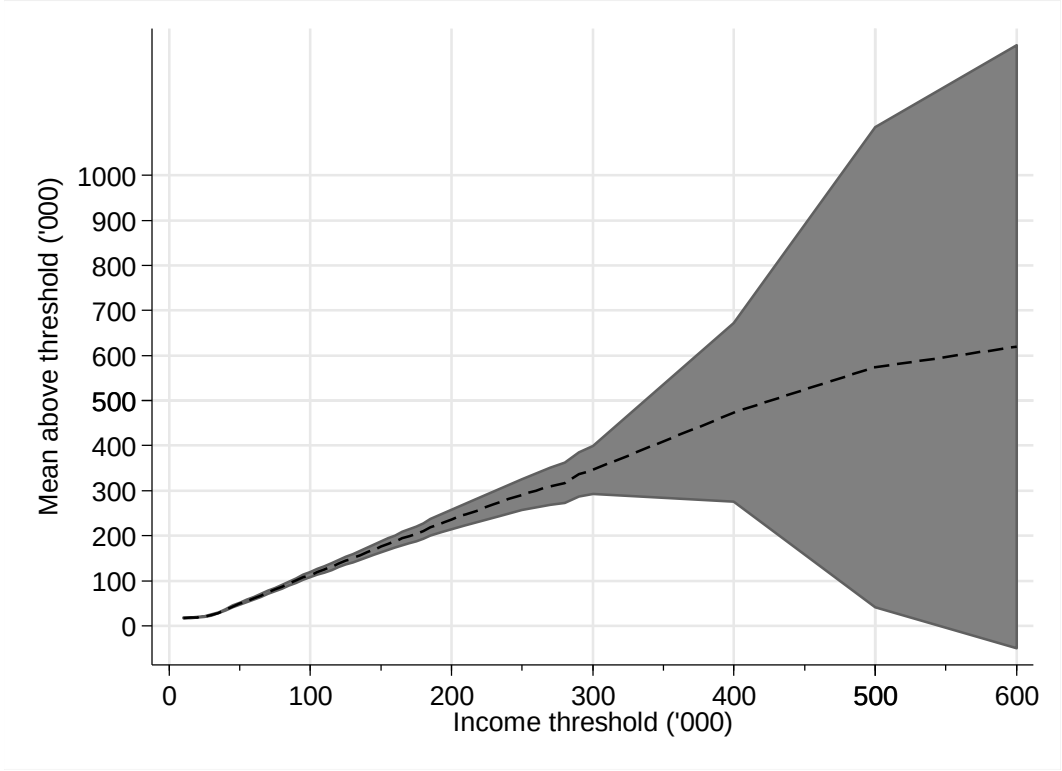


1996

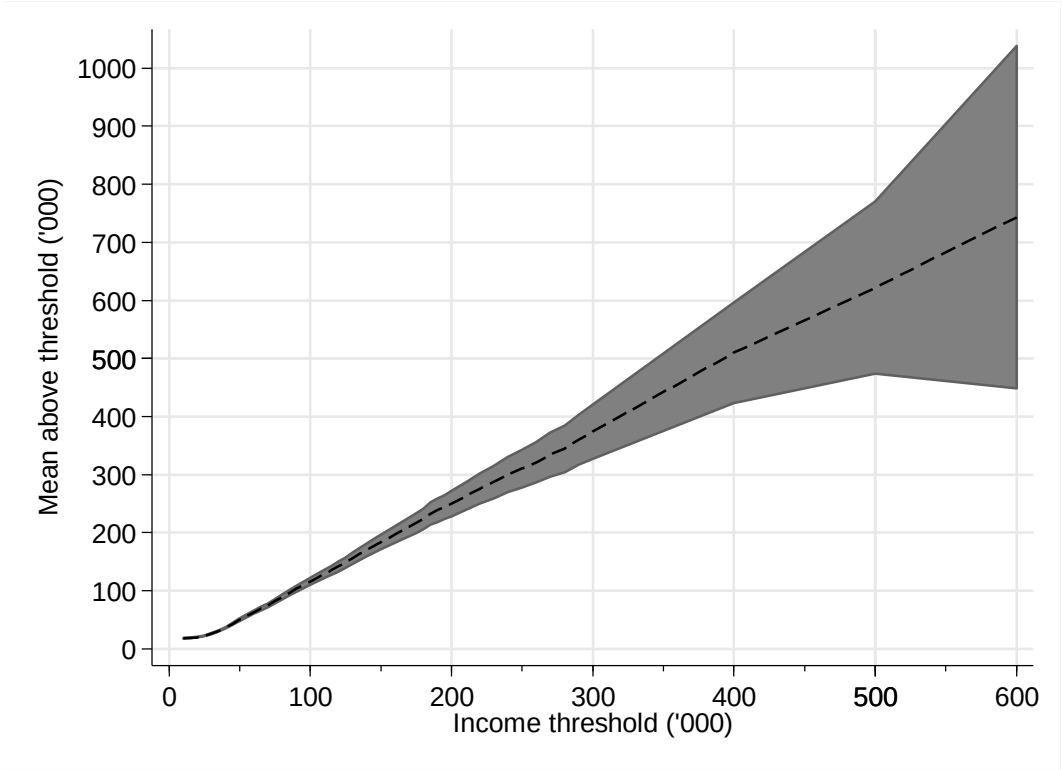


Mean excess plots, by year (pareto03)

1997

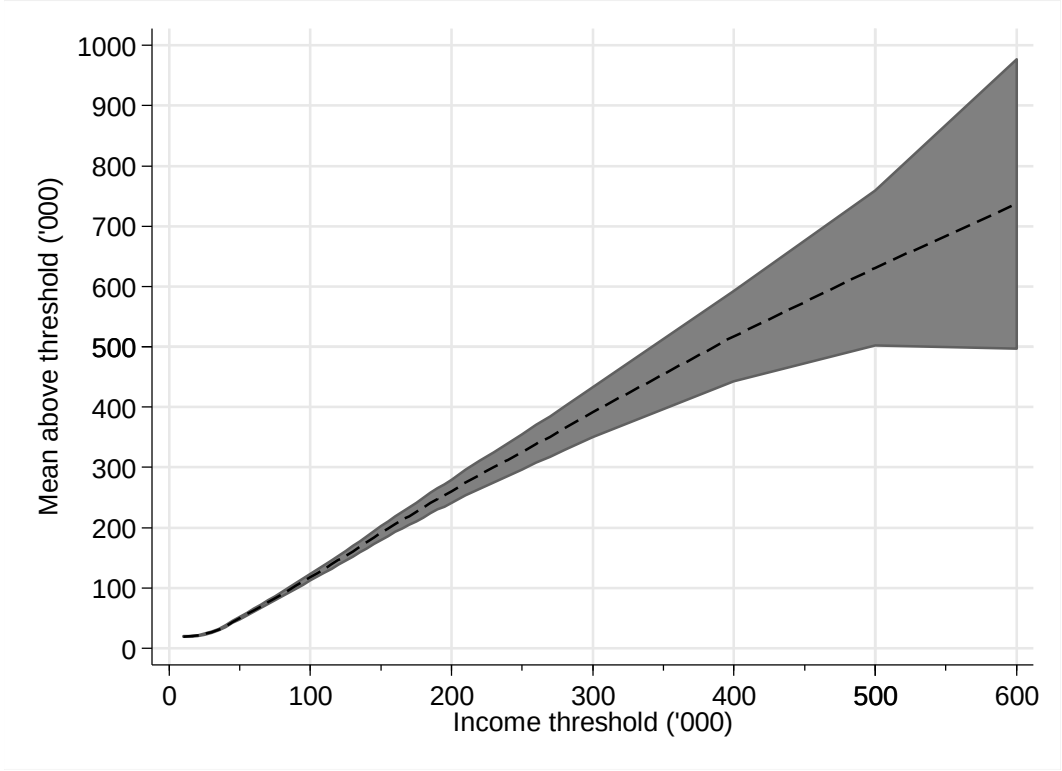


1998

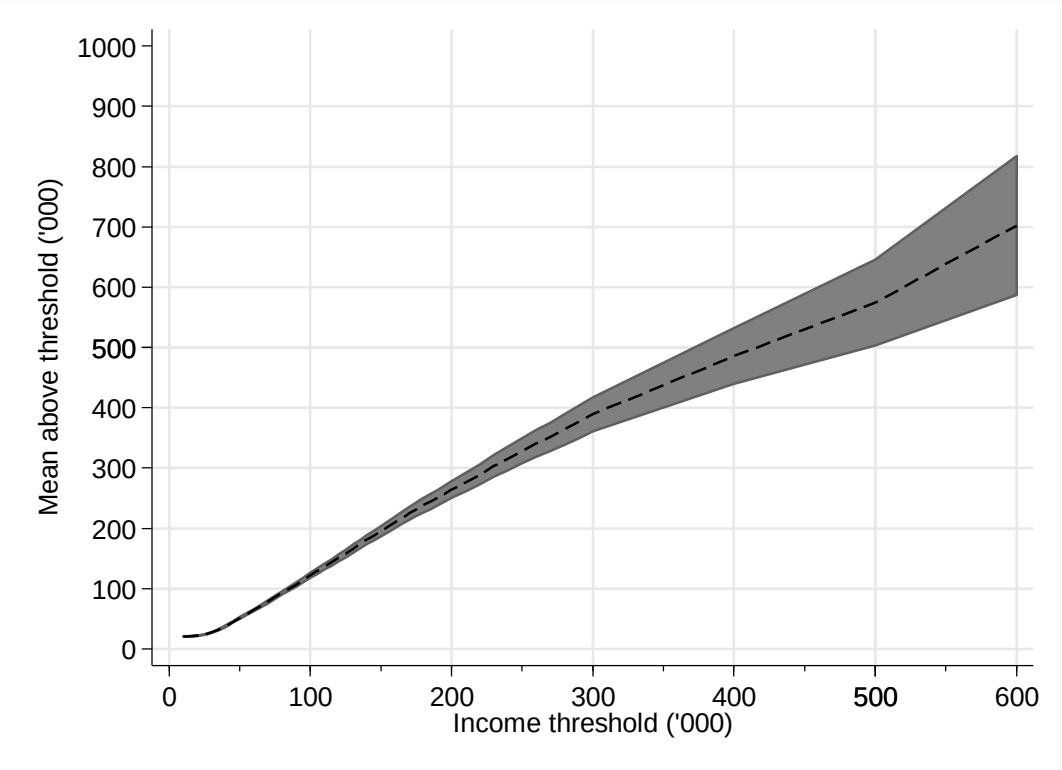


Mean excess plots, by year (pareto03)

1999

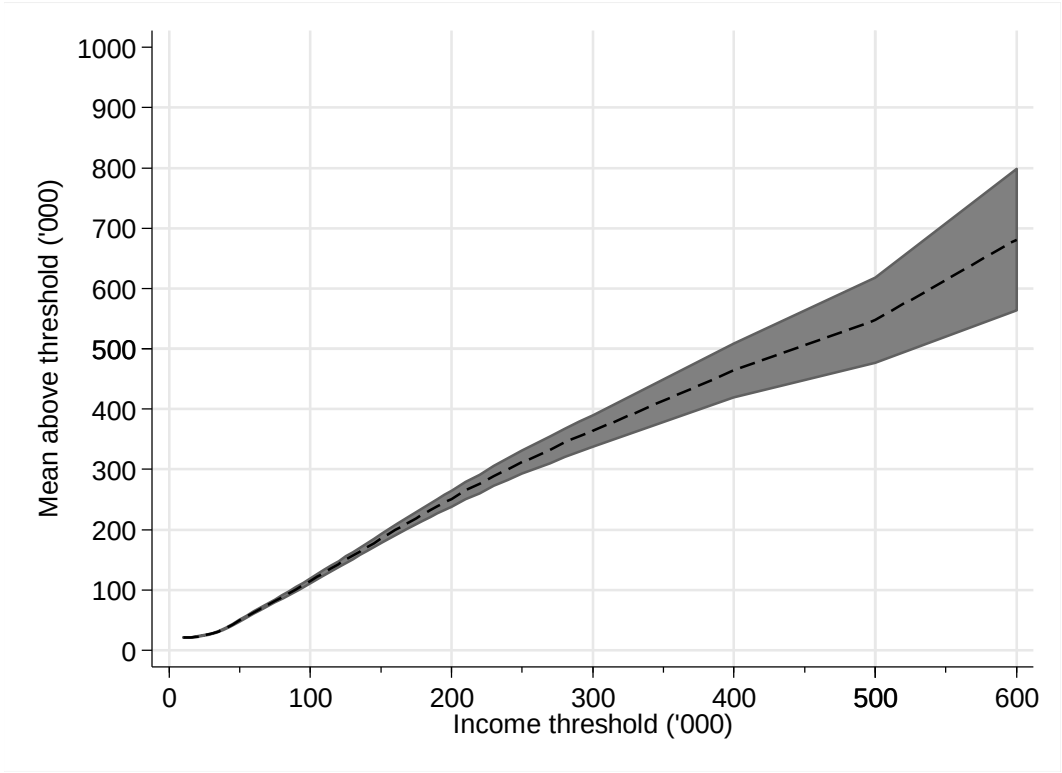


2000

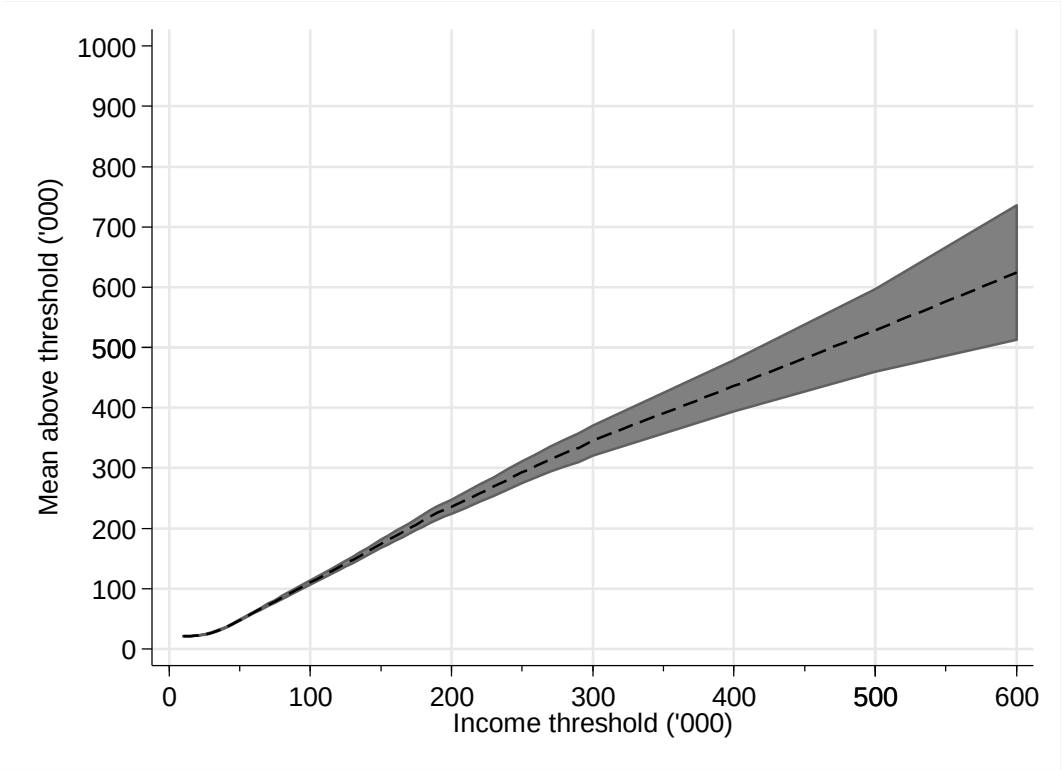


Mean excess plots, by year (pareto03)

2001

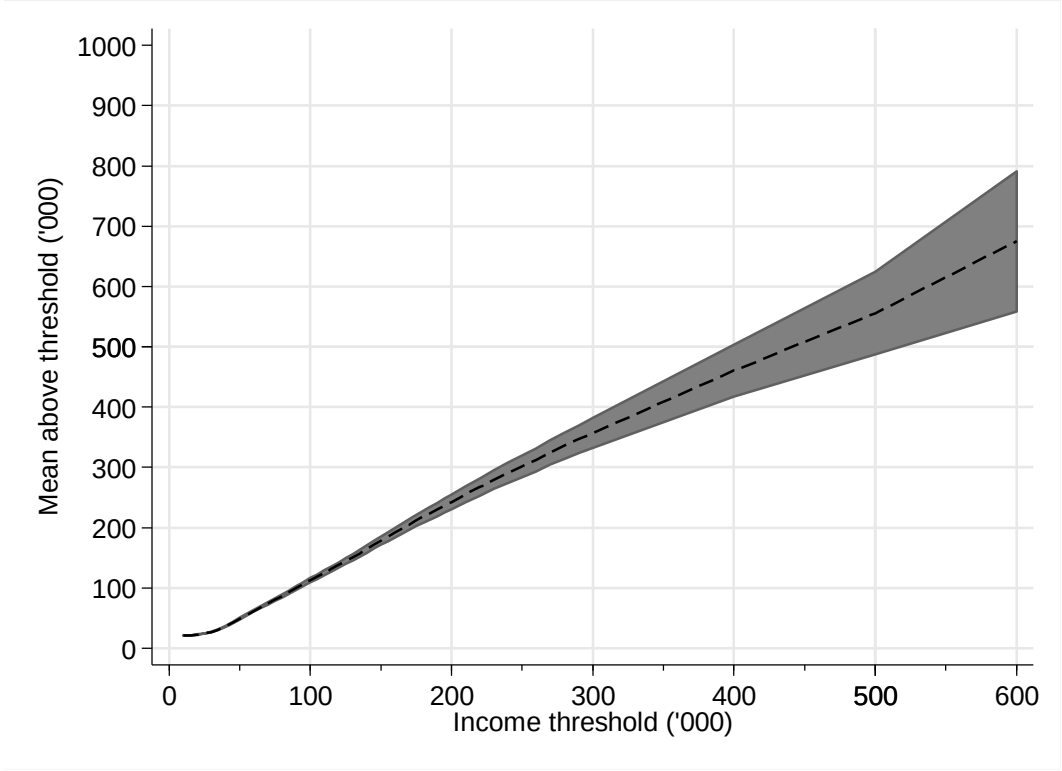


2002

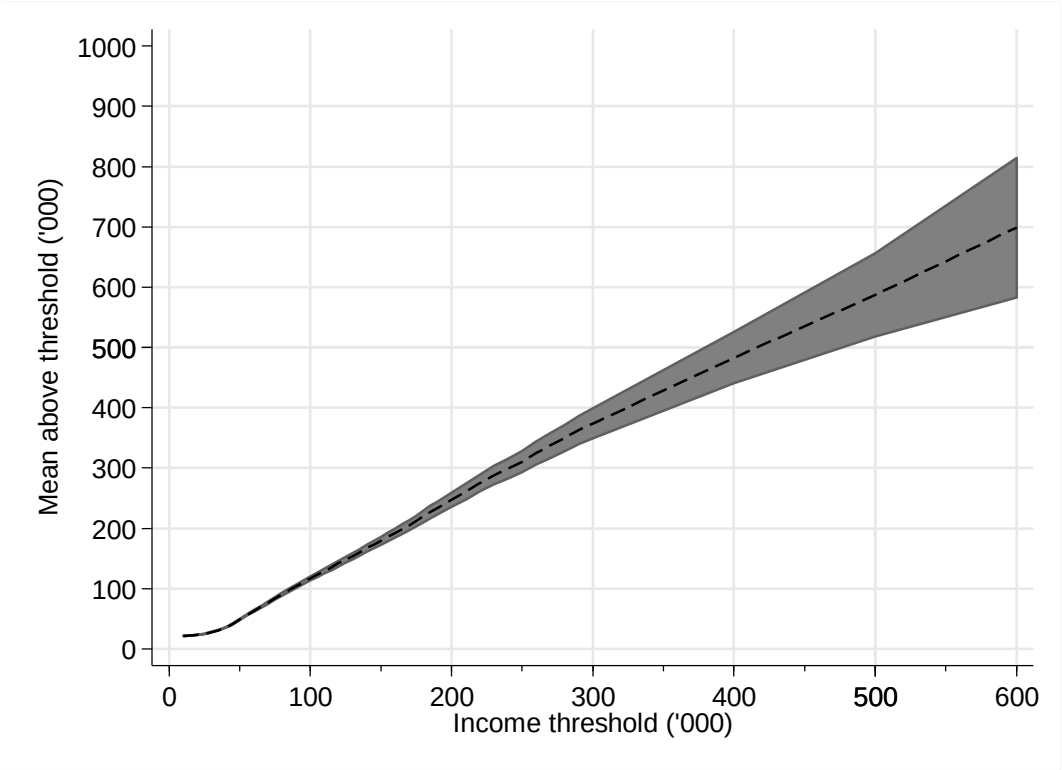


Mean excess plots, by year (pareto03)

2003

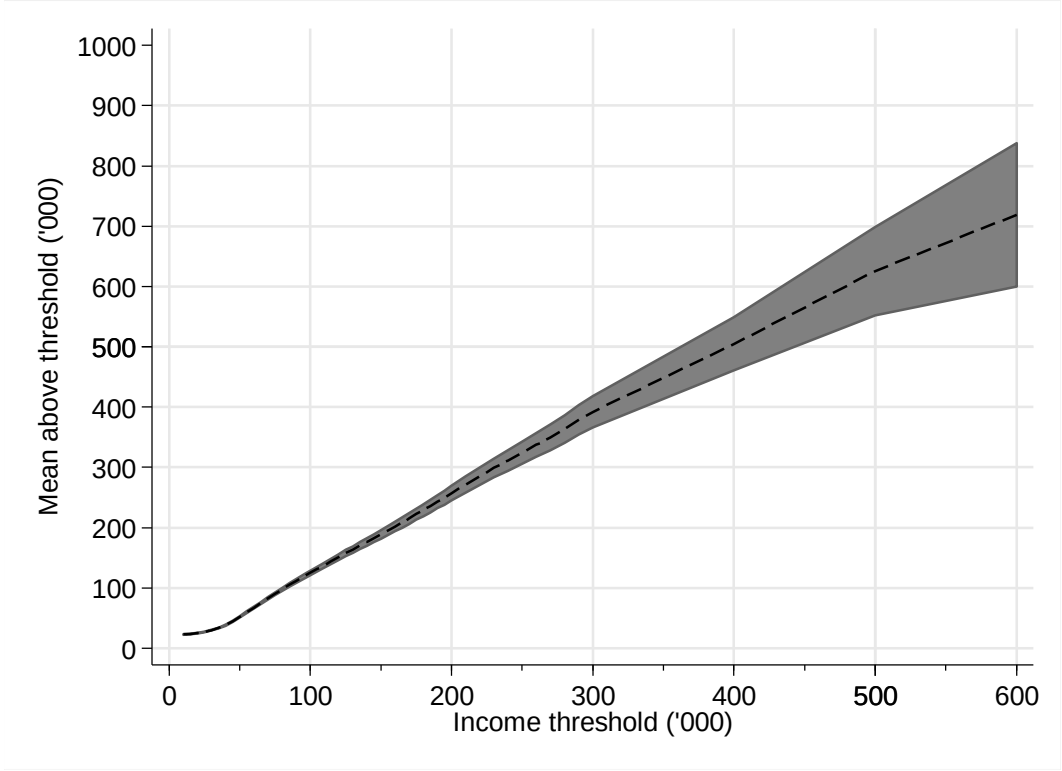


2004

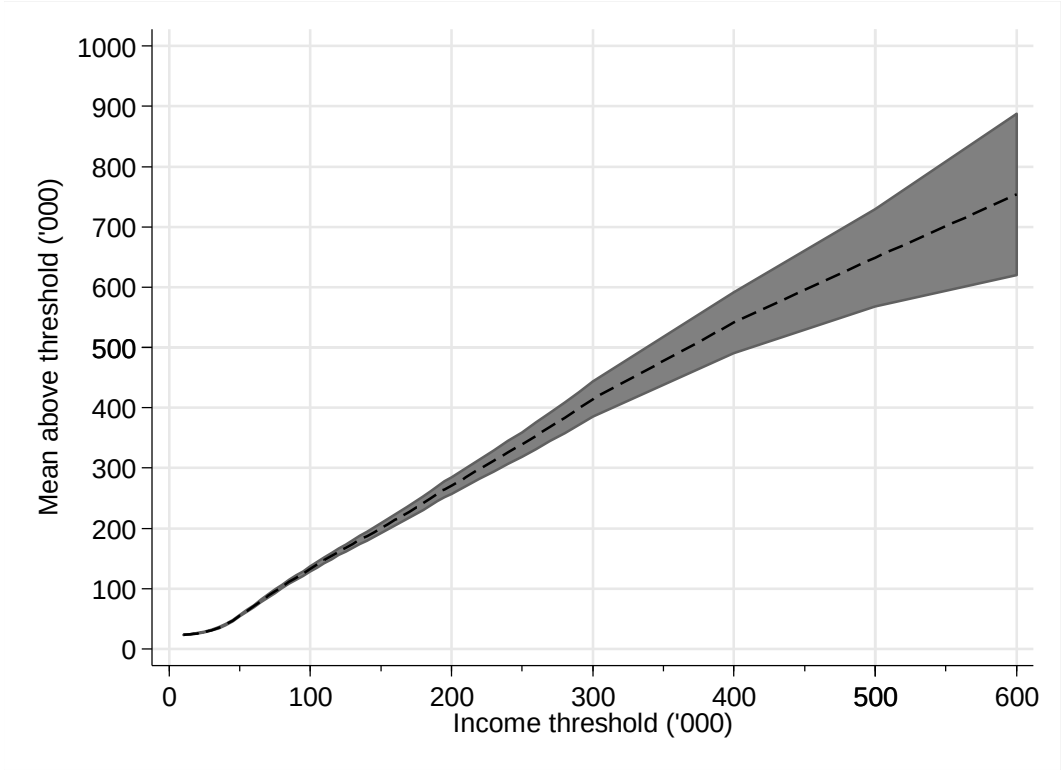


Mean excess plots, by year (pareto03)

2005

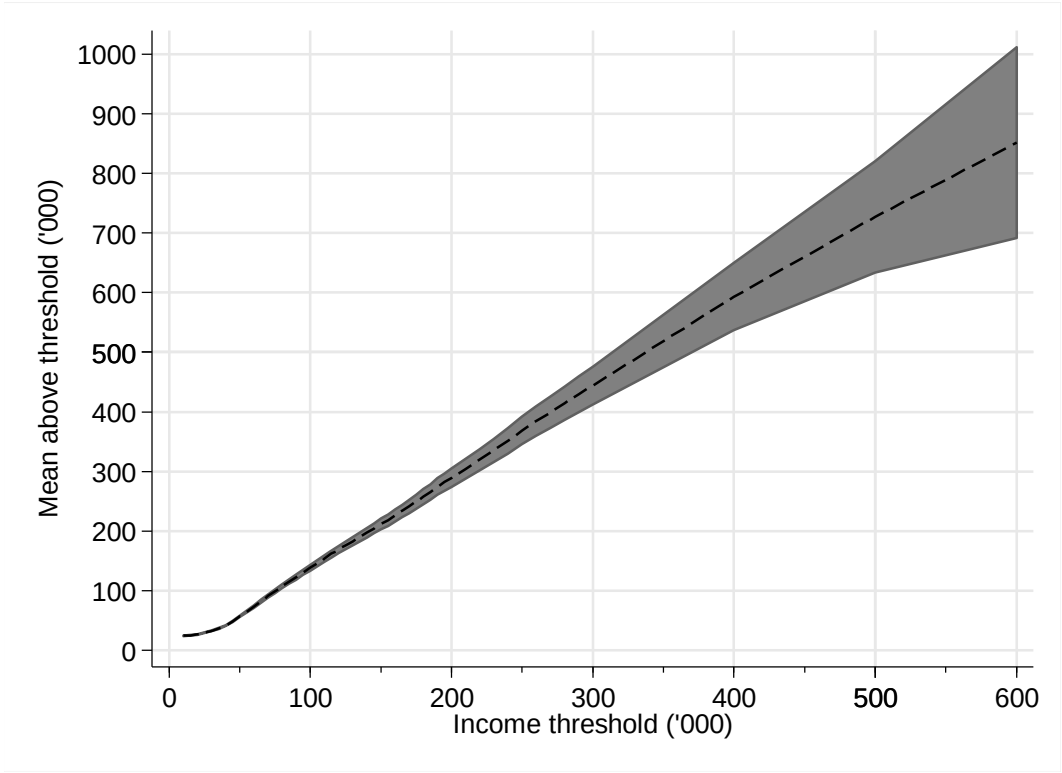


2006

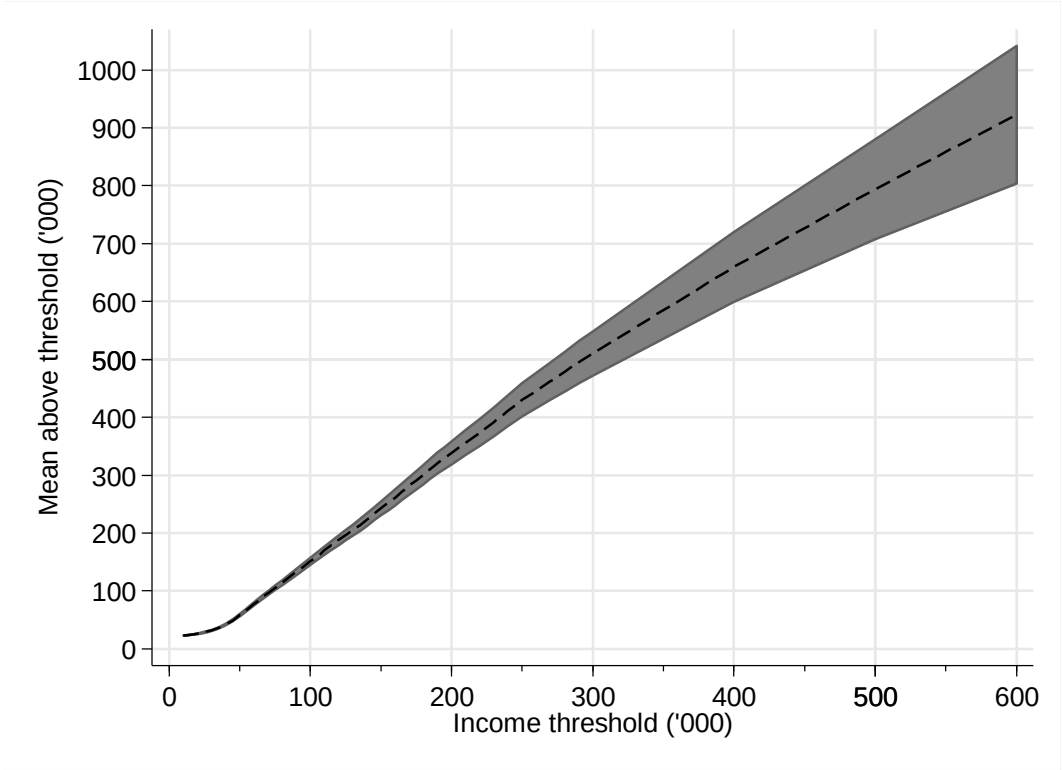


Mean excess plots, by year (pareto03)

2007

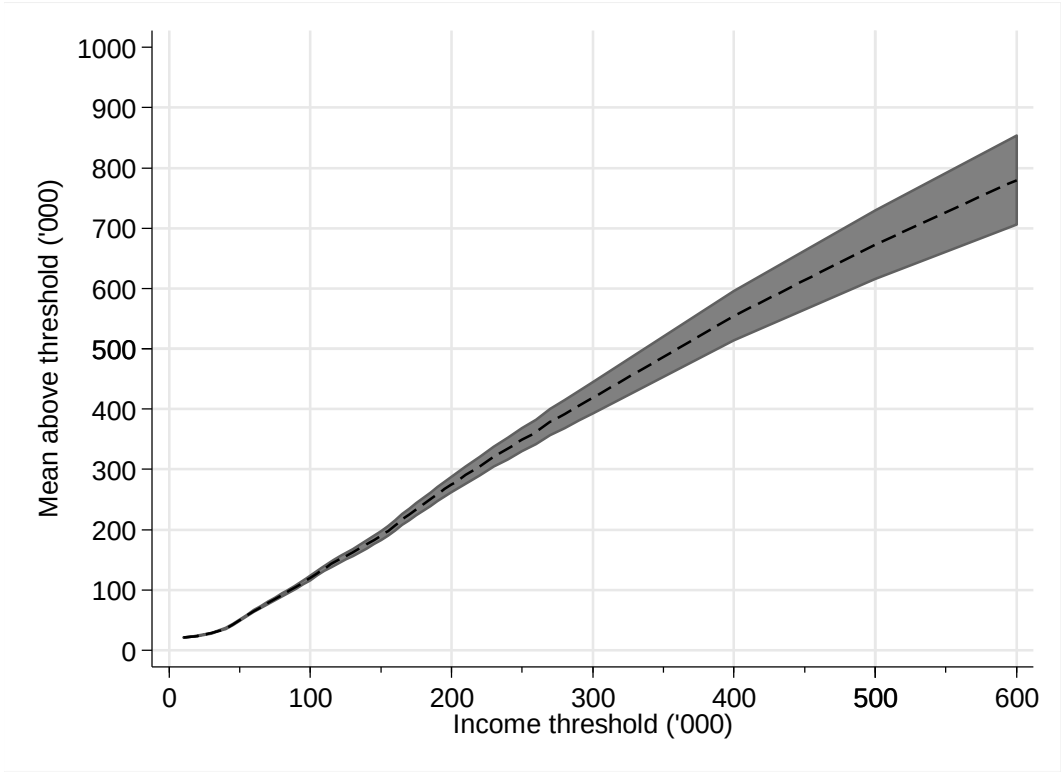


2009



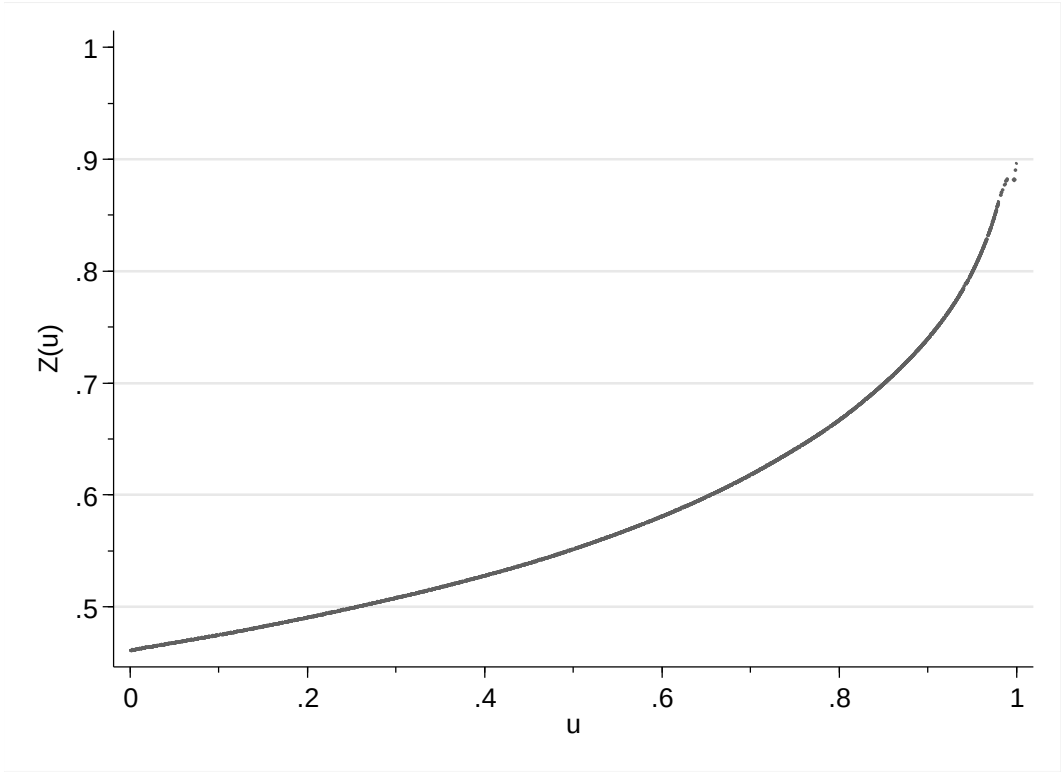
Mean excess plots, by year (pareto03)

2010

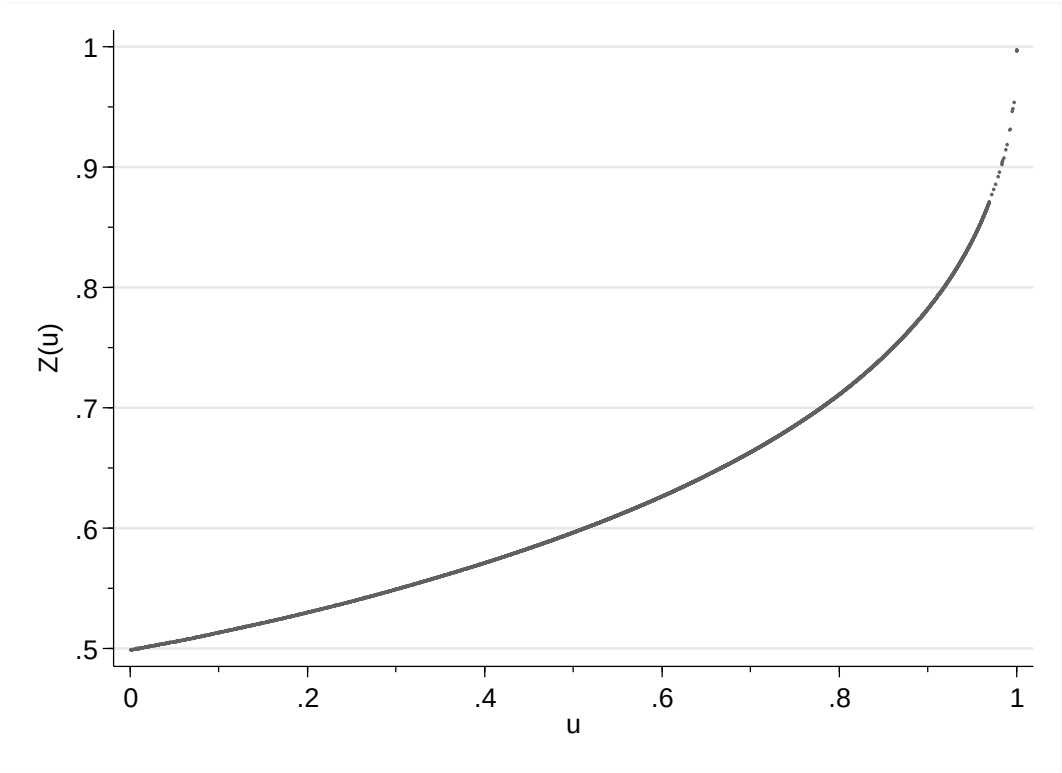


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

1995

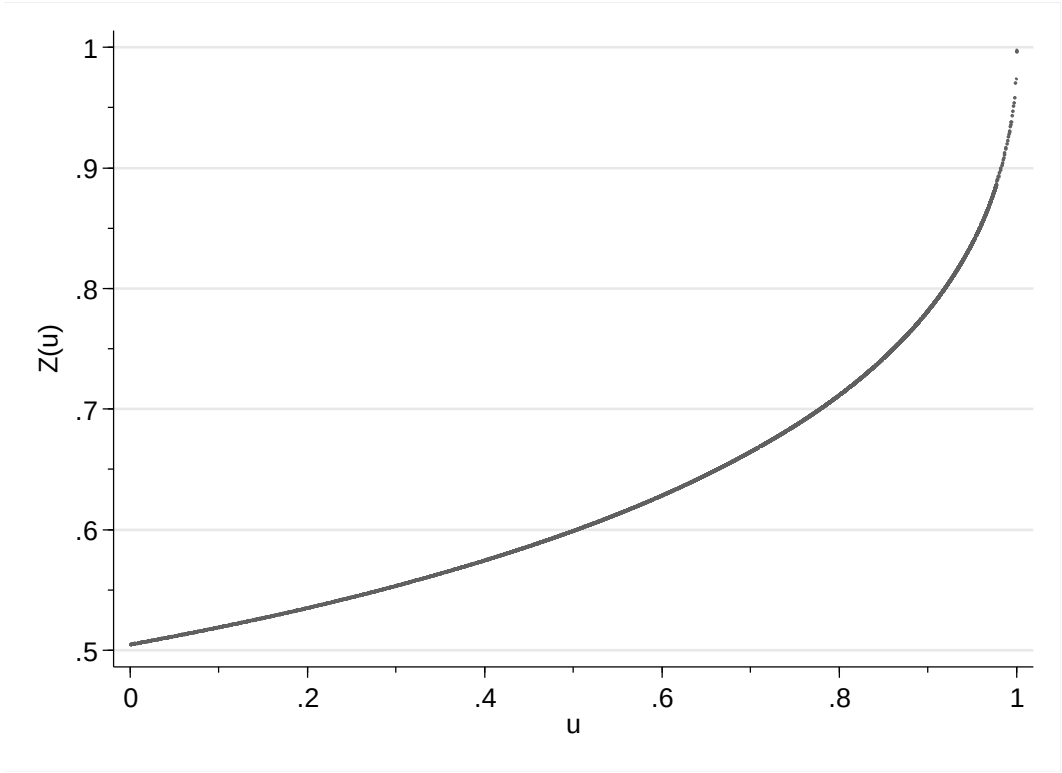


1996

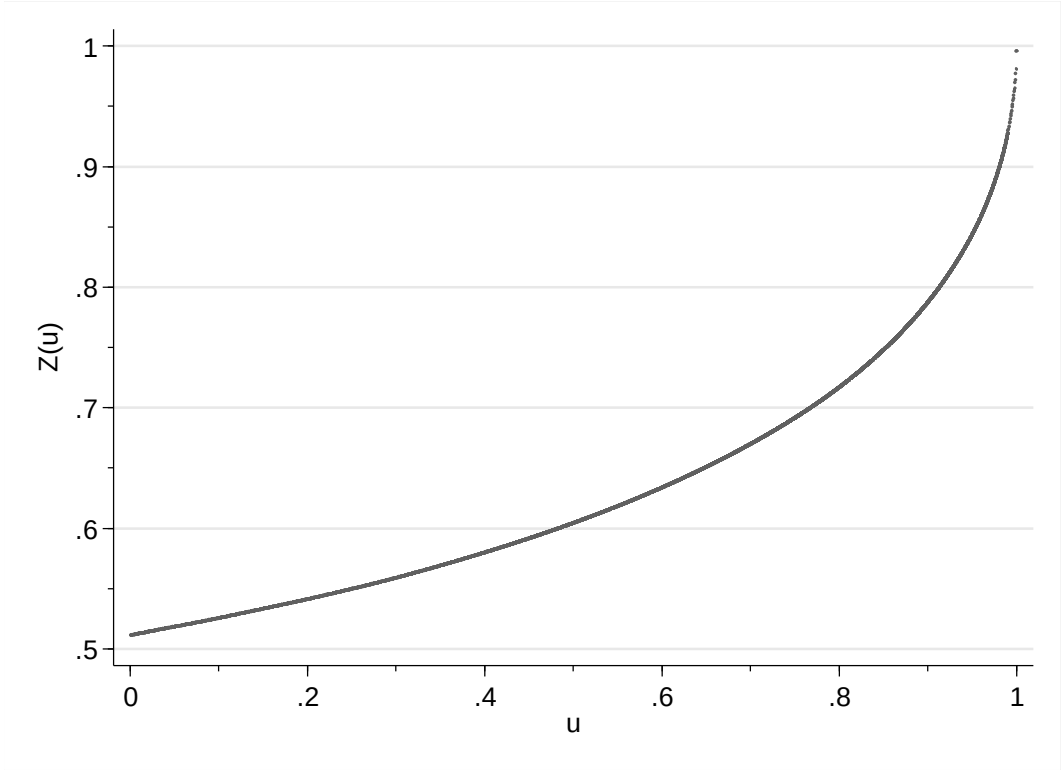


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

1997

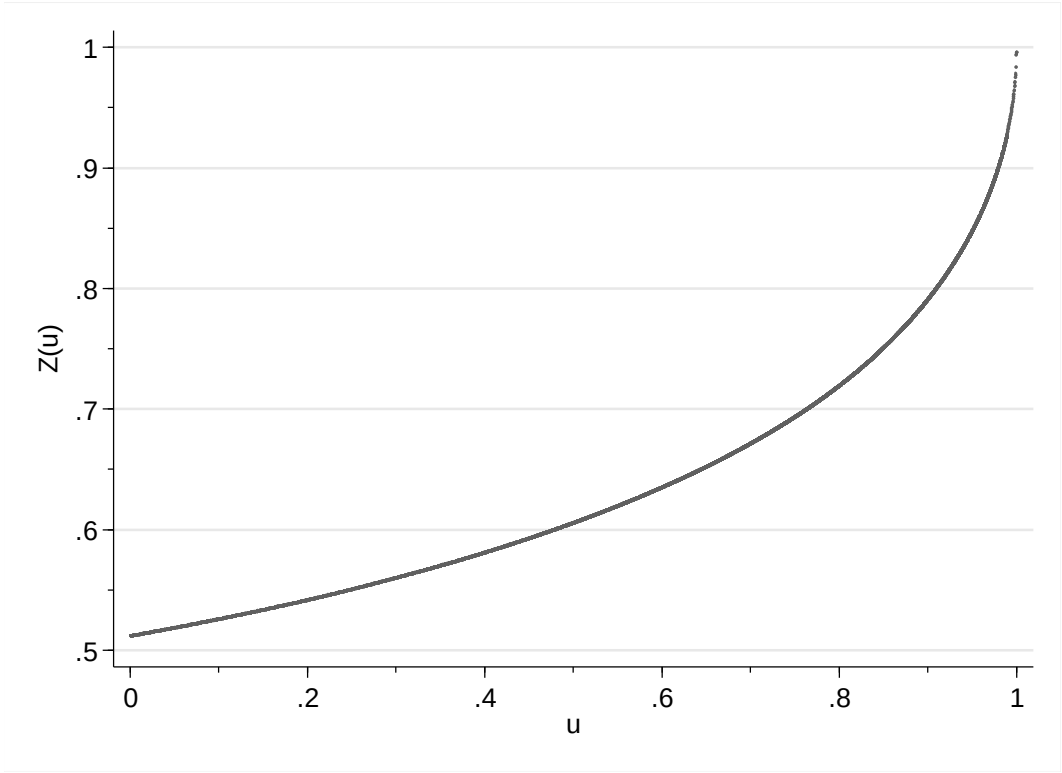


1998

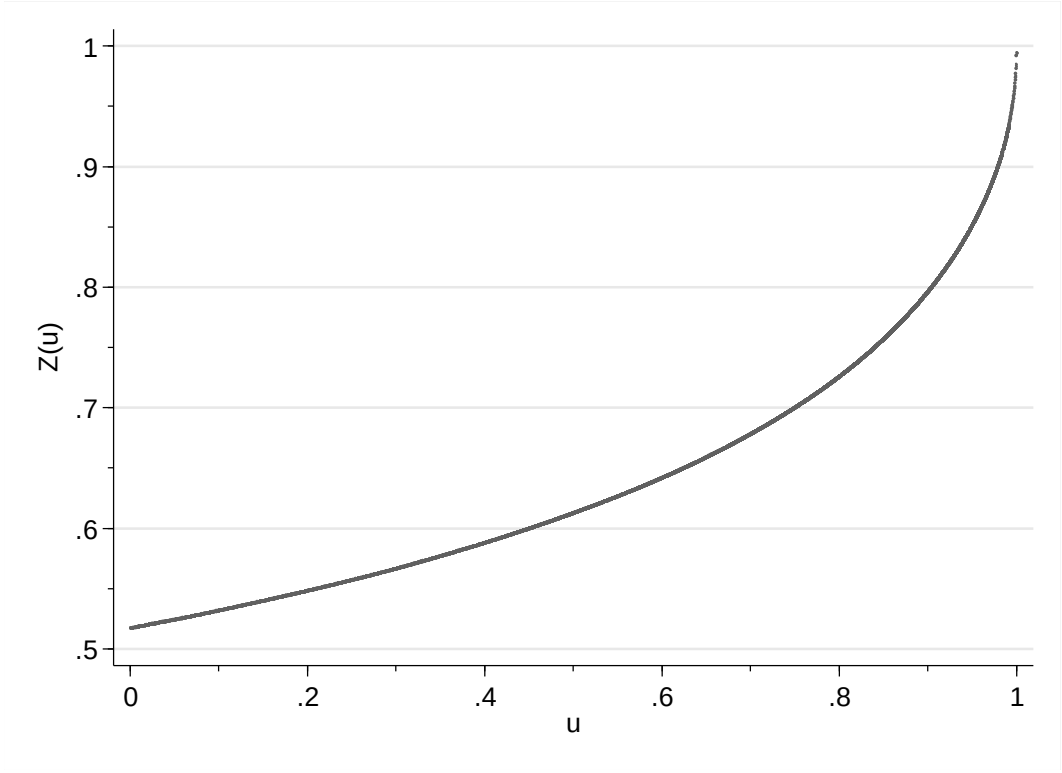


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

1999

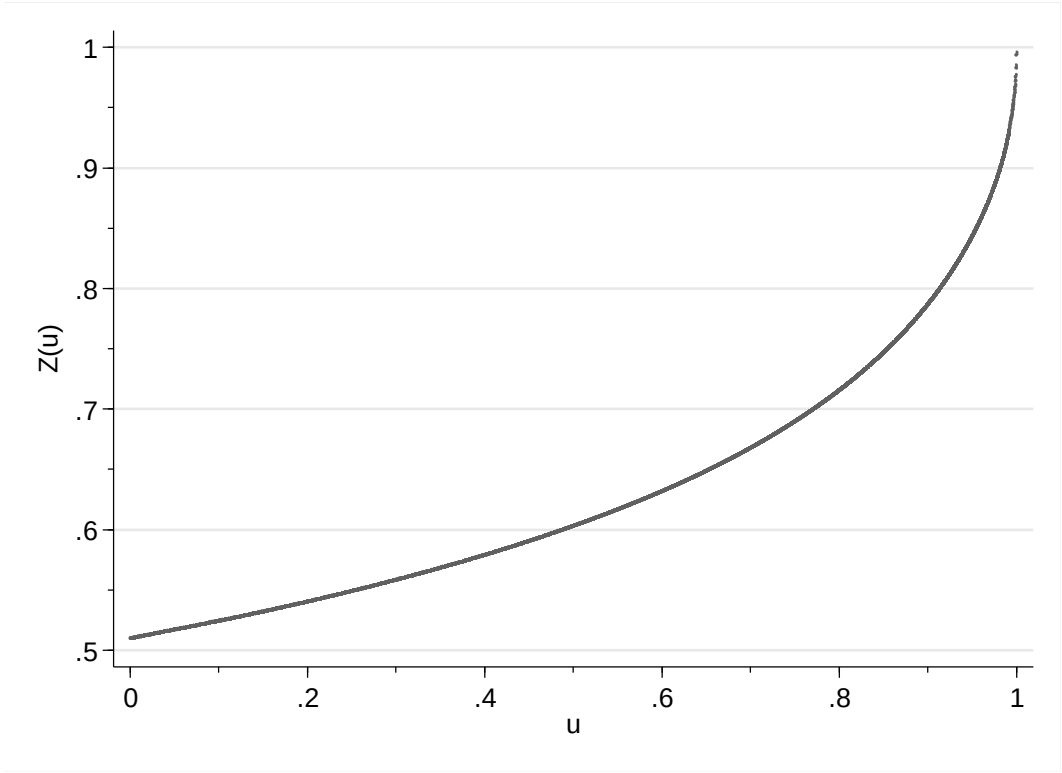


2000

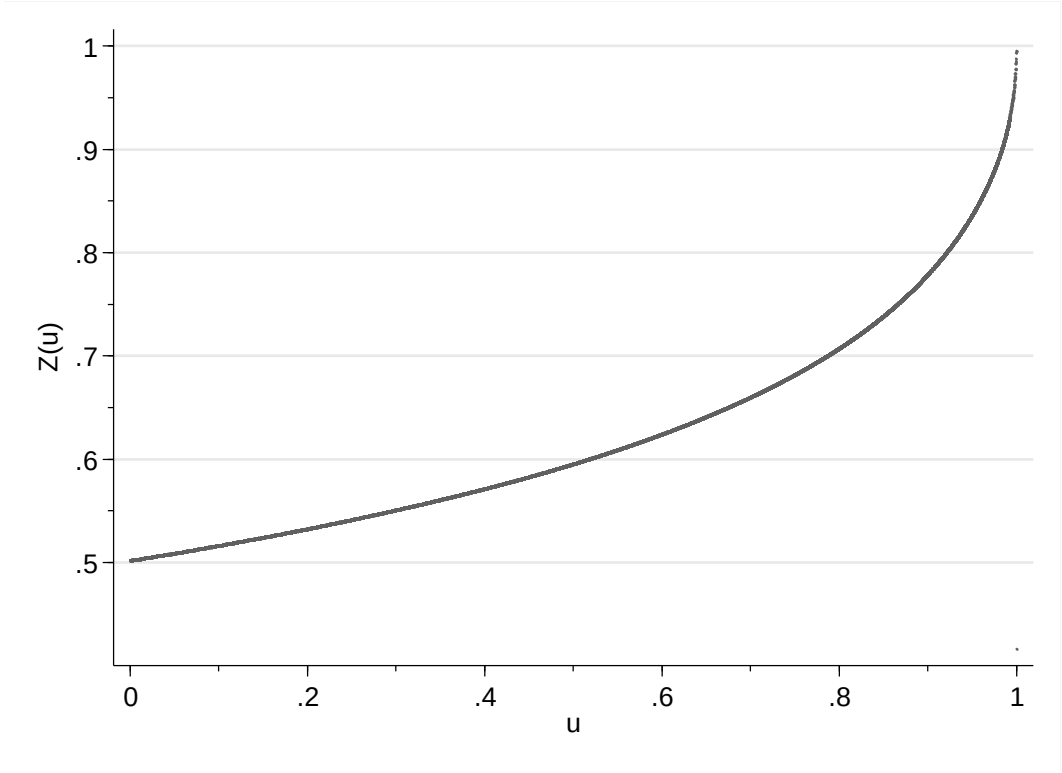


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

2001

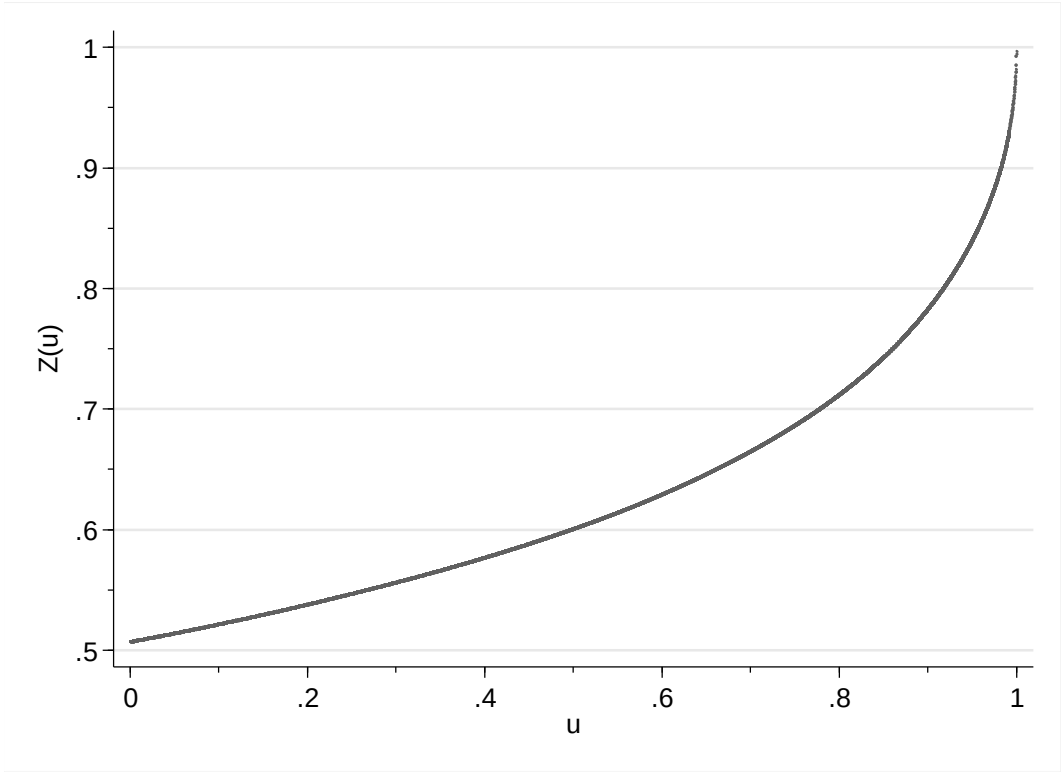


2002

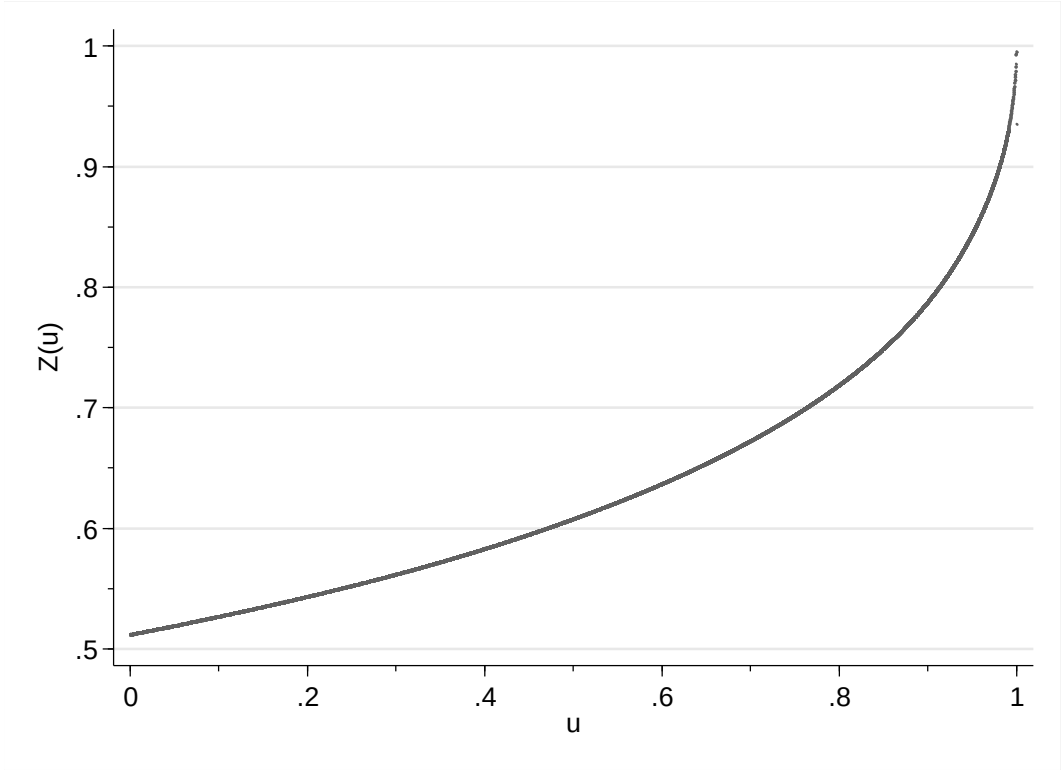


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

2003

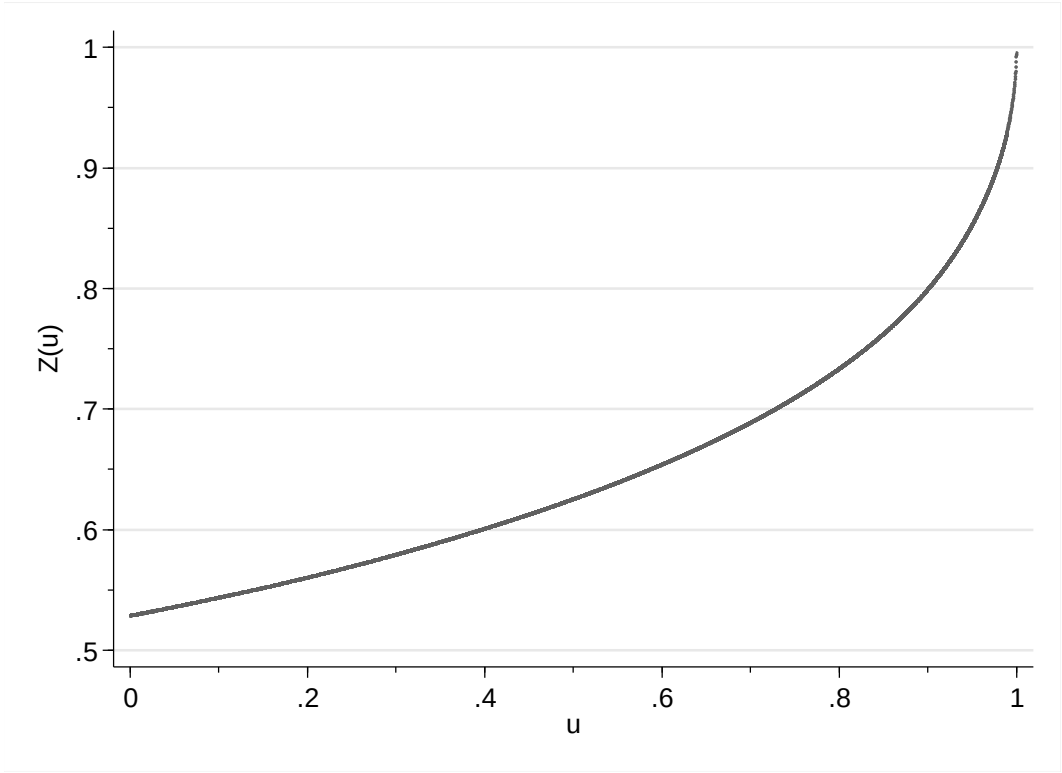


2004

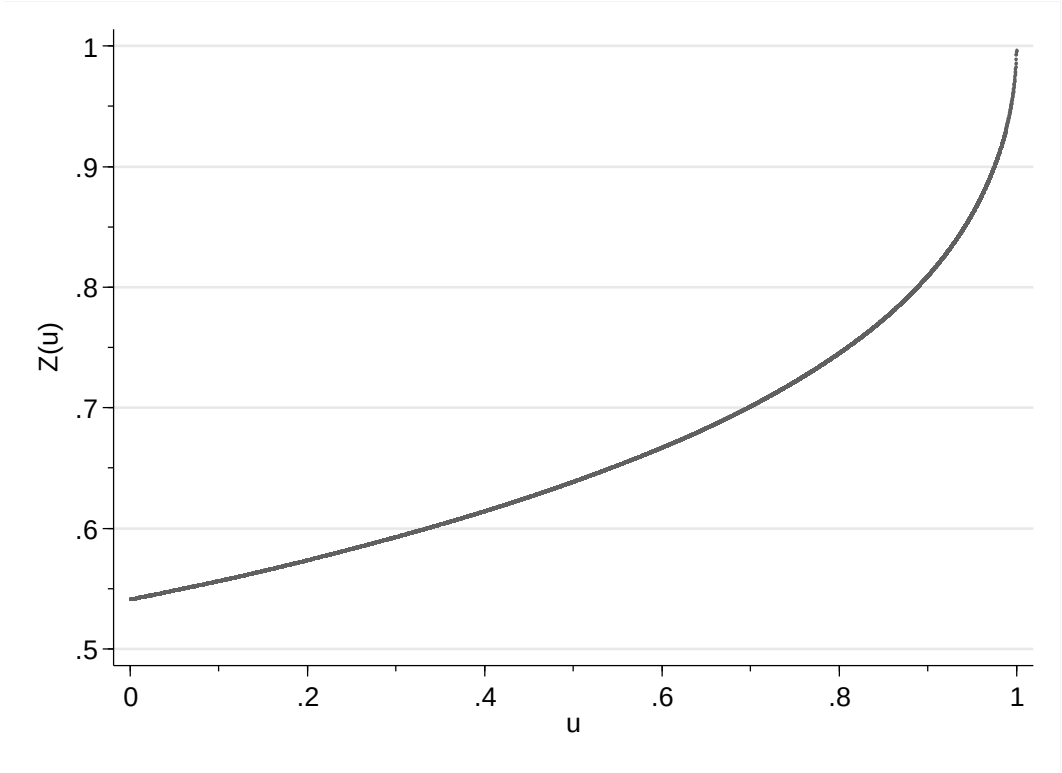


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

2005

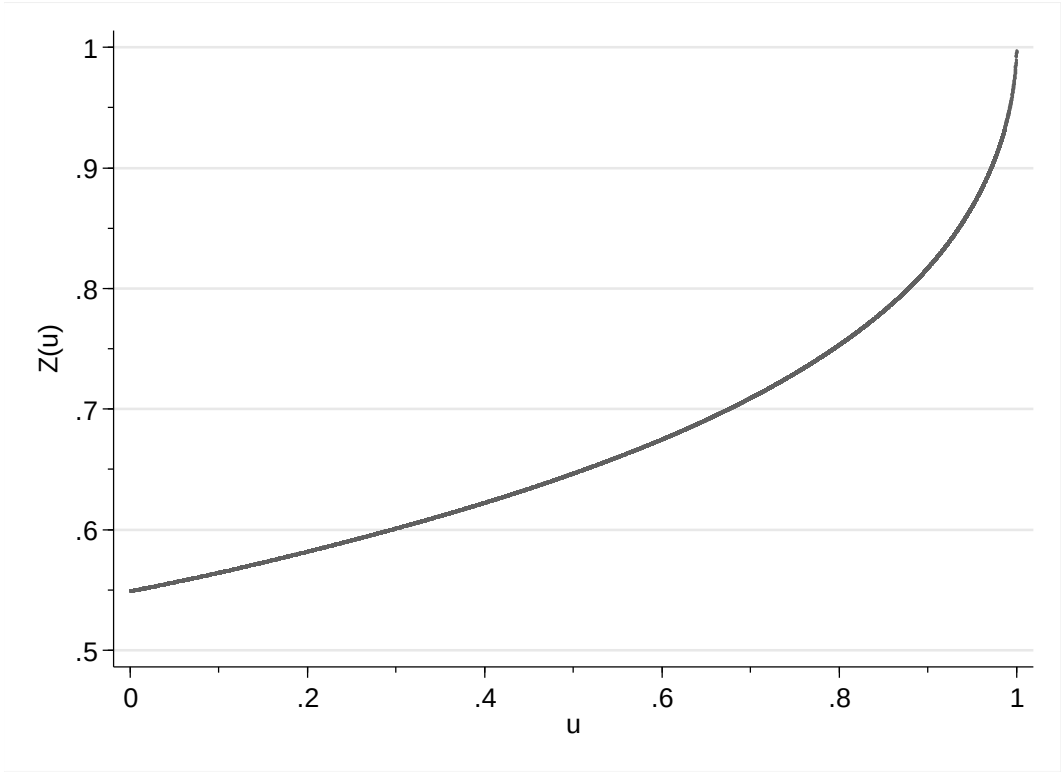


2006

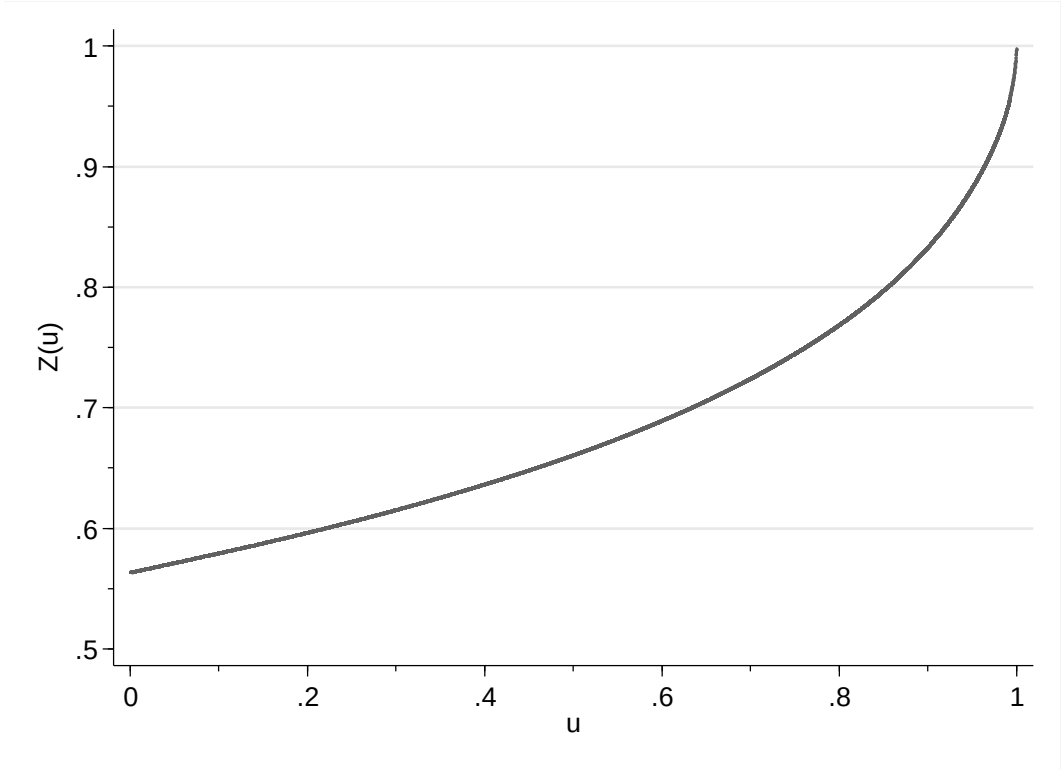


Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

2007

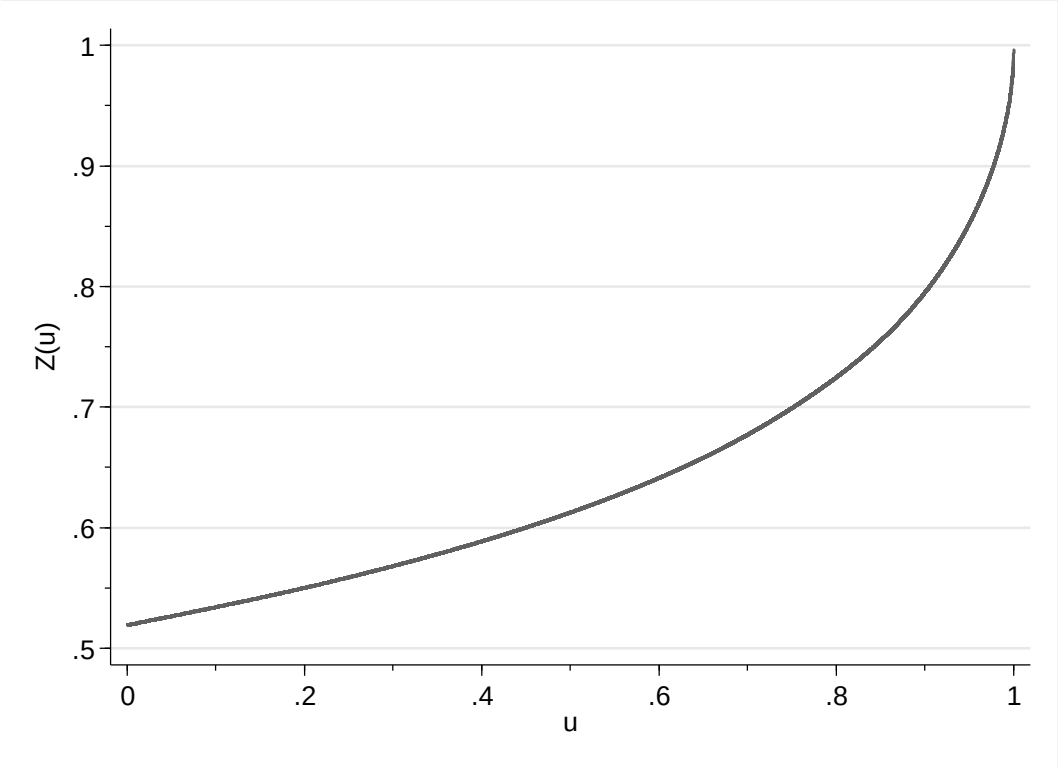


2009



Zenga plots, by year, threshold = £60k p.a. (pareto03; plots for other thresholds available)

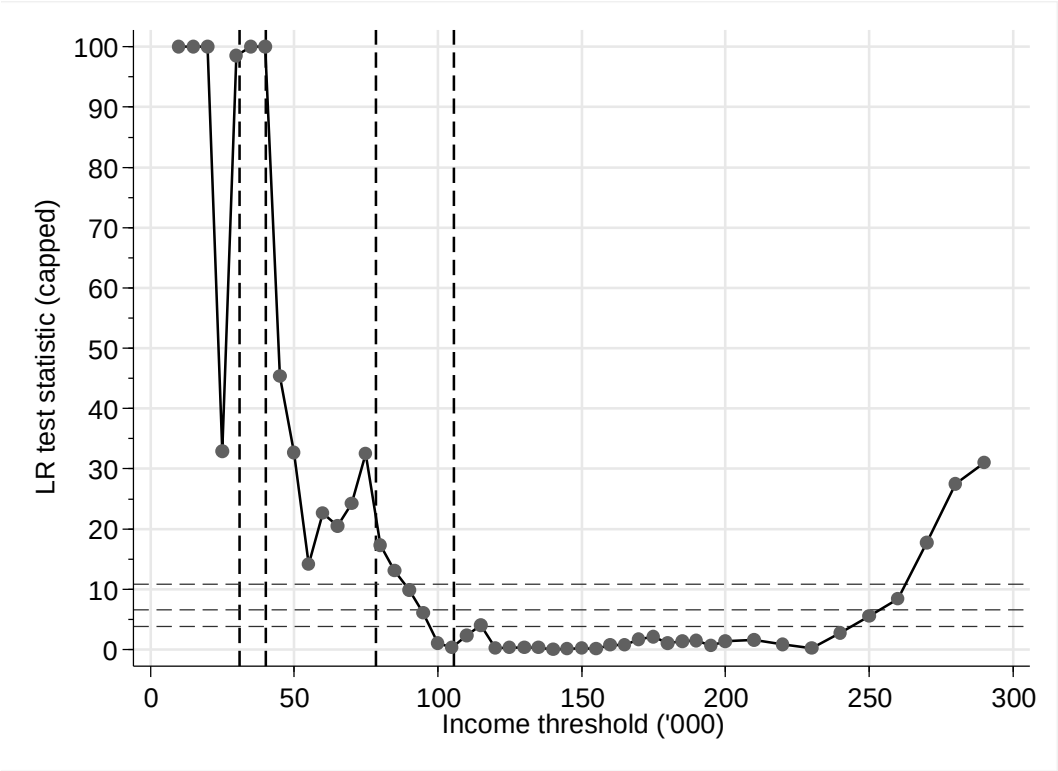
2010



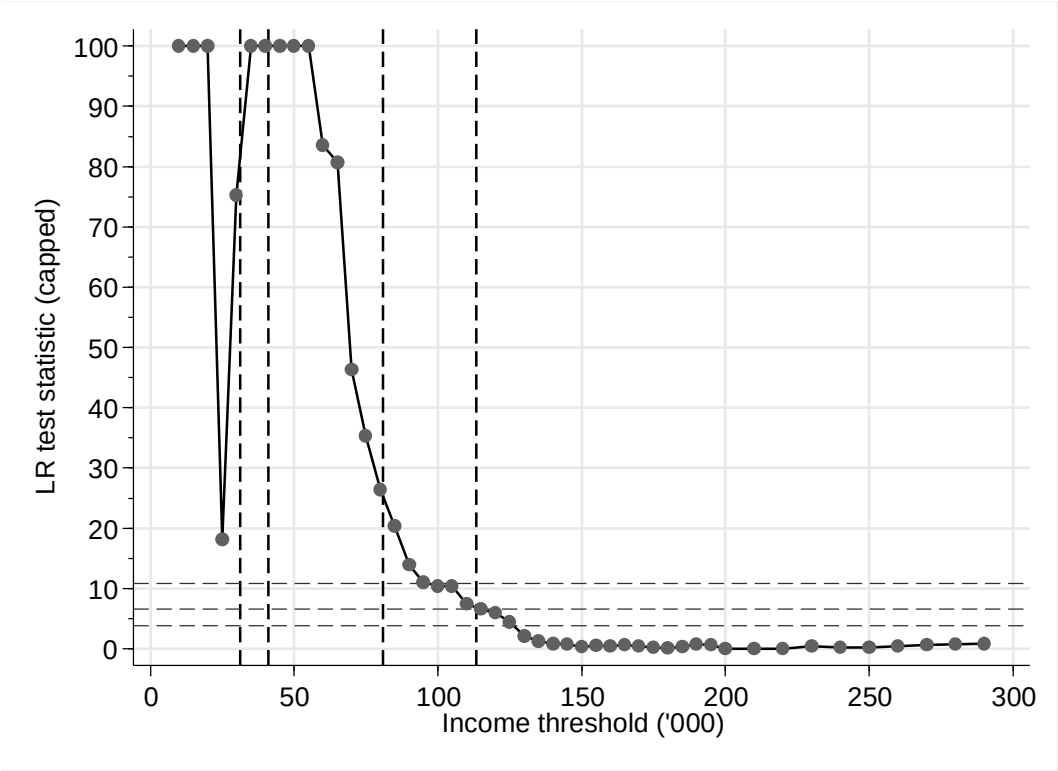
Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

Vertical dashed lines show (from left to right) *p*90, *p*95, *p*99, *p*99.5

1995

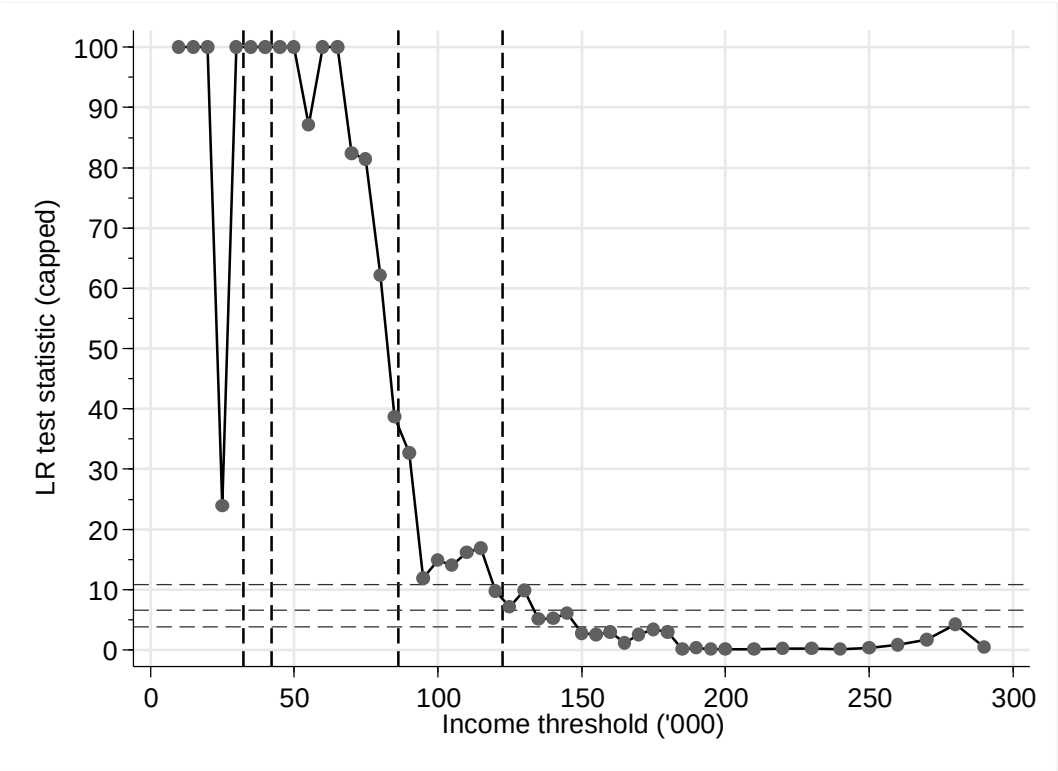


1996

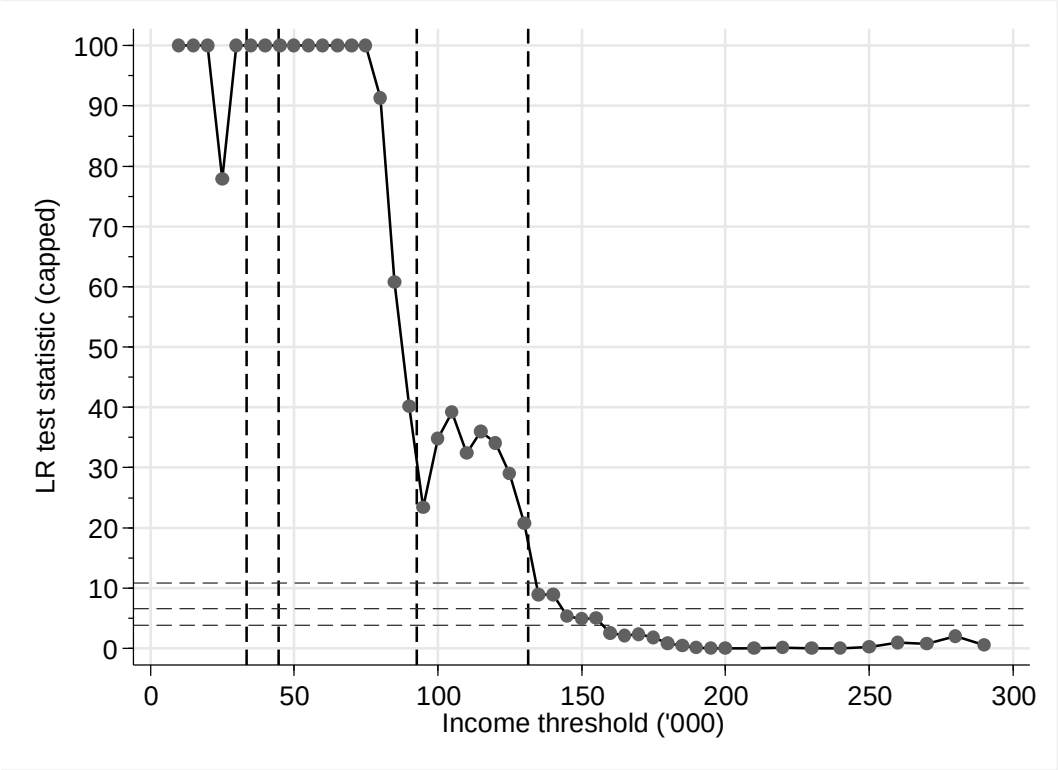


Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

1997

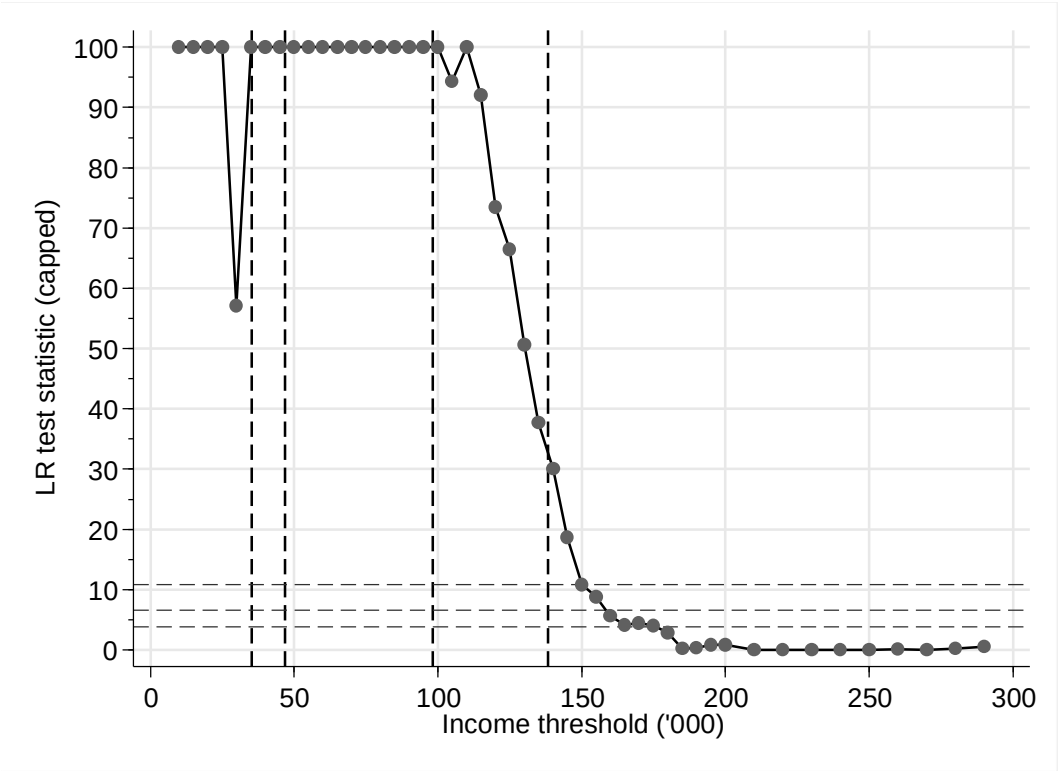


1998

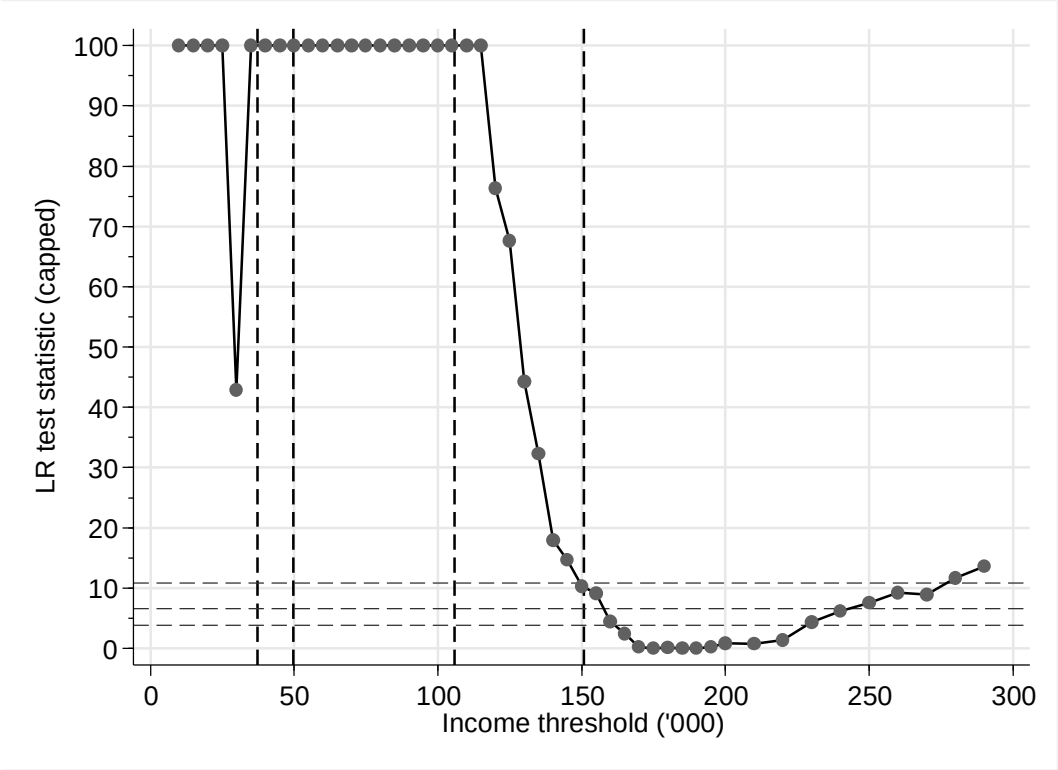


Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

1999

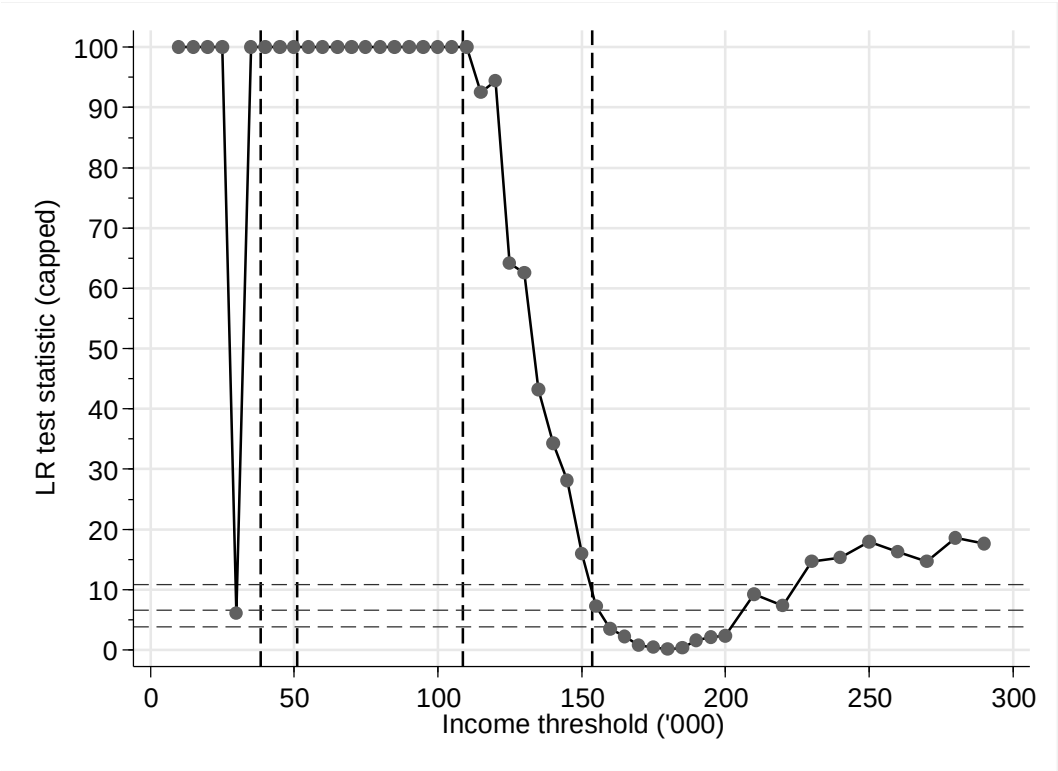


2000

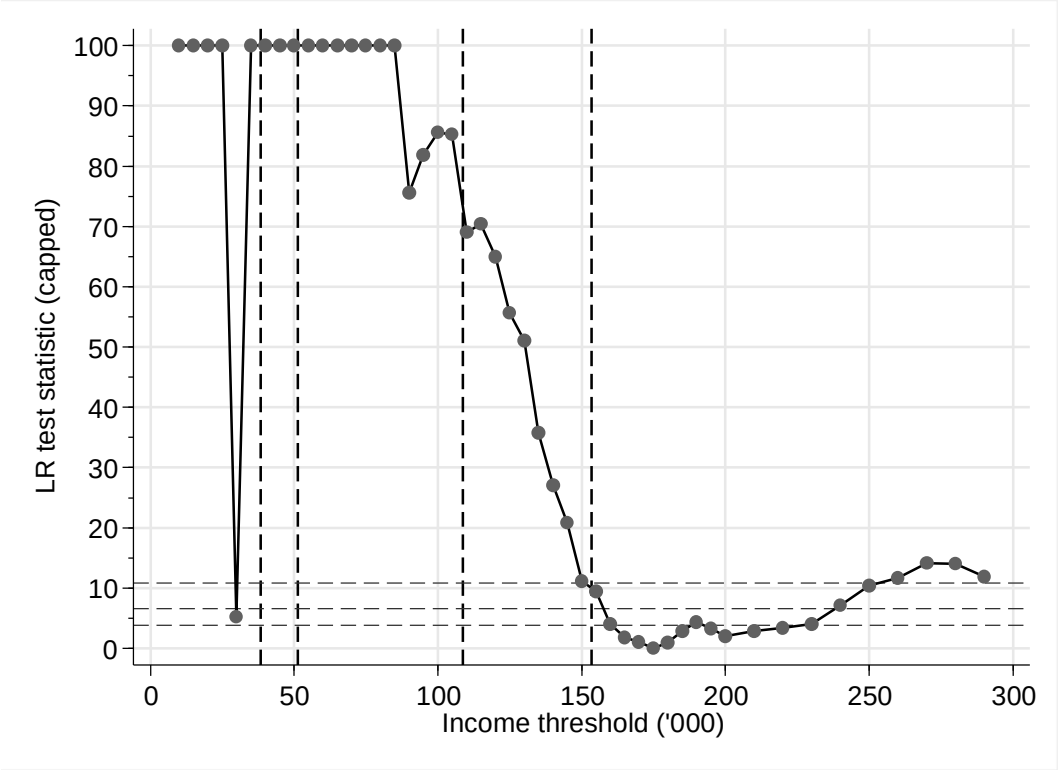


Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

2001

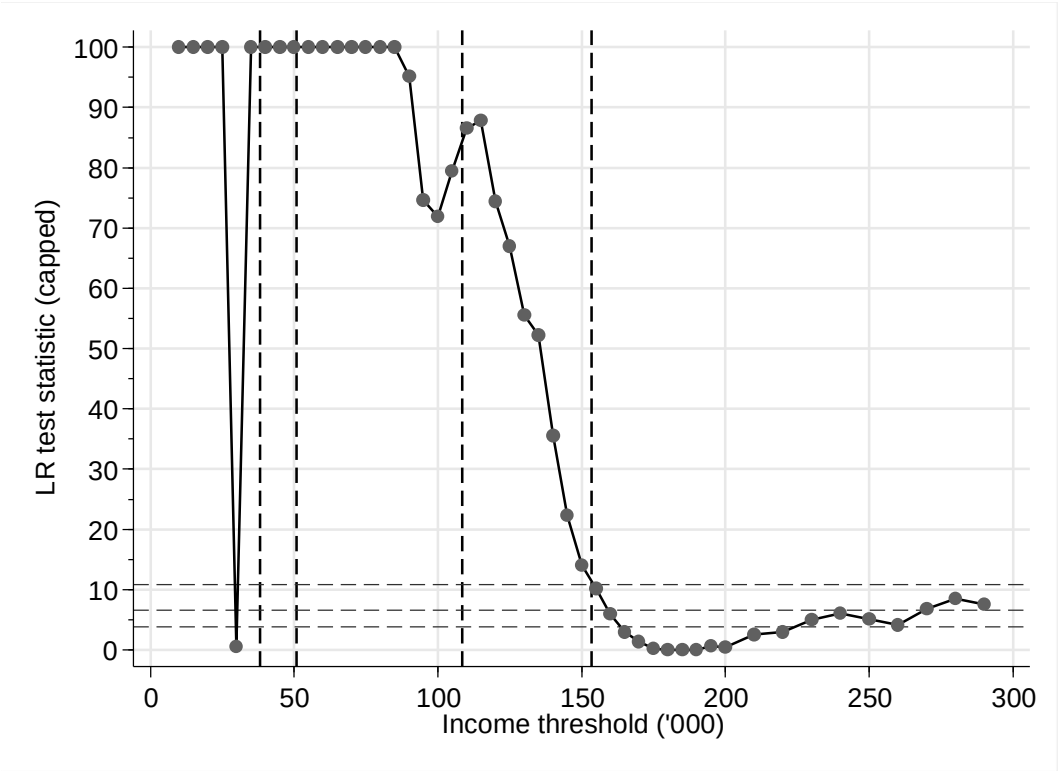


2002

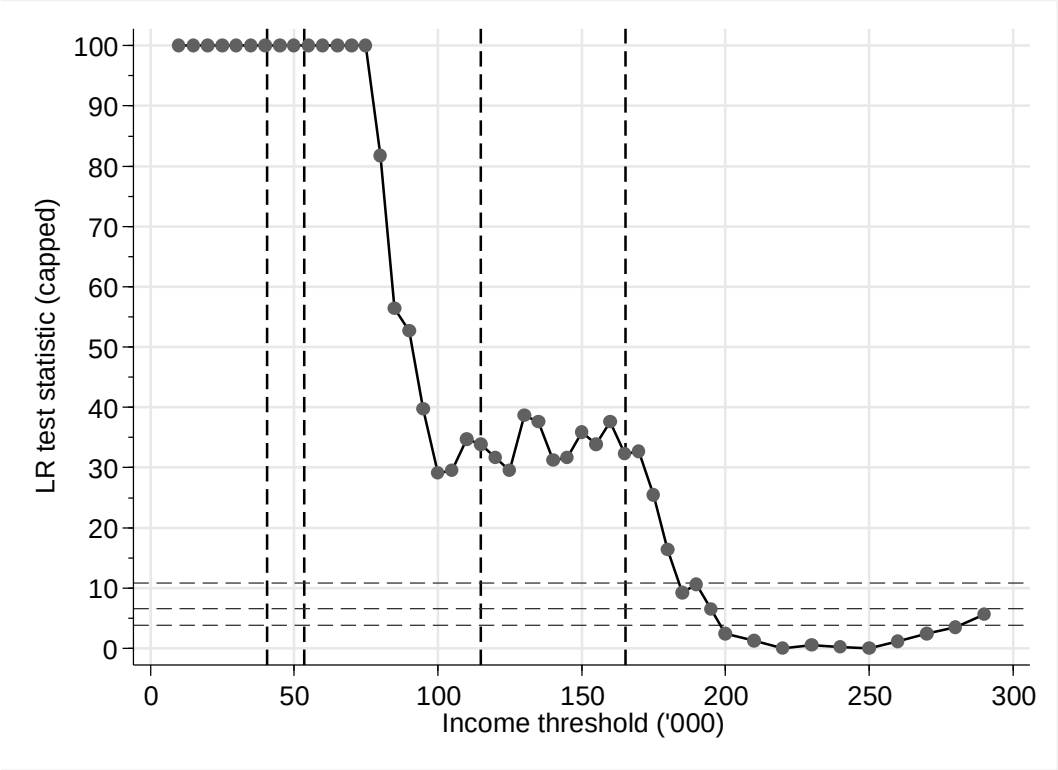


Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

2003

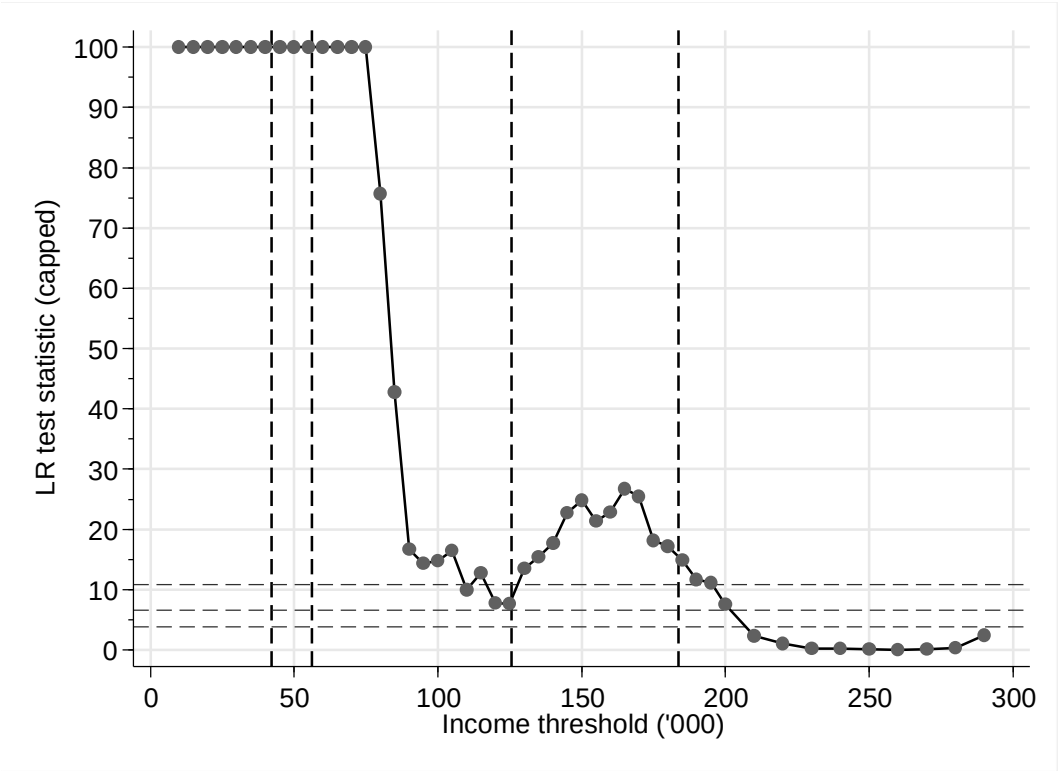


2004

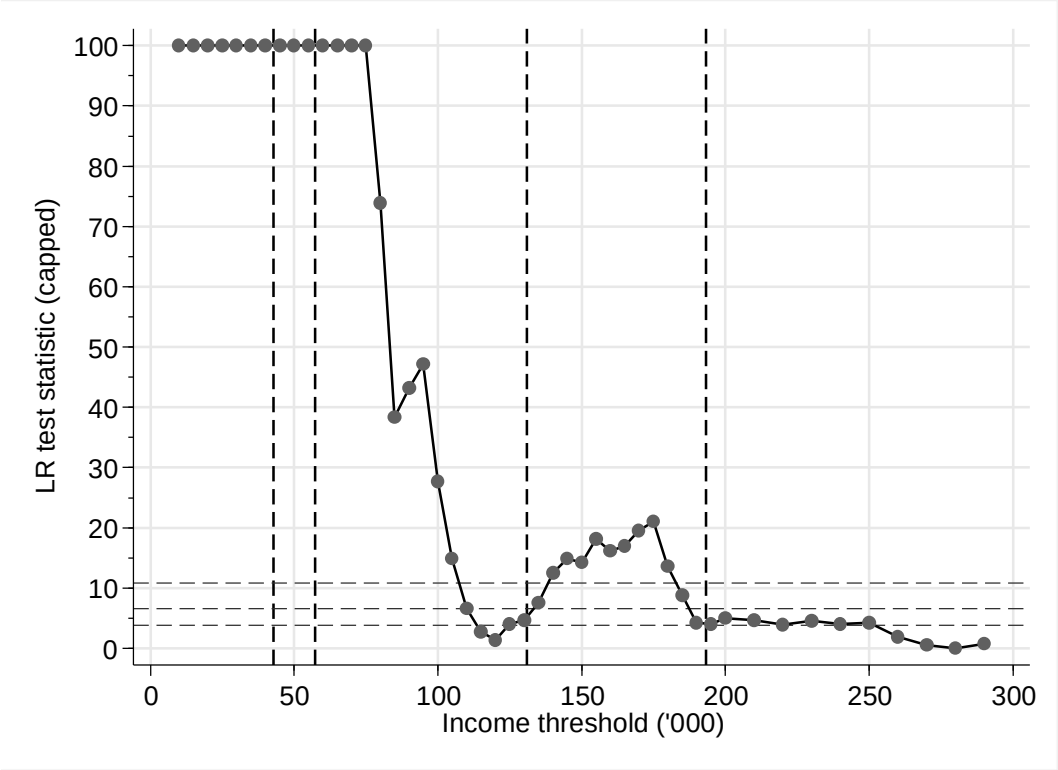


Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

2005

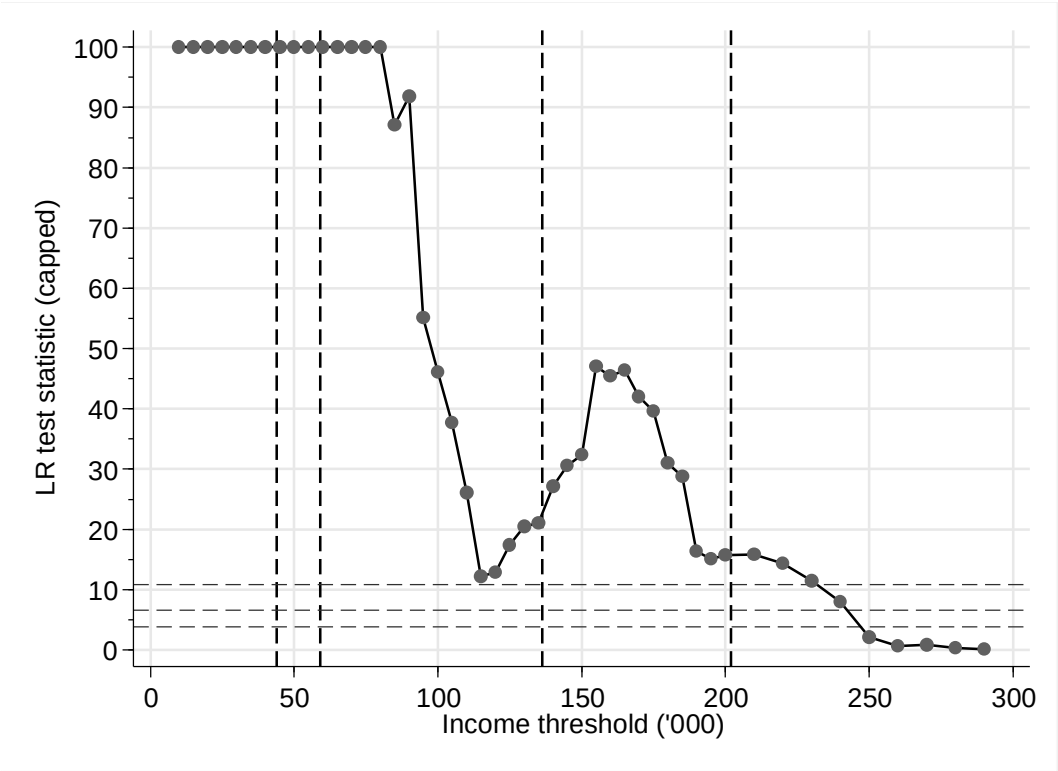


2006

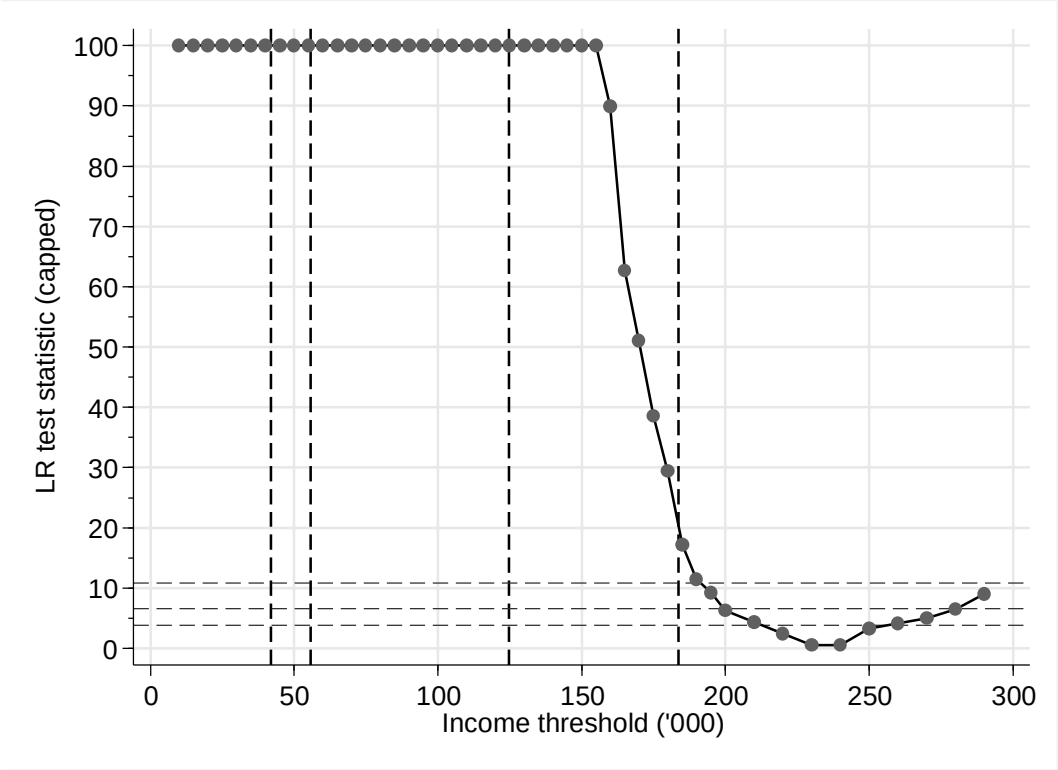


Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

2007

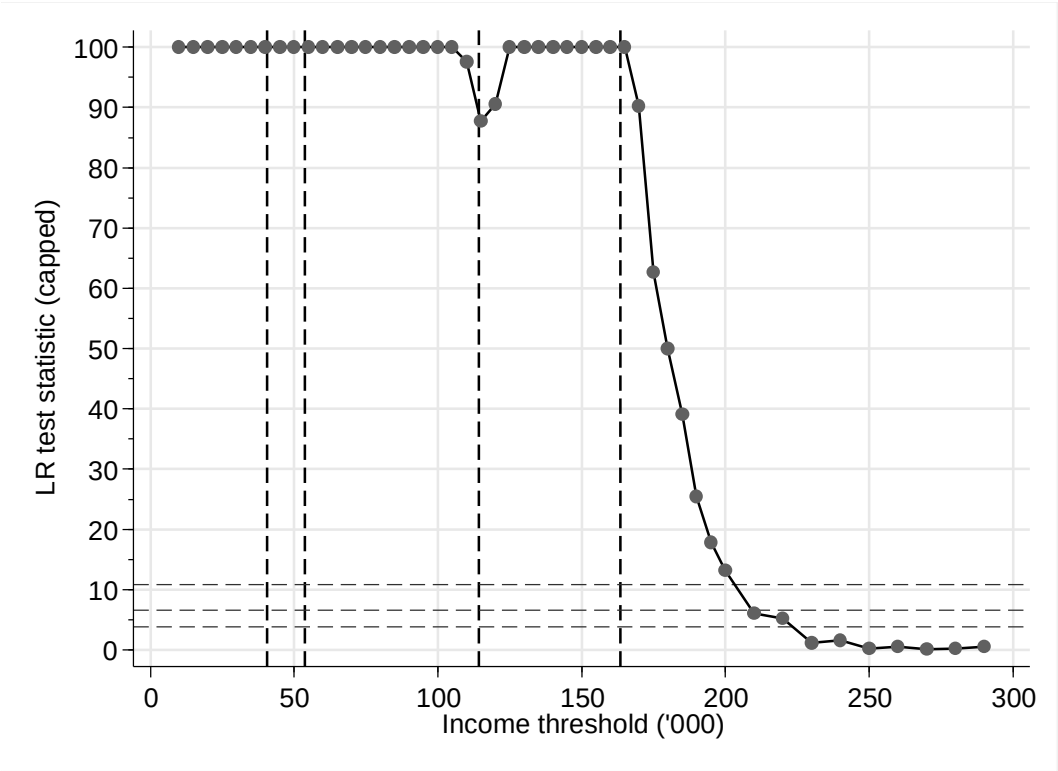


2009



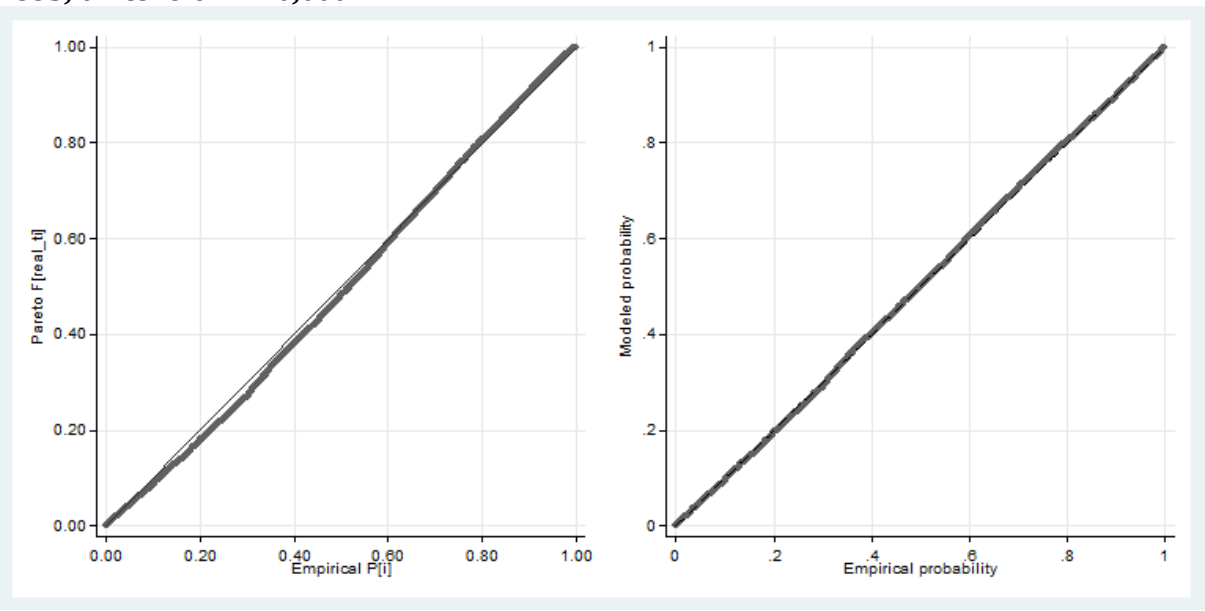
Likelihood ratio test statistics (Pareto I versus Pareto II), by year and threshold
(pareto08)

2010

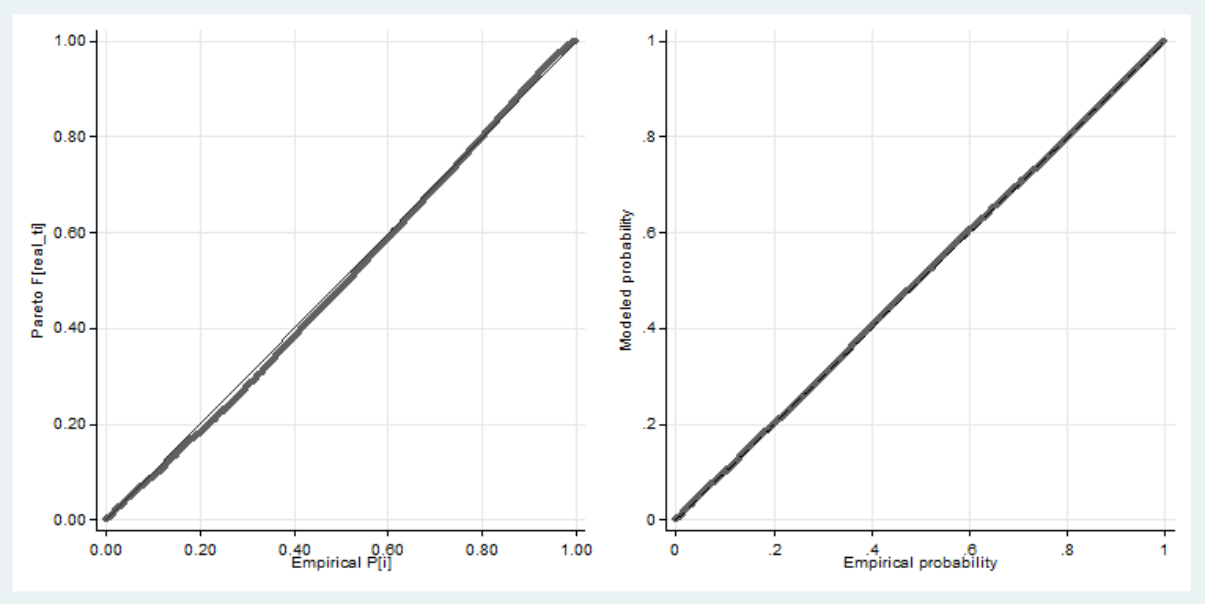


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1995, threshold = £40,000

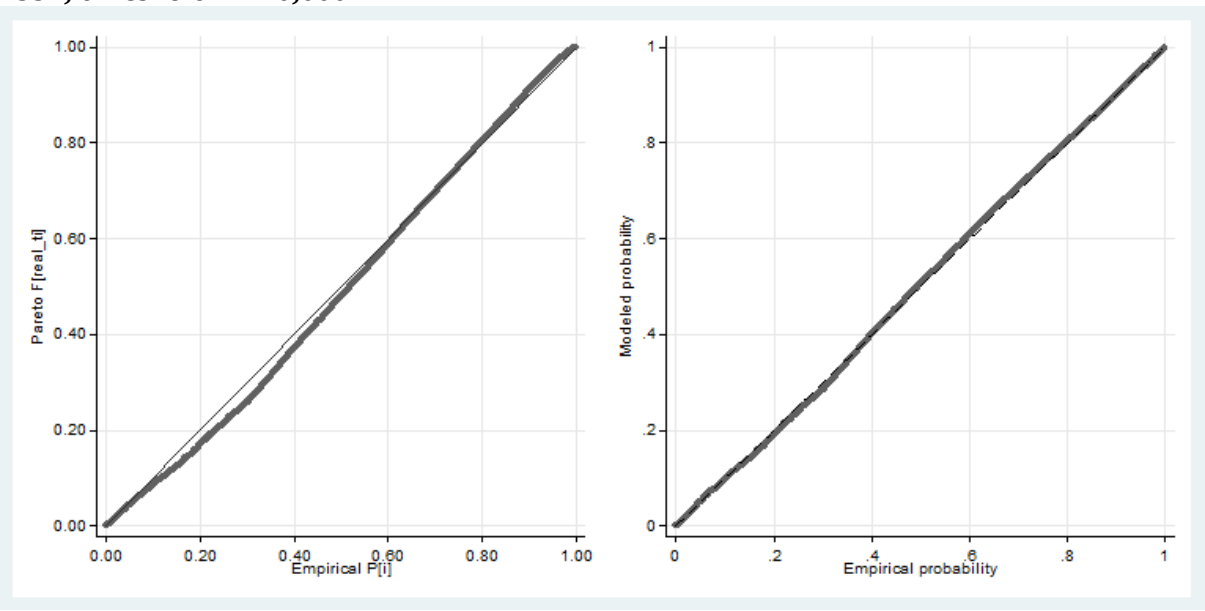


1996, threshold = £40,000

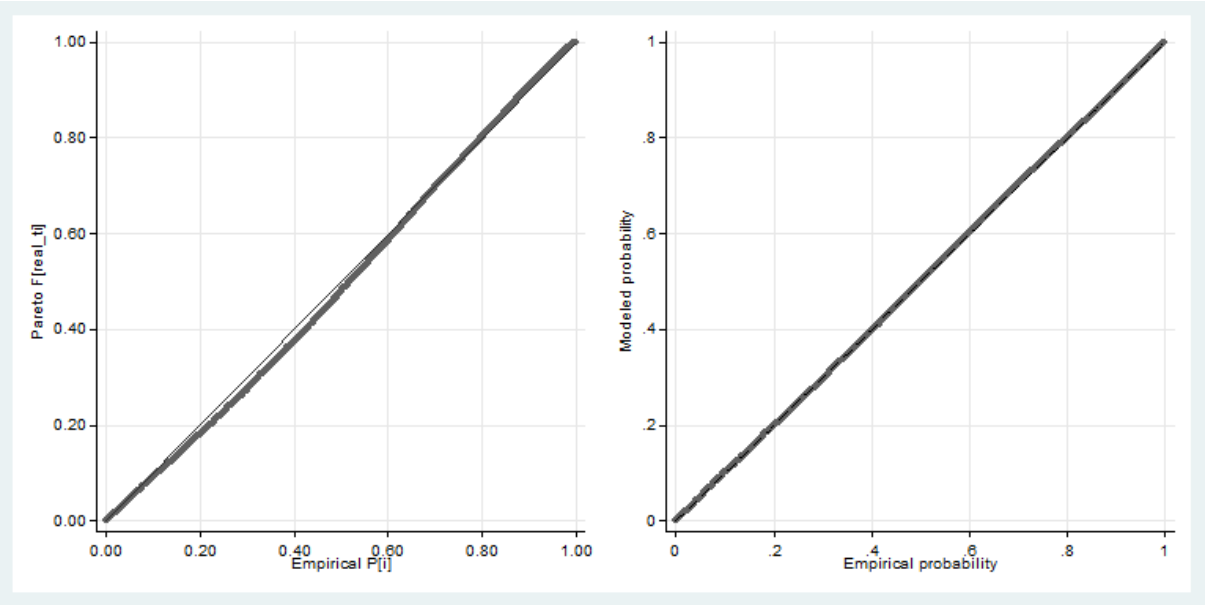


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1997, threshold = £40,000

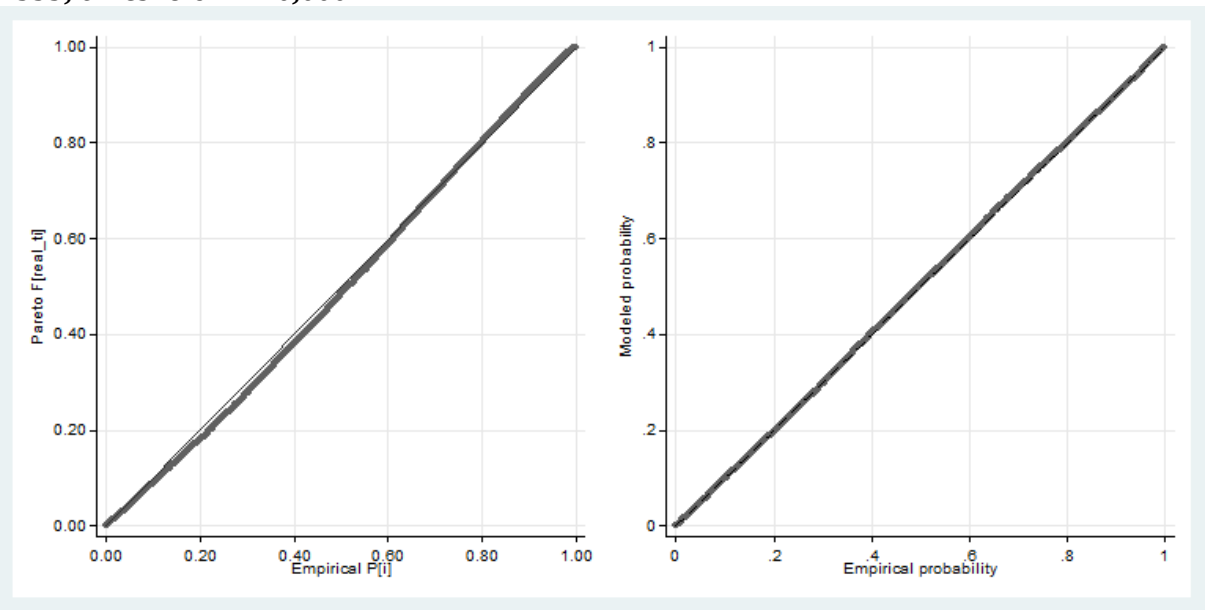


1998, threshold = £40,000

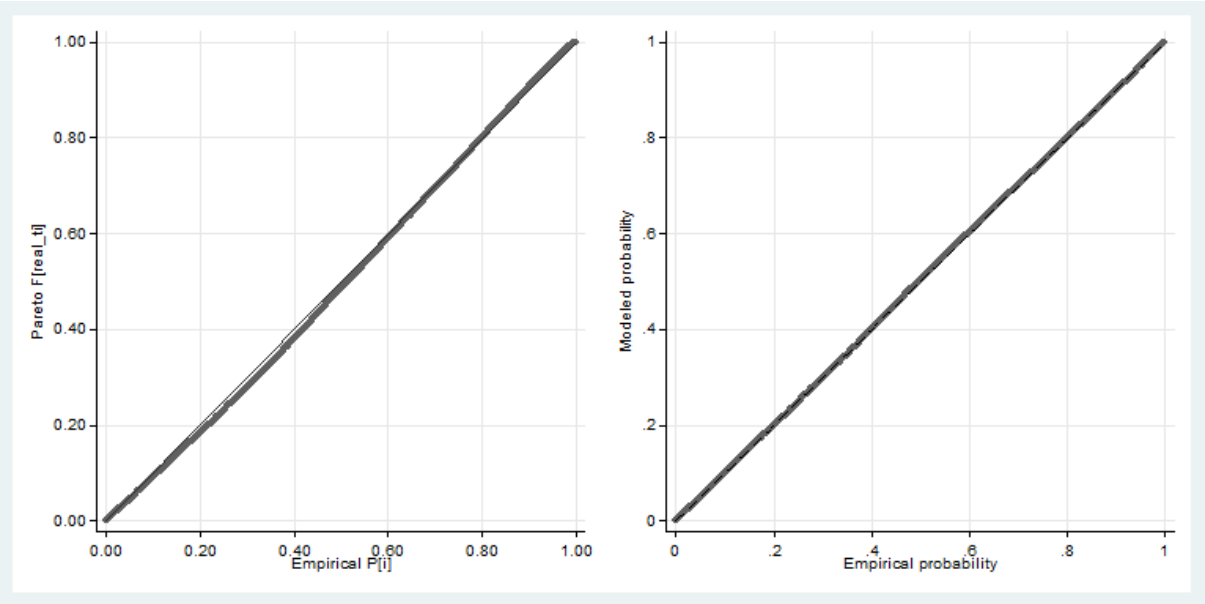


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1999, threshold = £40,000

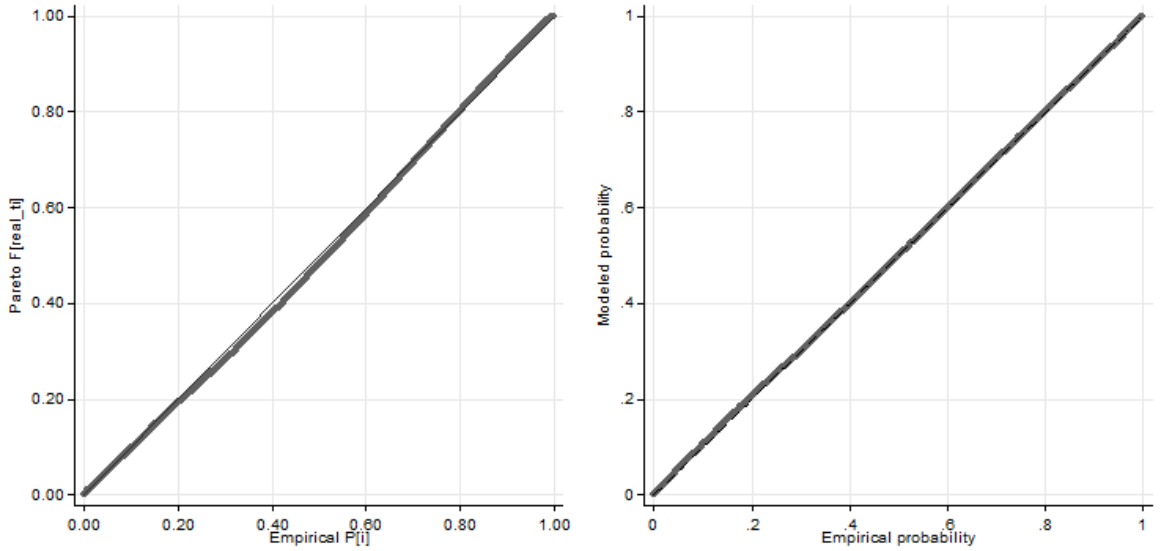


2000, threshold = £40,000

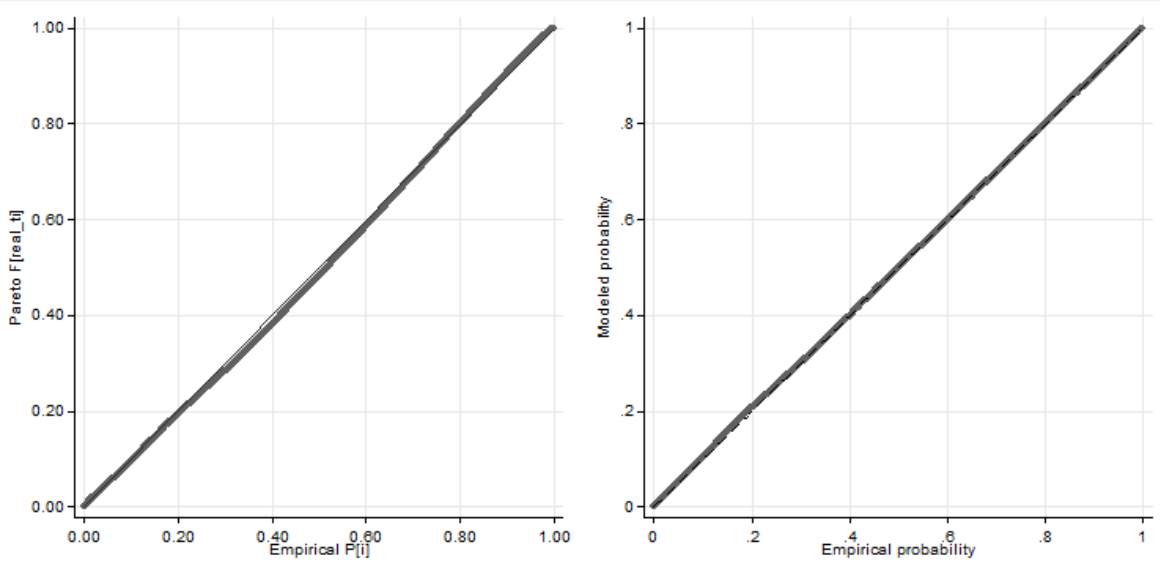


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2001, threshold = £40,000

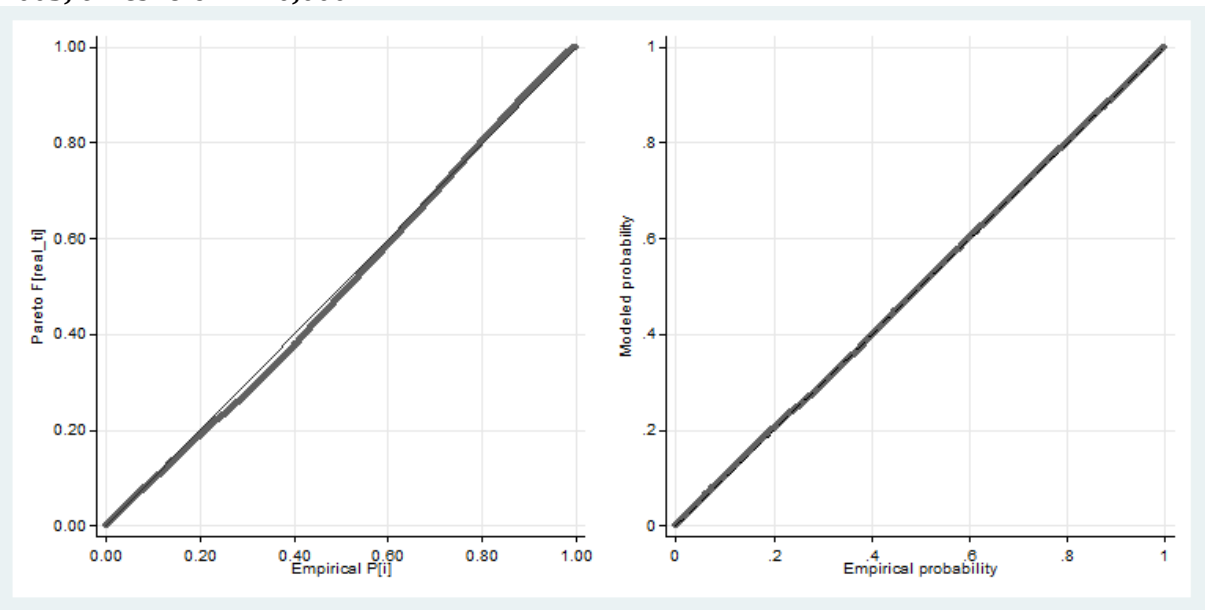


2002, threshold = £40,000

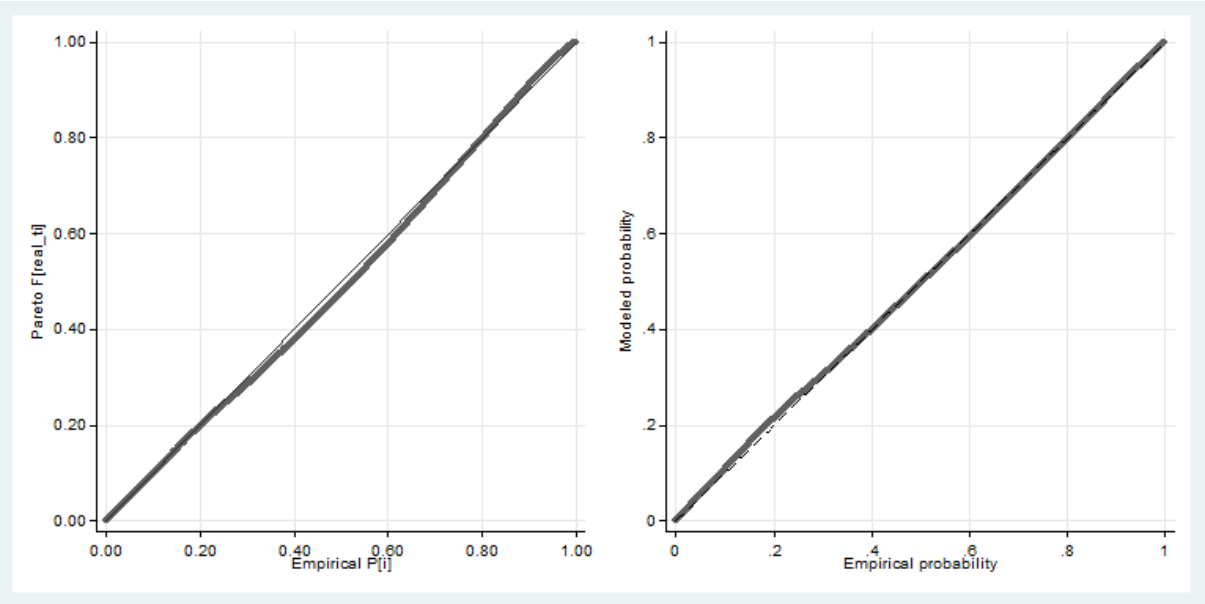


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2003, threshold = £40,000

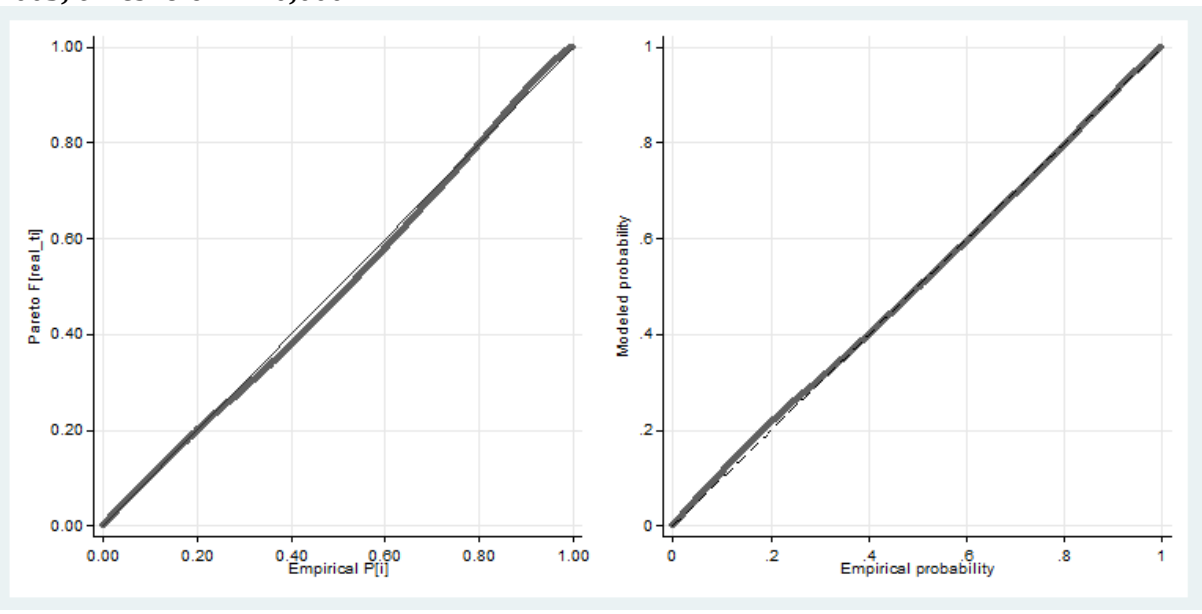


2004, threshold = £40,000

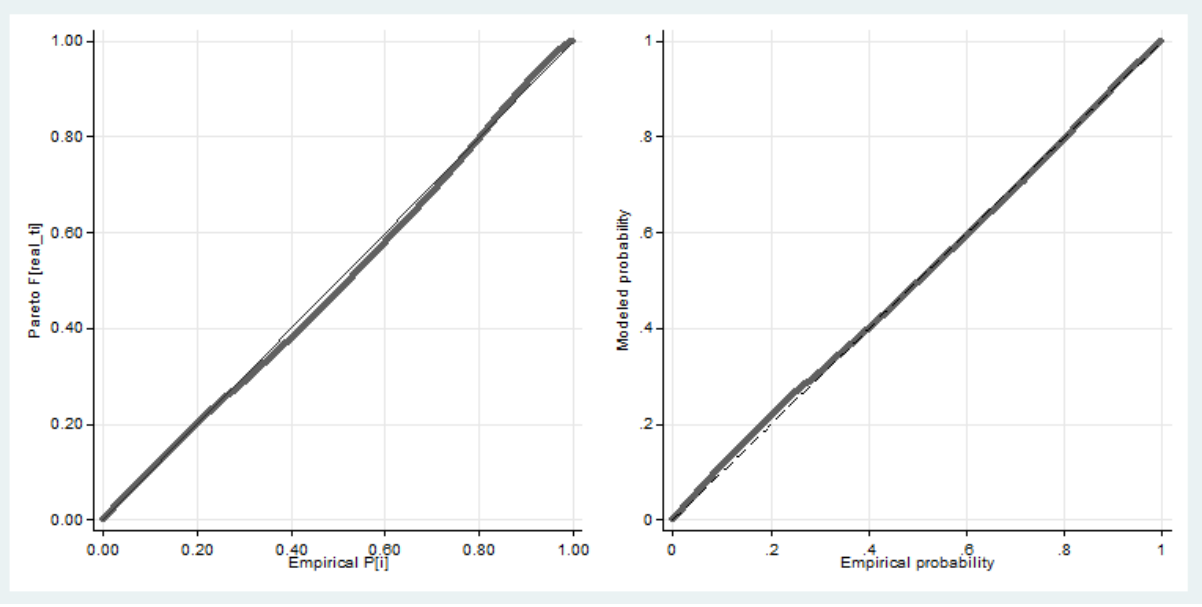


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2005, threshold = £40,000

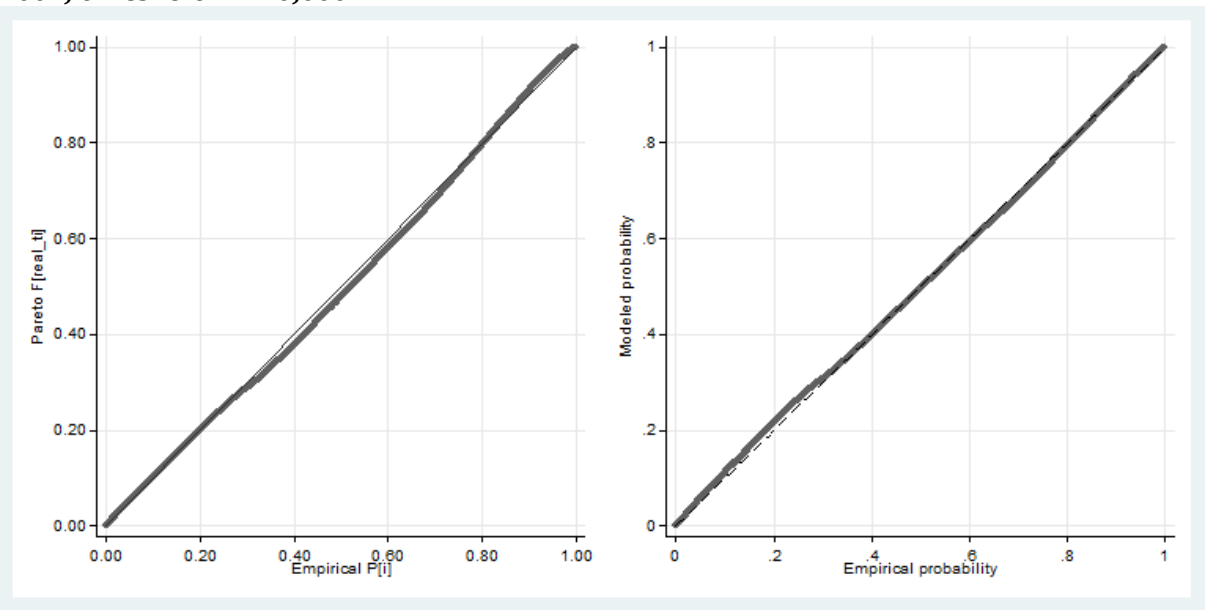


2006, threshold = £40,000

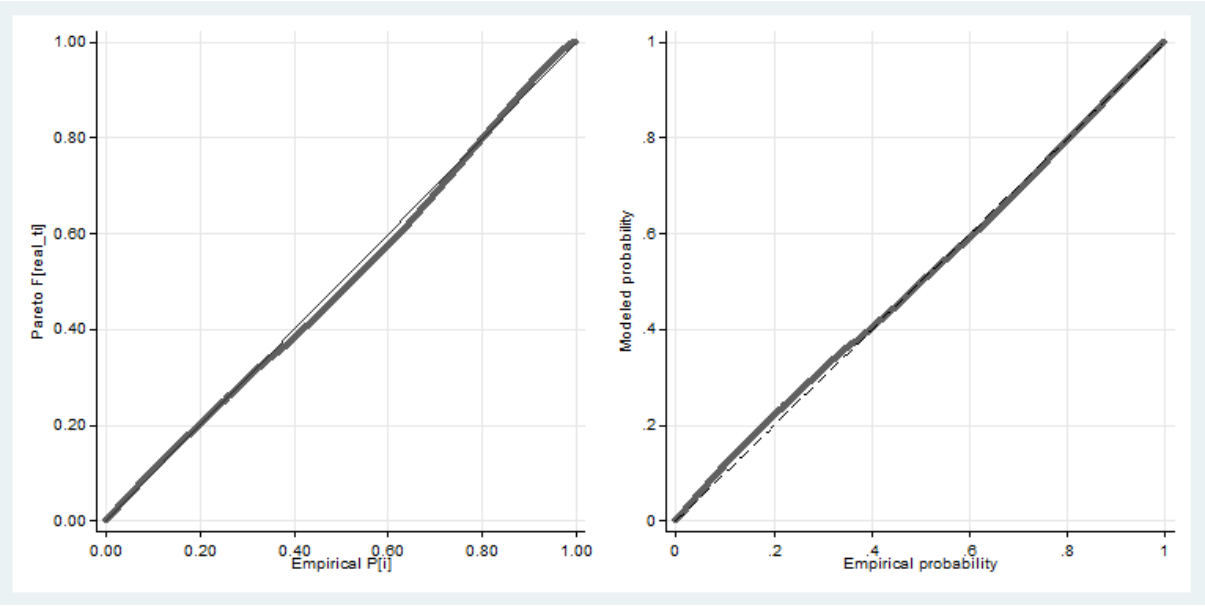


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2007, threshold = £40,000

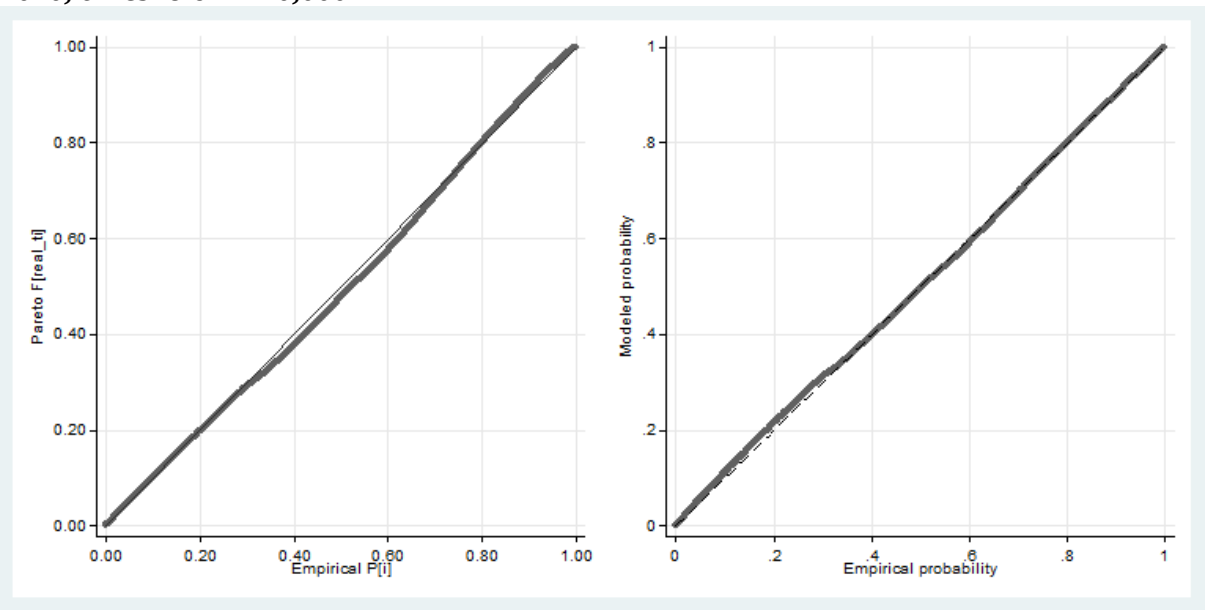


2009, threshold = £40,000



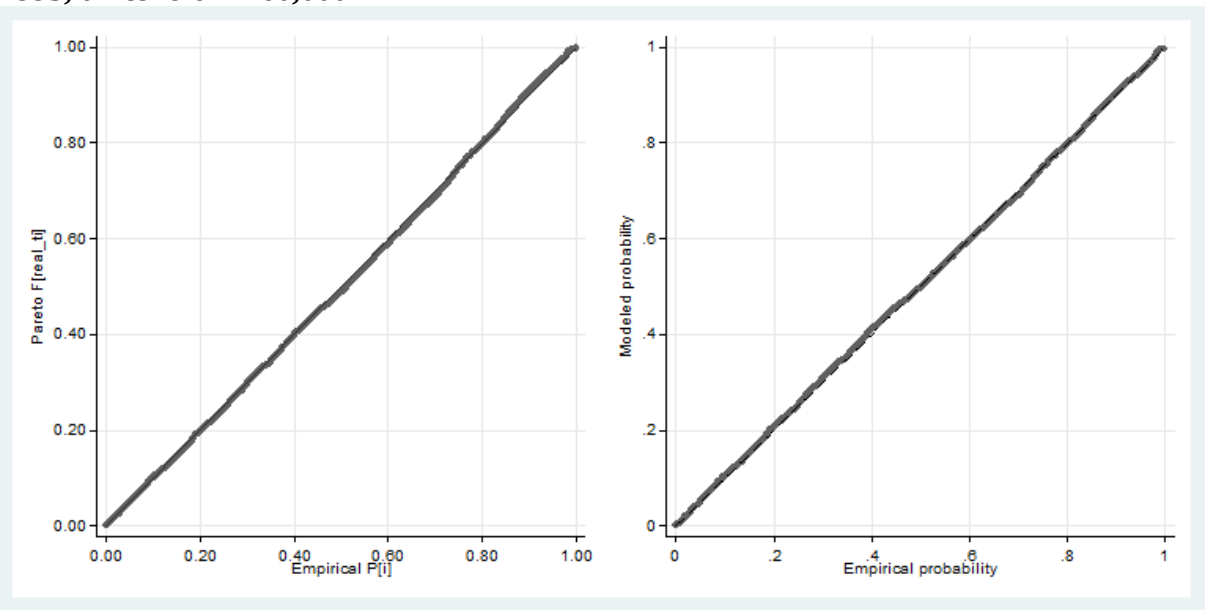
PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2010, threshold = £40,000

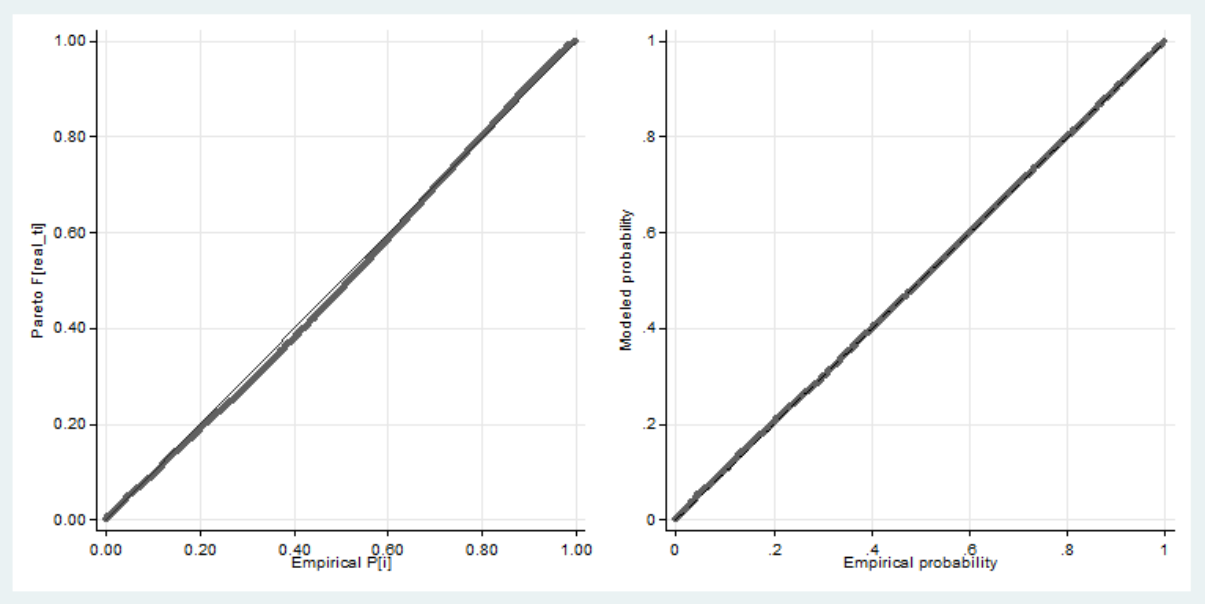


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1995, threshold = £60,000

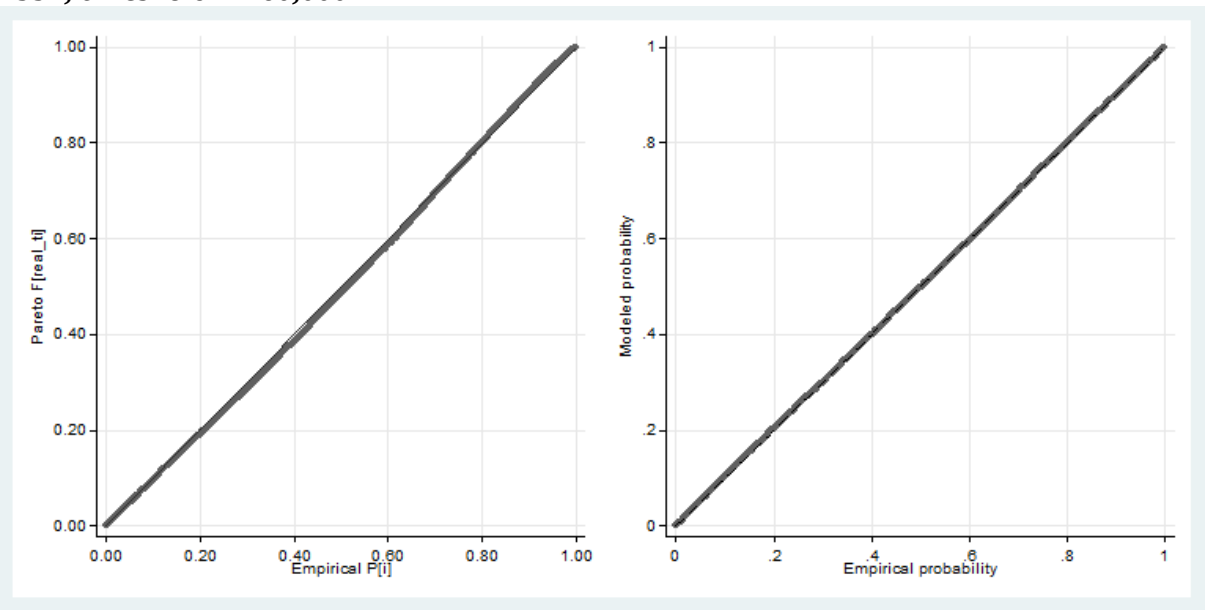


1996, threshold = £60,000

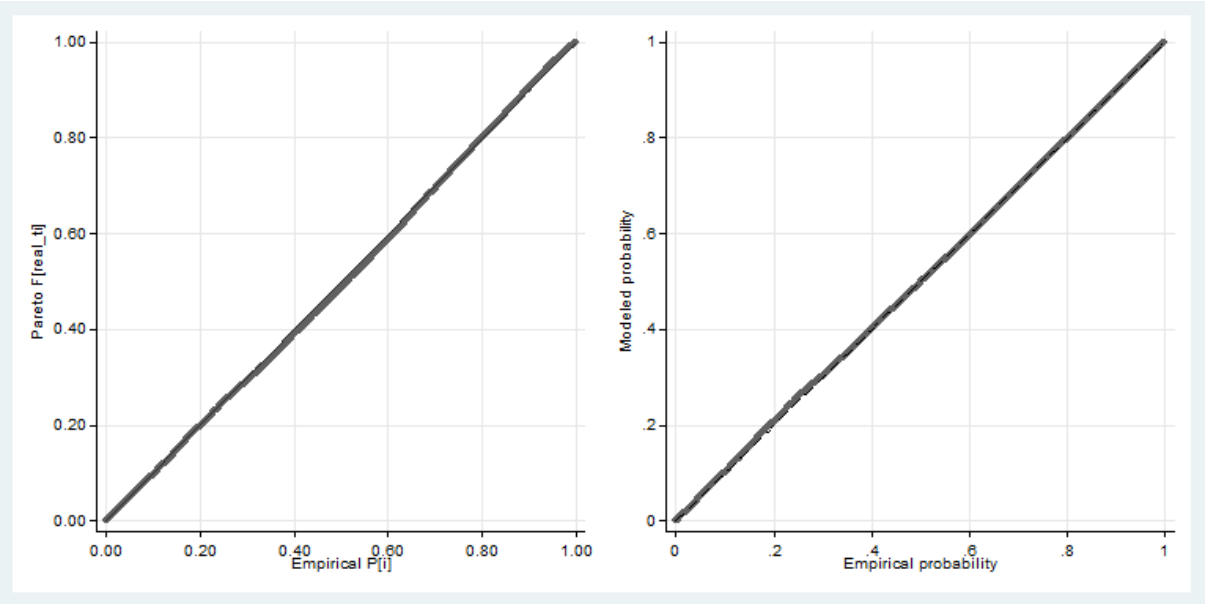


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1997, threshold = £60,000

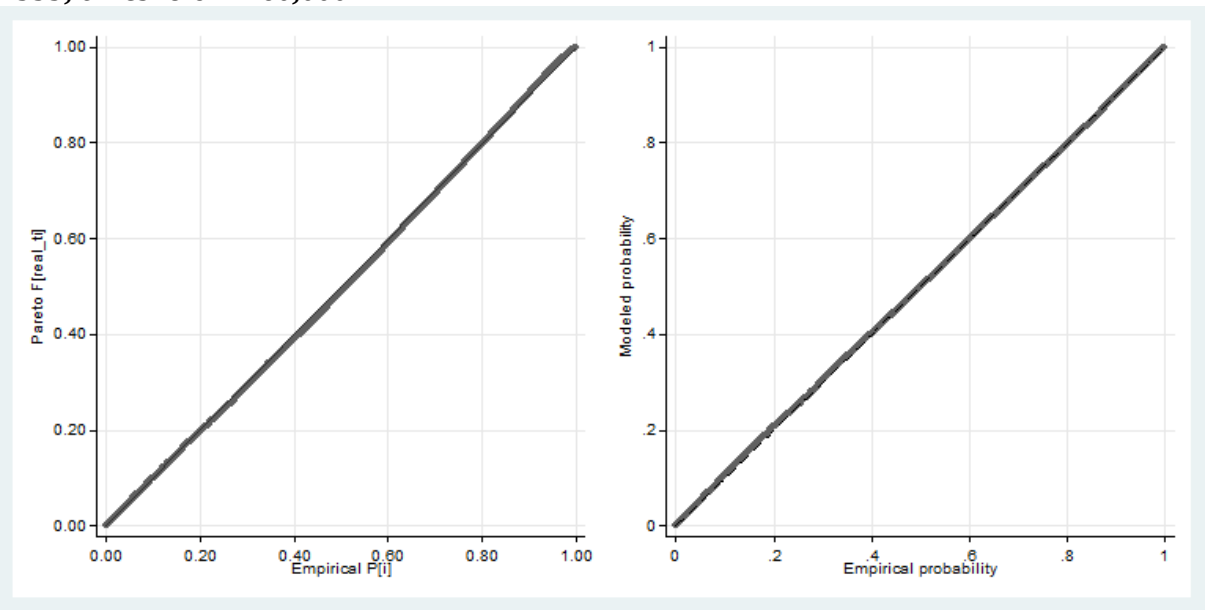


1998, threshold = £60,000

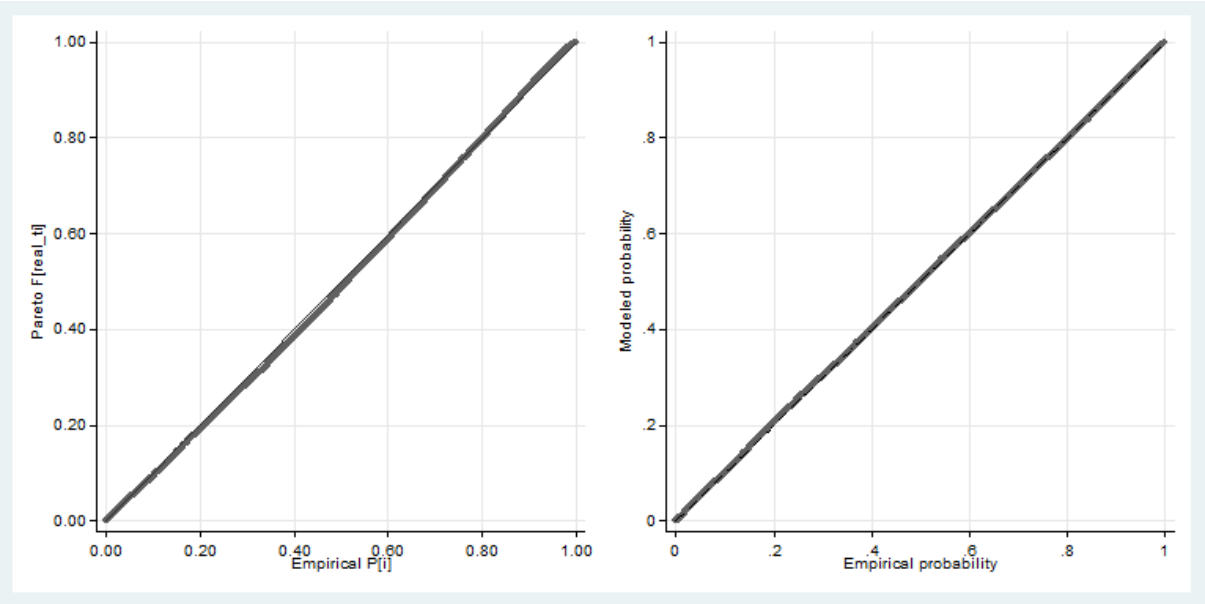


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1999, threshold = £60,000

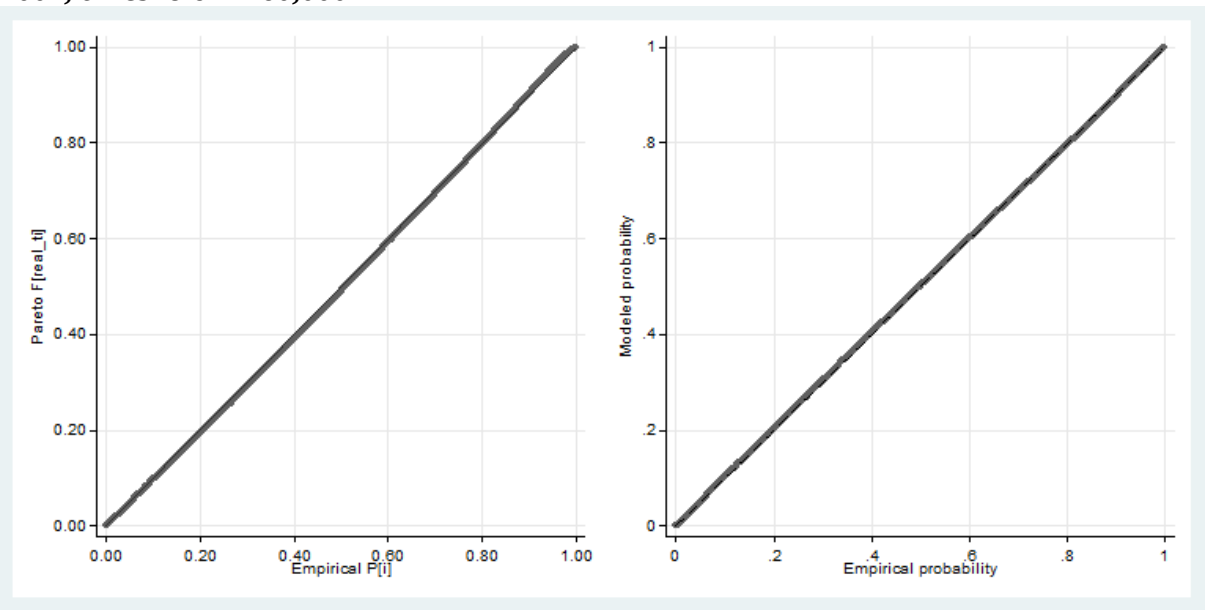


2000, threshold = £60,000

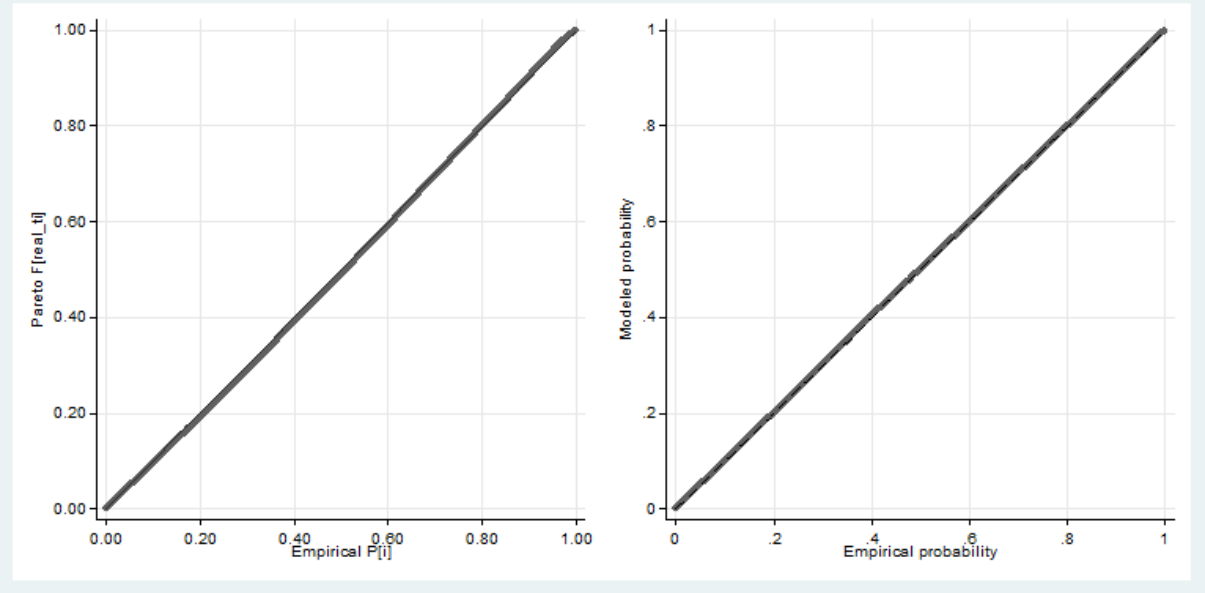


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2001, threshold = £60,000

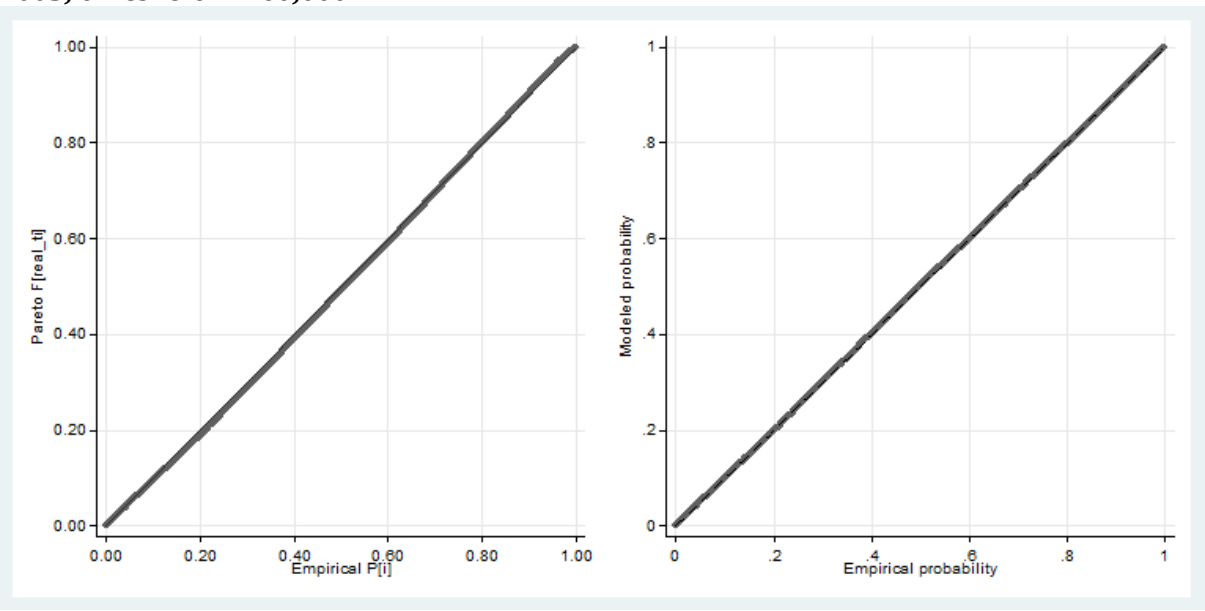


2002, threshold = £60,000

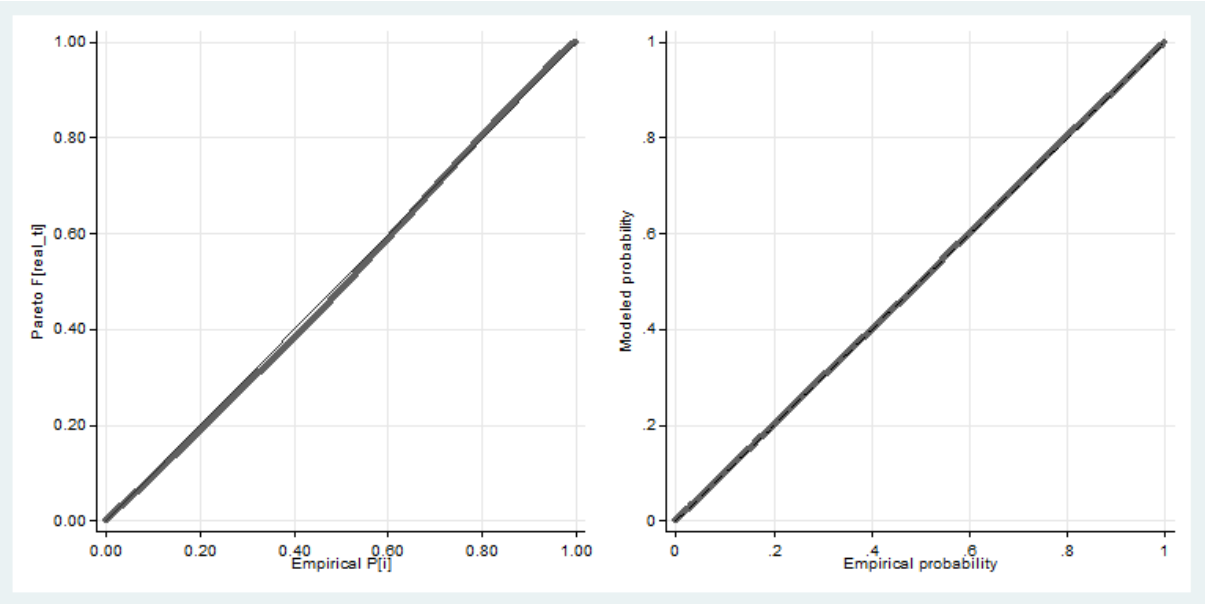


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2003, threshold = £60,000

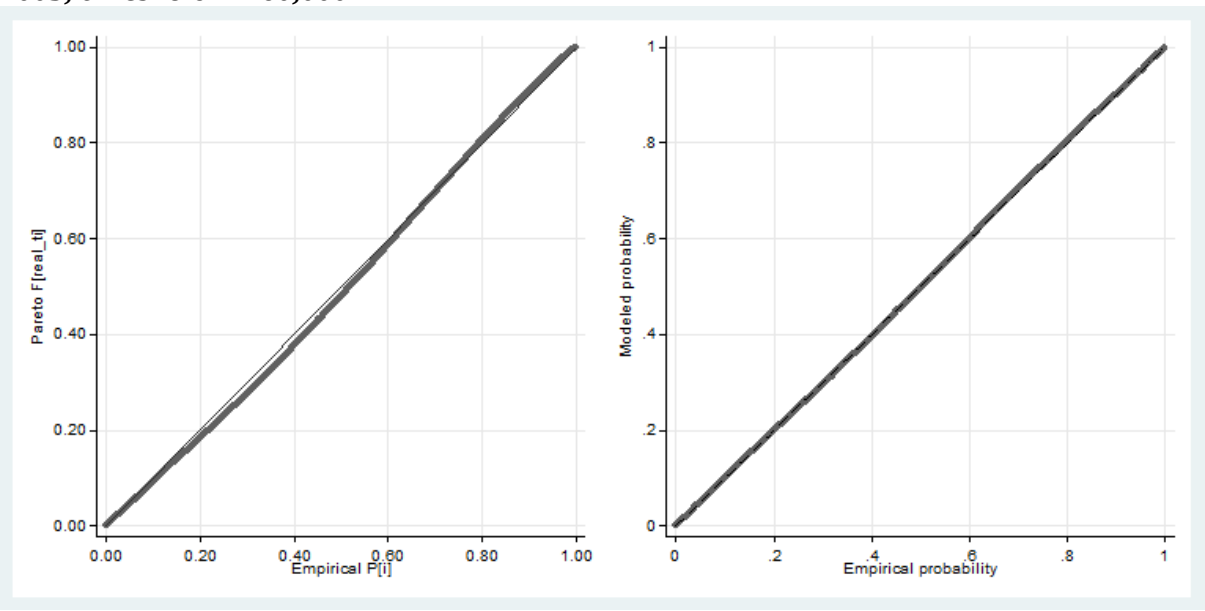


2004, threshold = £60,000

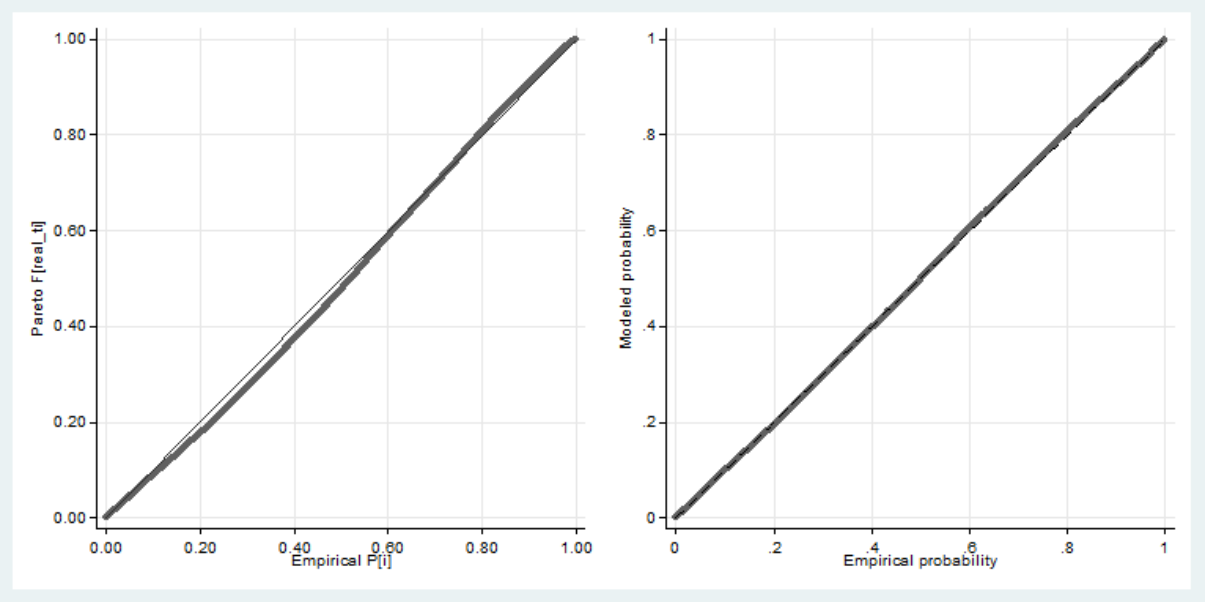


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2005, threshold = £60,000

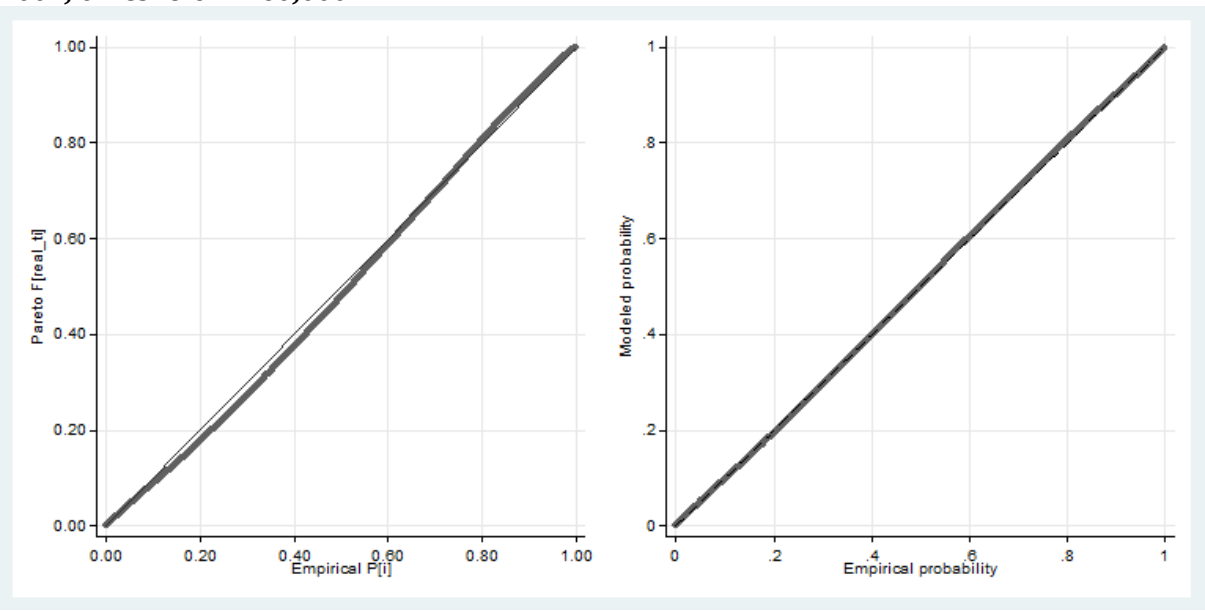


2006, threshold = £60,000

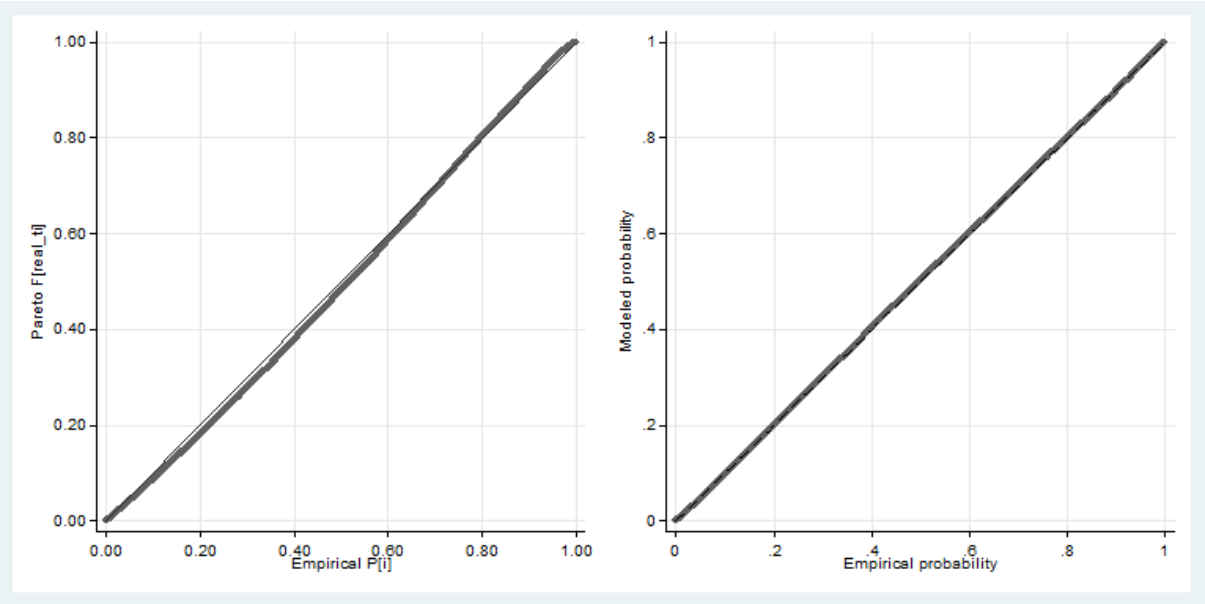


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2007, threshold = £60,000

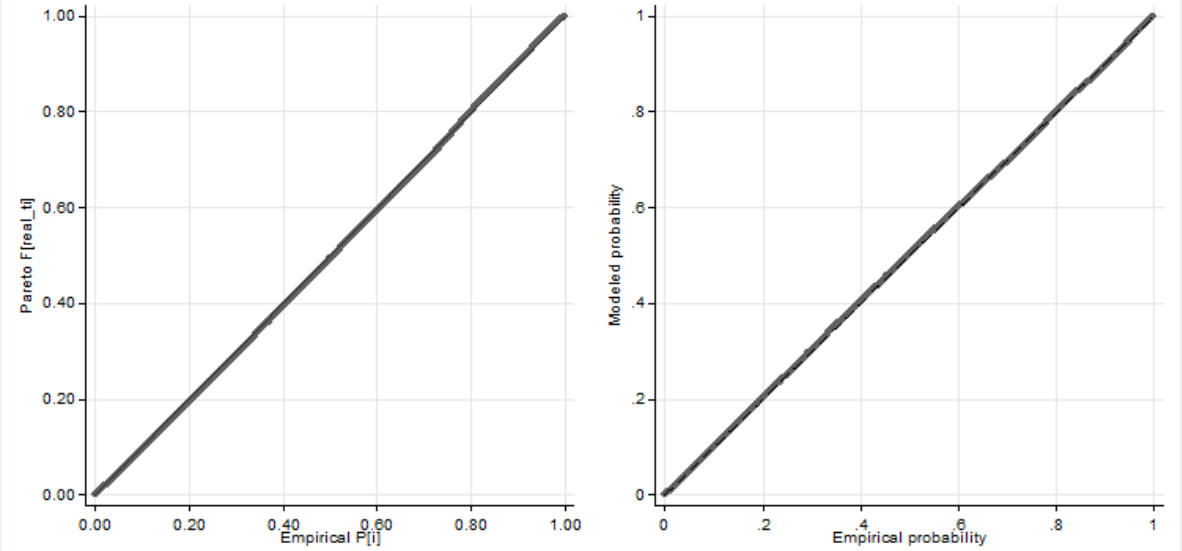


2009, threshold = £60,000



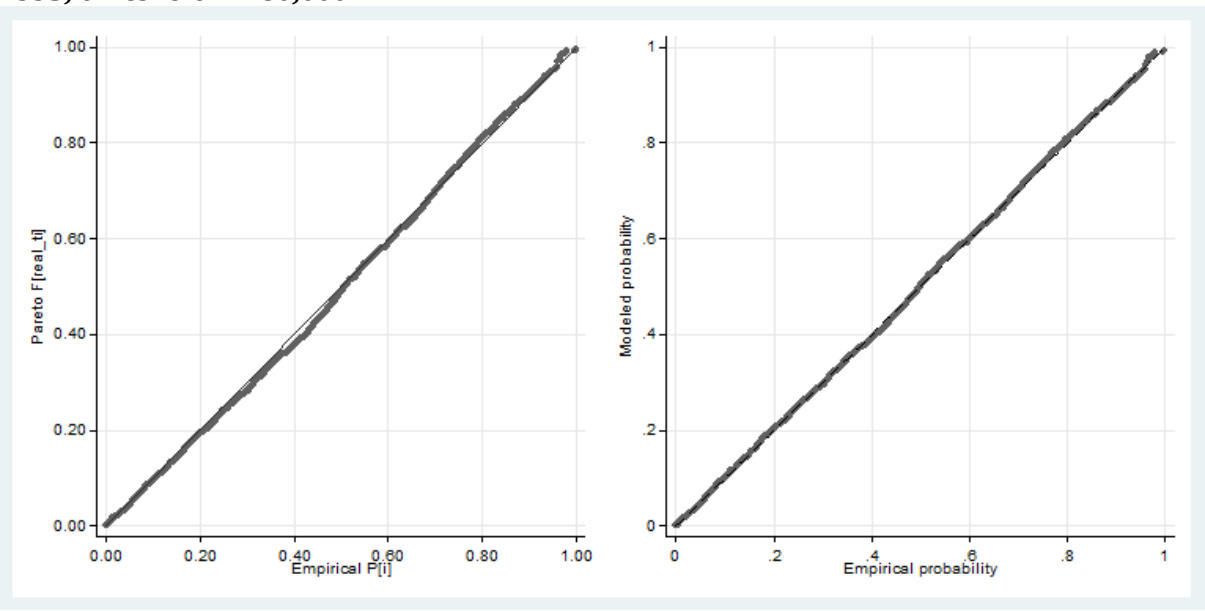
PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2010, threshold = £60,000

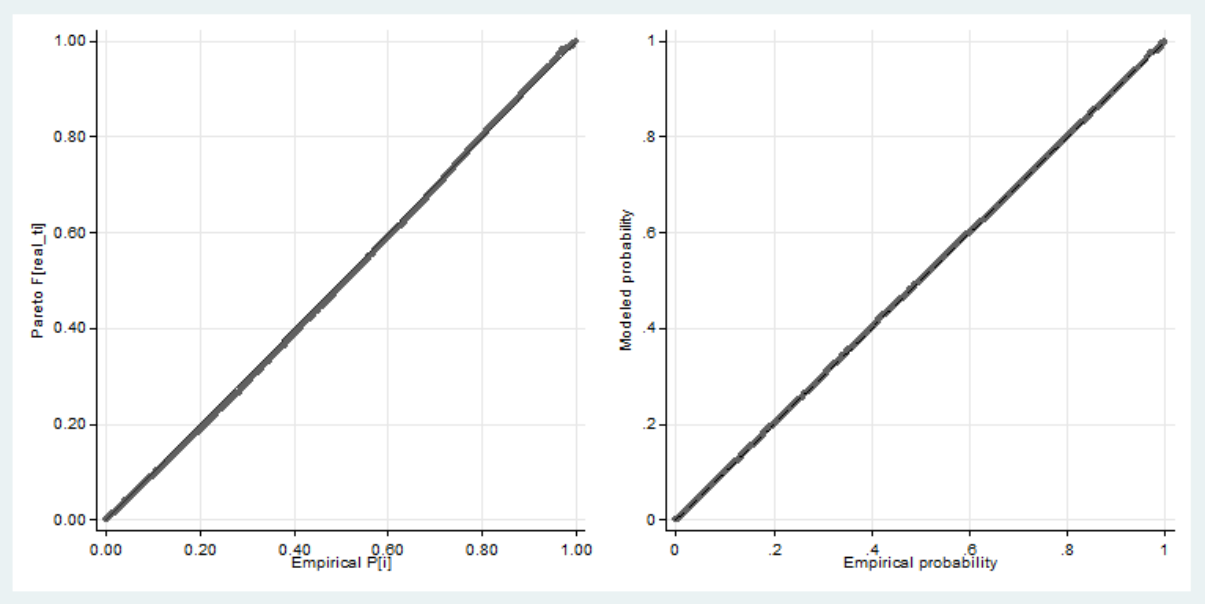


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1995, threshold = £80,000

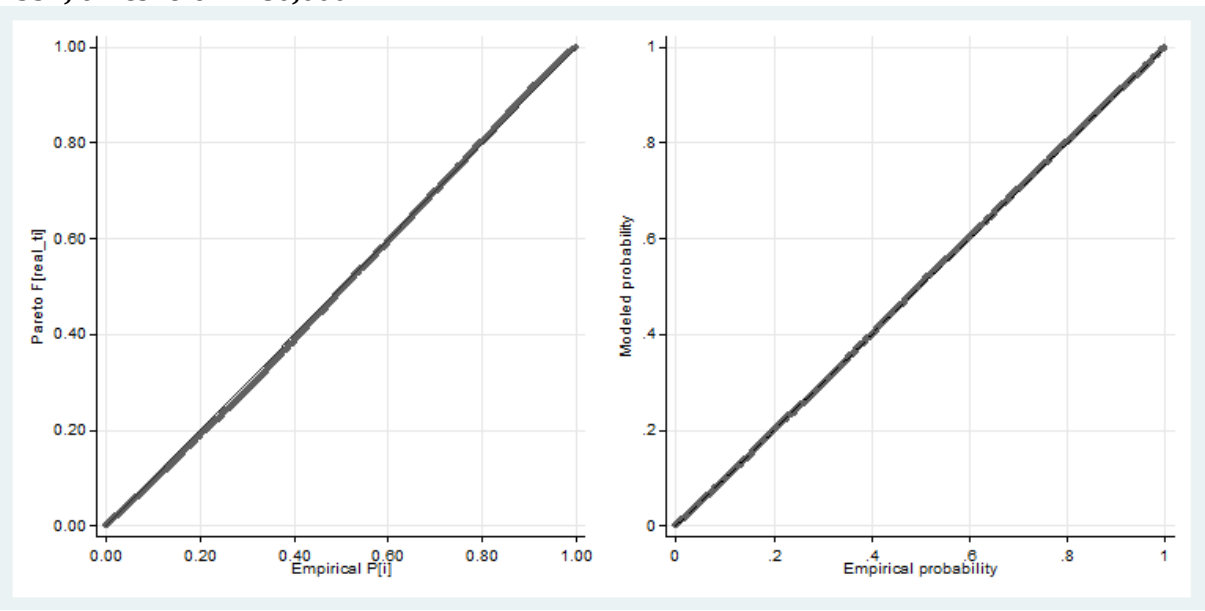


1996, threshold = £80,000

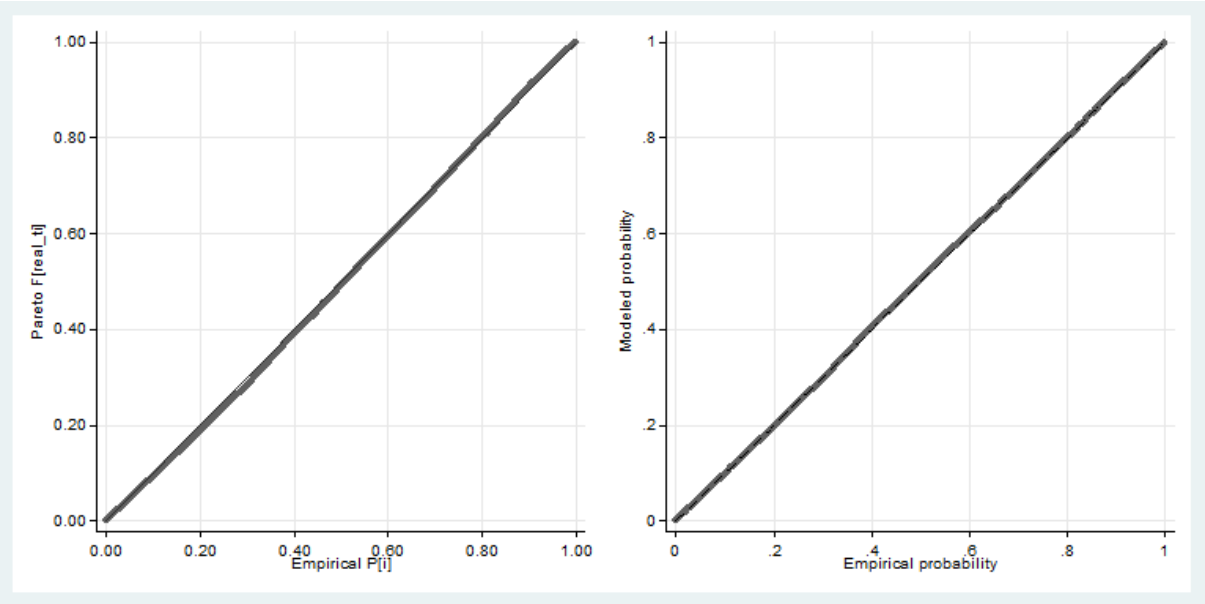


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1997, threshold = £80,000

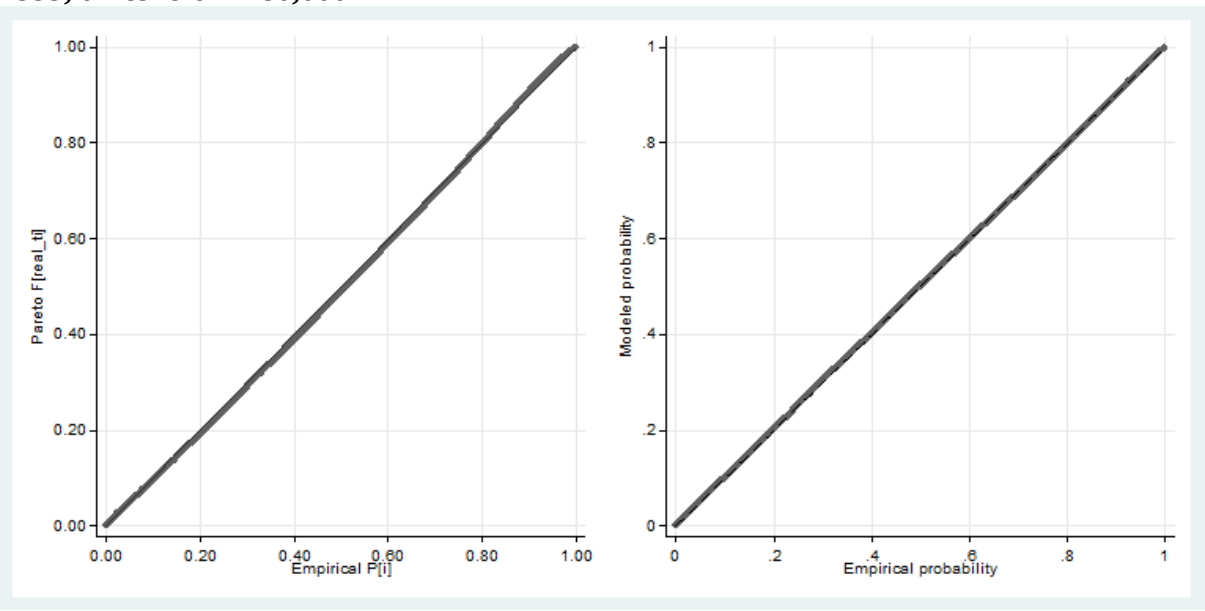


1998, threshold = £80,000

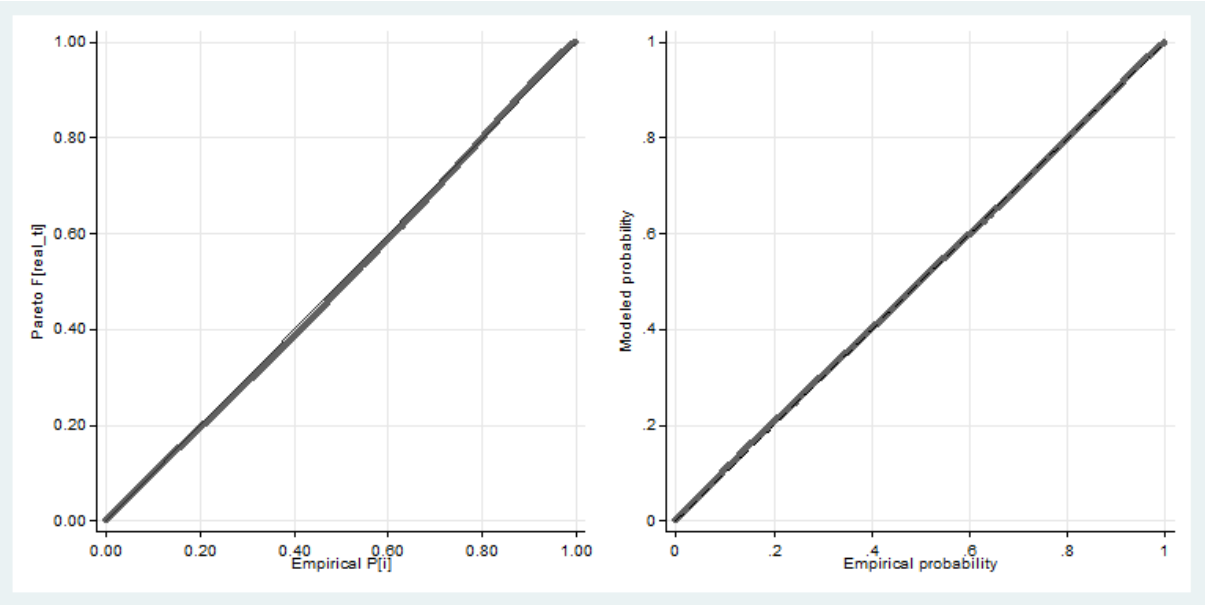


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1999, threshold = £80,000

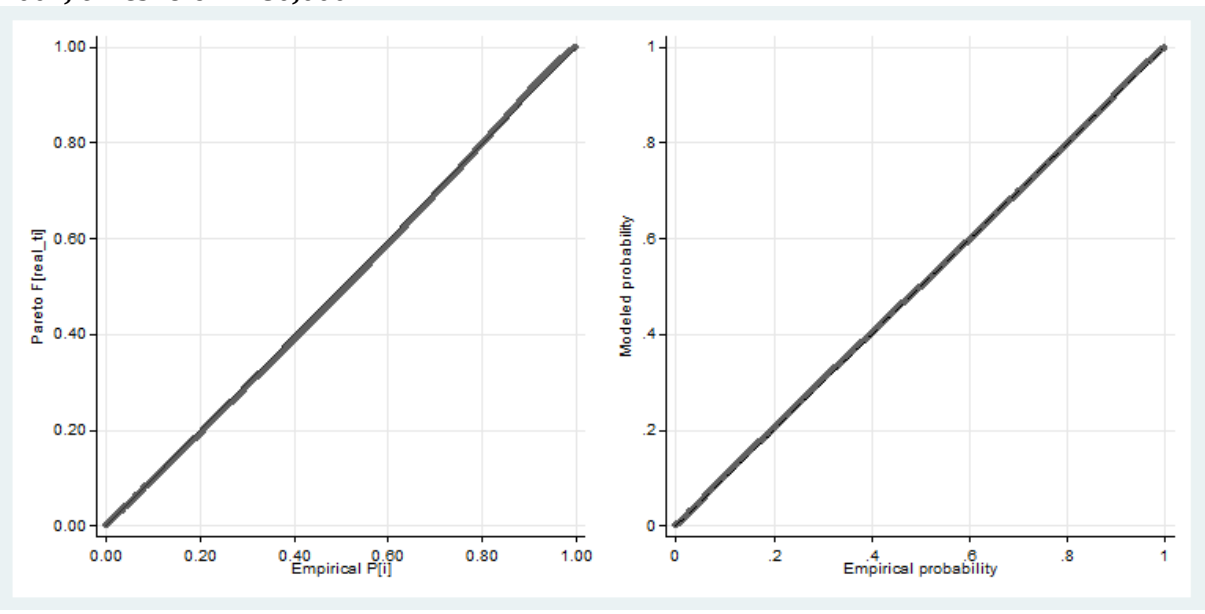


2000, threshold = £80,000

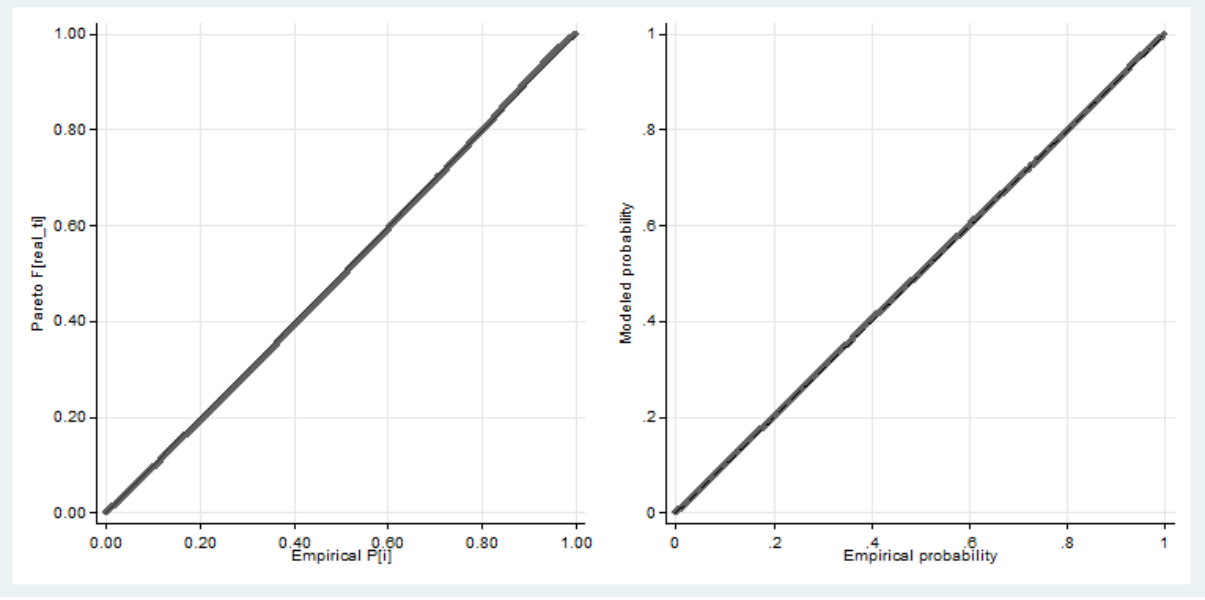


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2001, threshold = £80,000

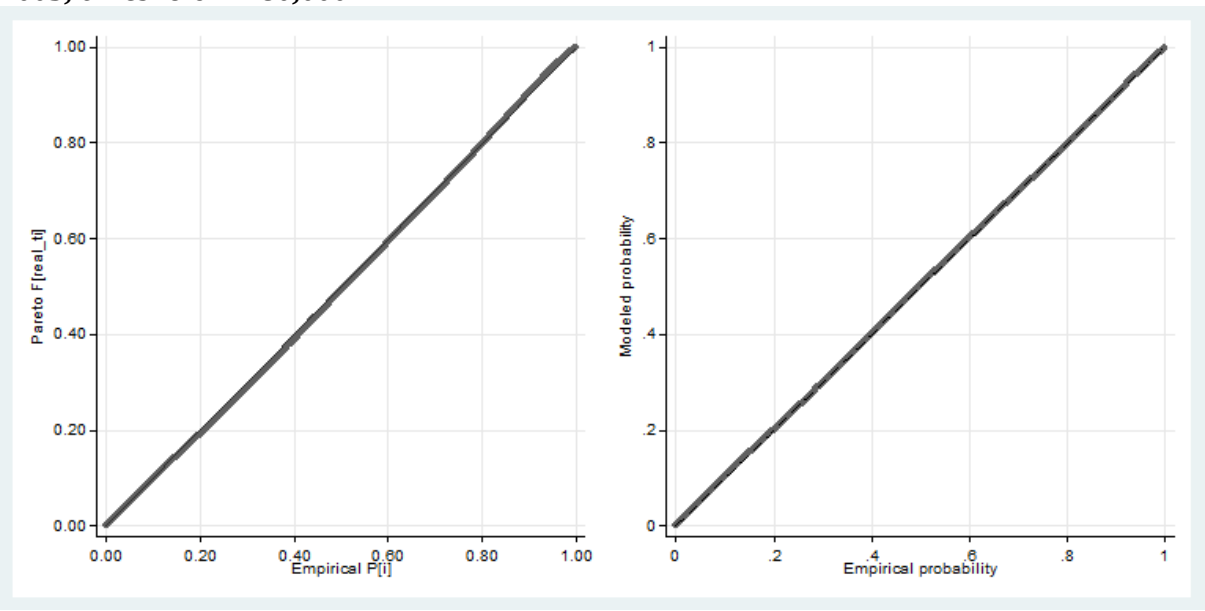


2002, threshold = £80,000

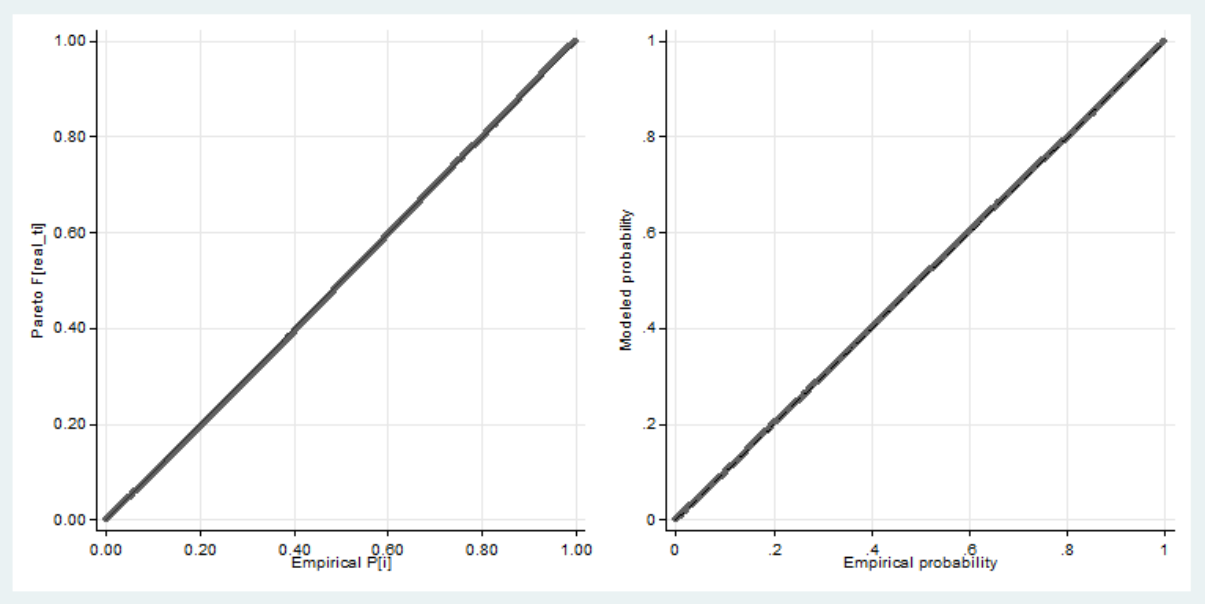


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2003, threshold = £80,000

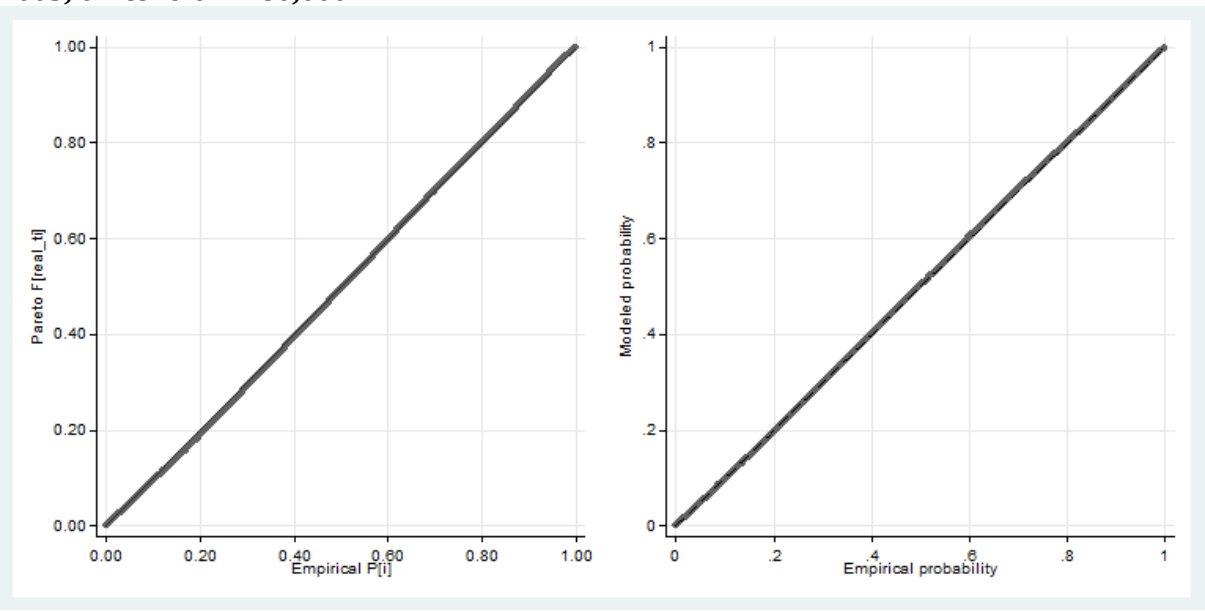


2004, threshold = £80,000

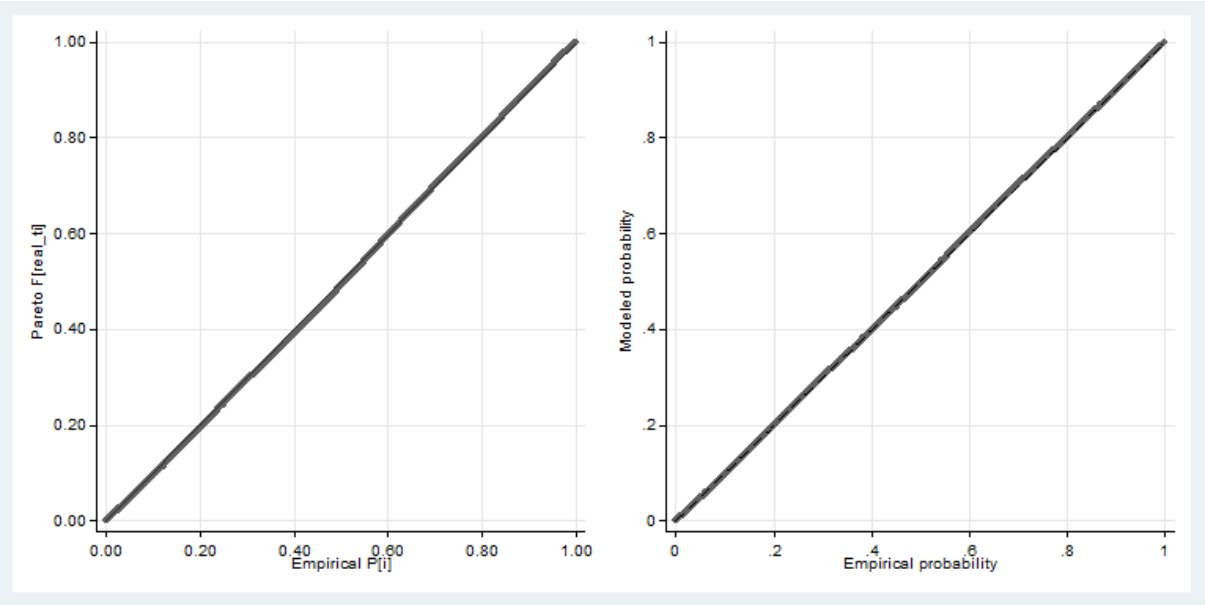


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2005, threshold = £80,000

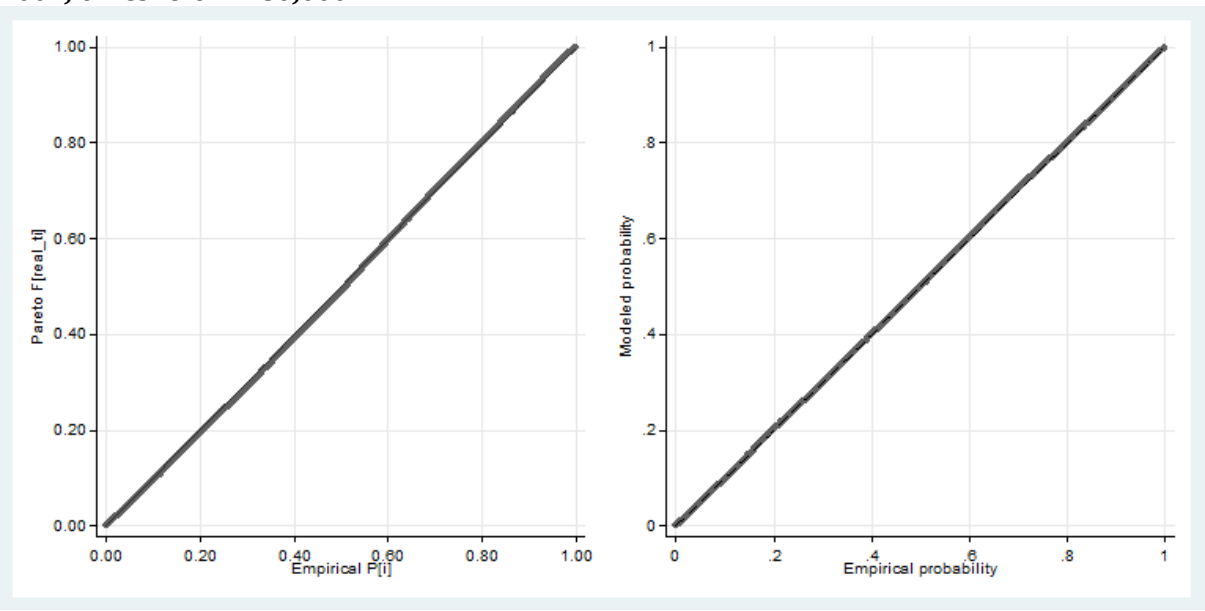


2006, threshold = £80,000

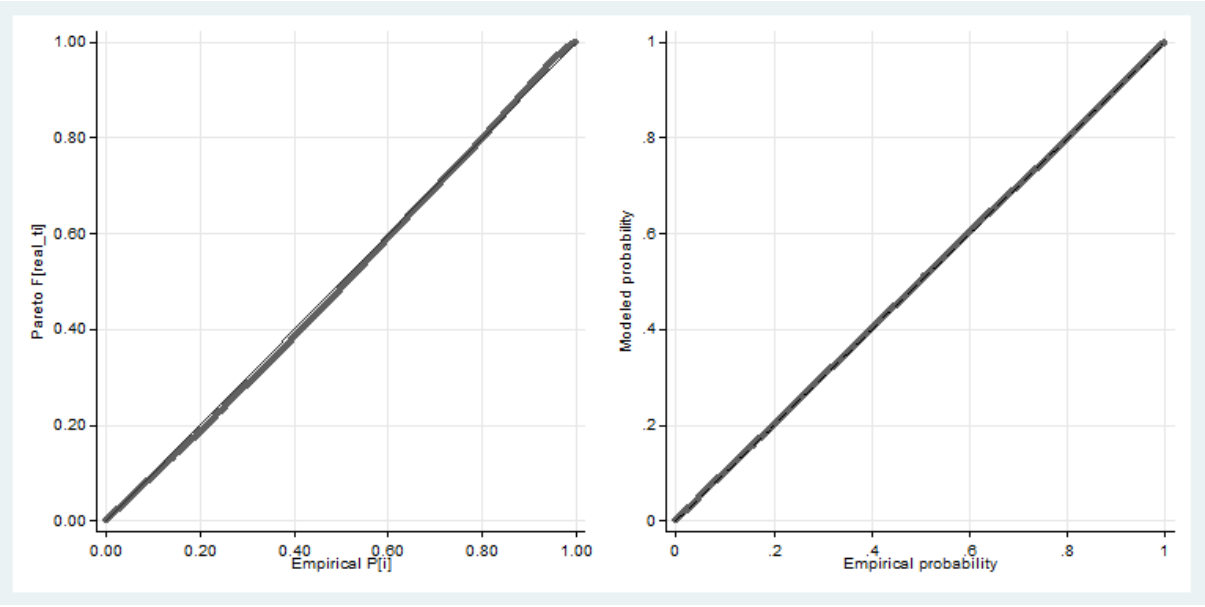


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2007, threshold = £80,000

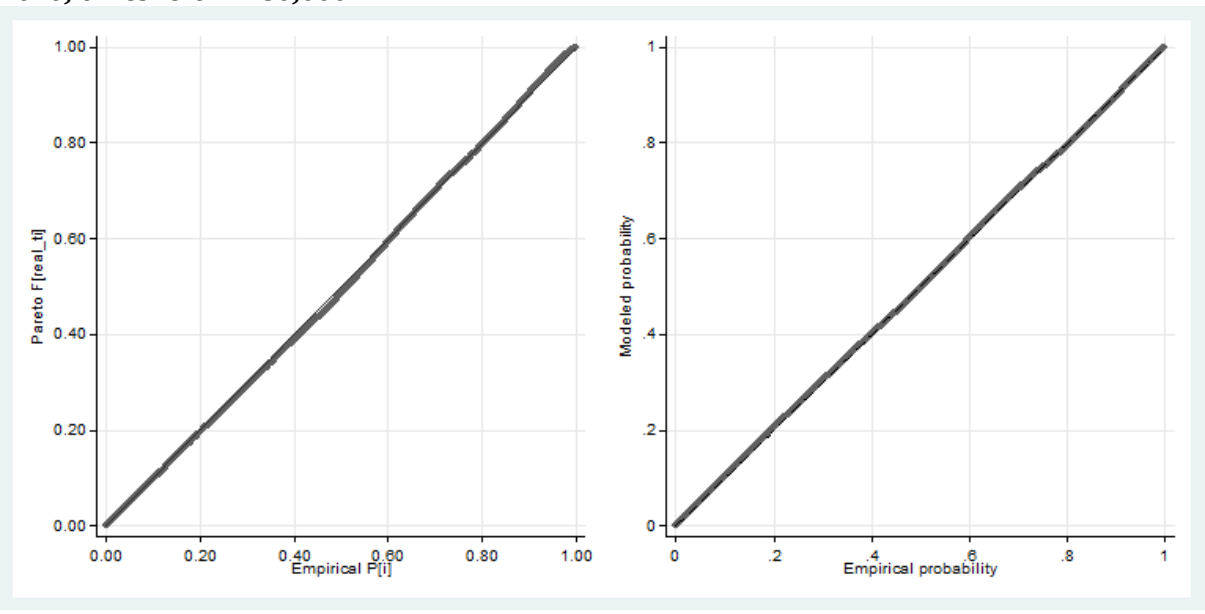


2009, threshold = £80,000



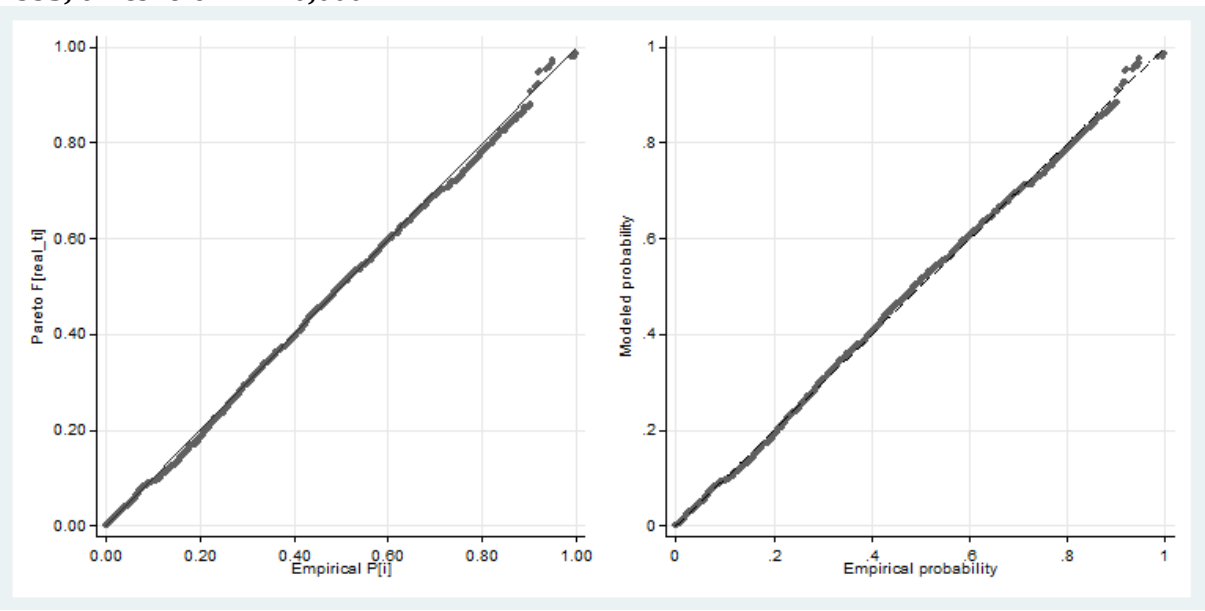
PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2010, threshold = £80,000

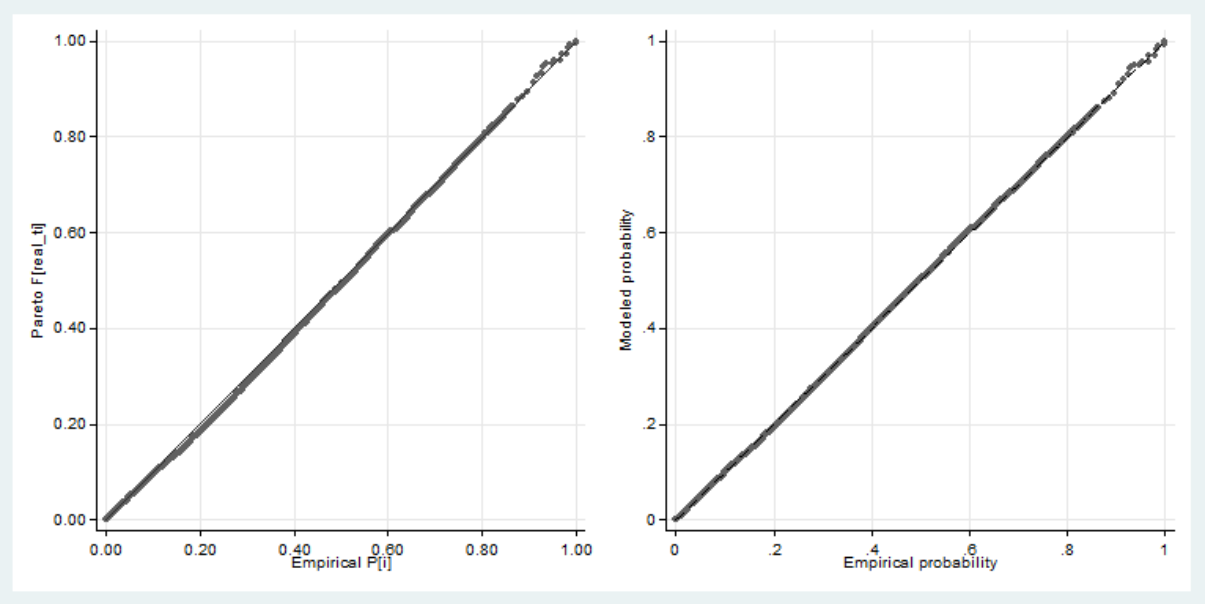


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1995, threshold = £120,000

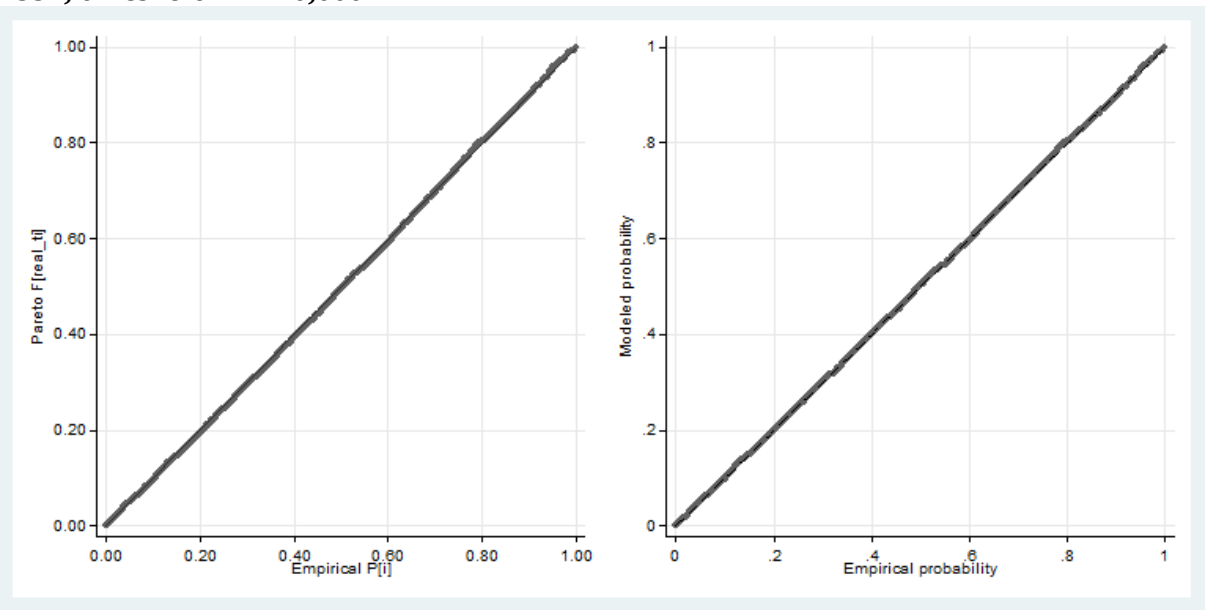


1996, threshold = £120,000

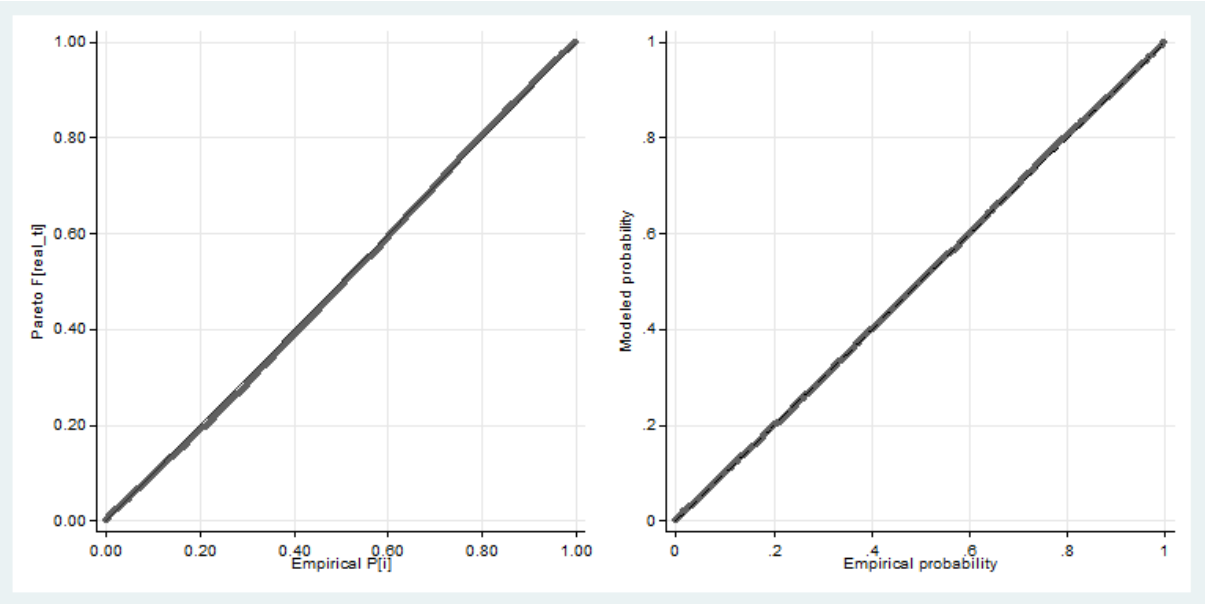


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1997, threshold = £120,000

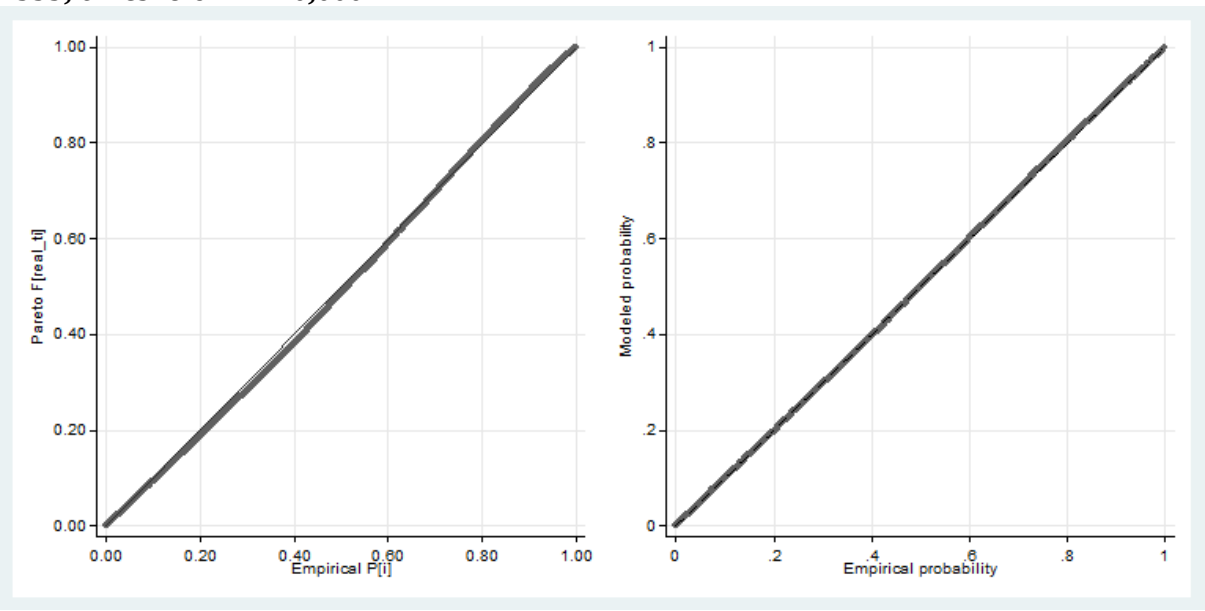


1998, threshold = £120,000

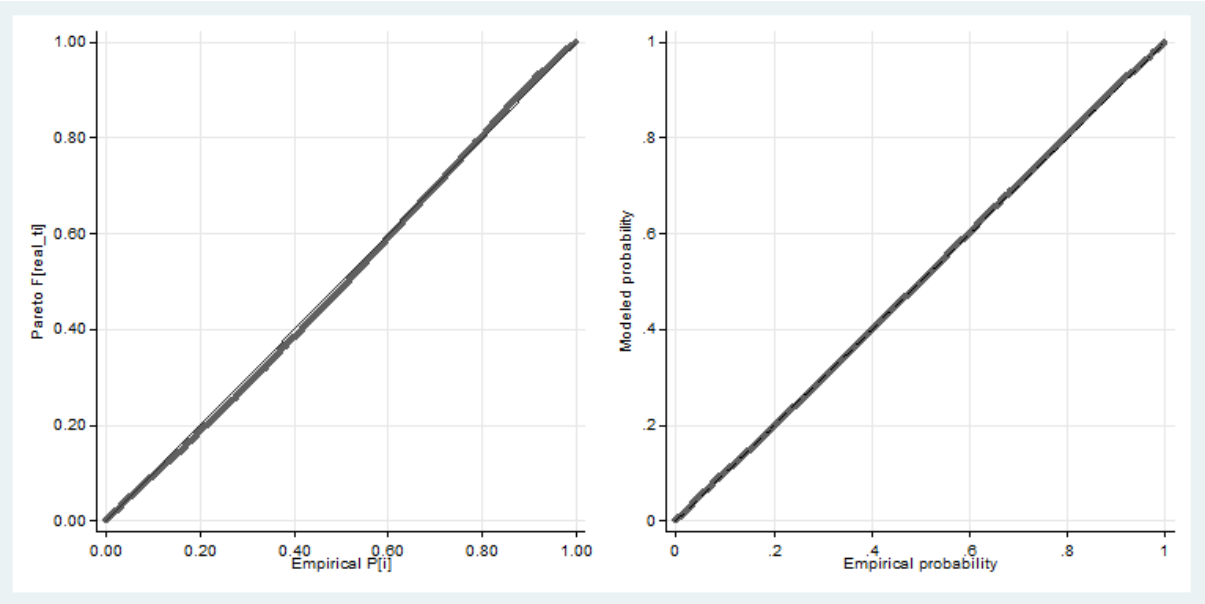


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

1999, threshold = £120,000

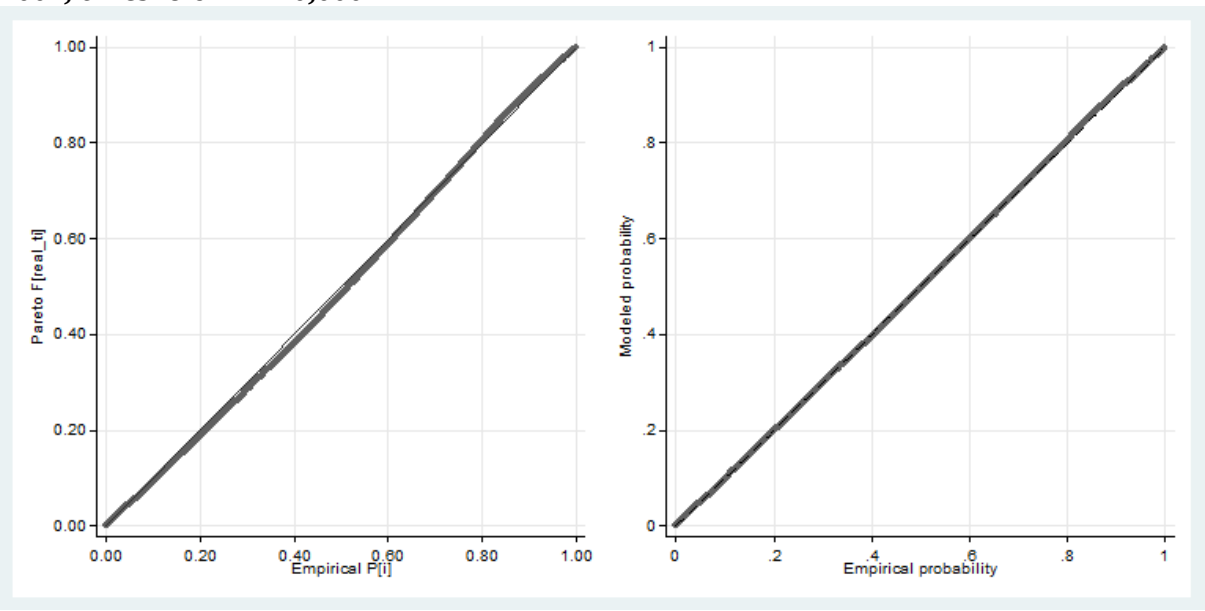


2000, threshold = £120,000

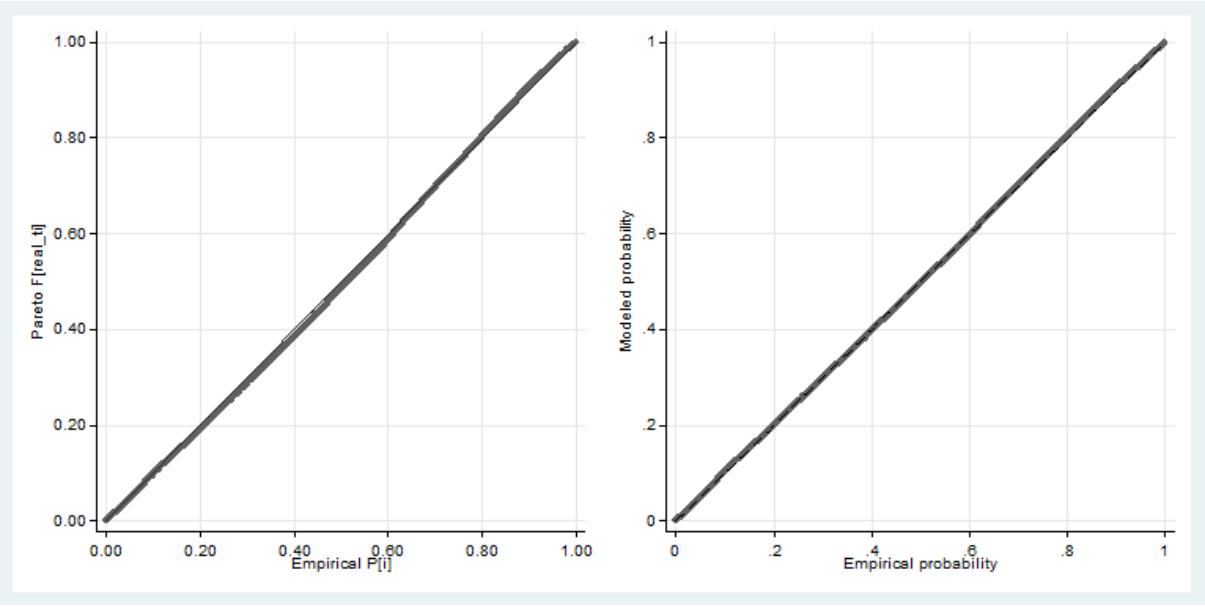


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2001, threshold = £120,000

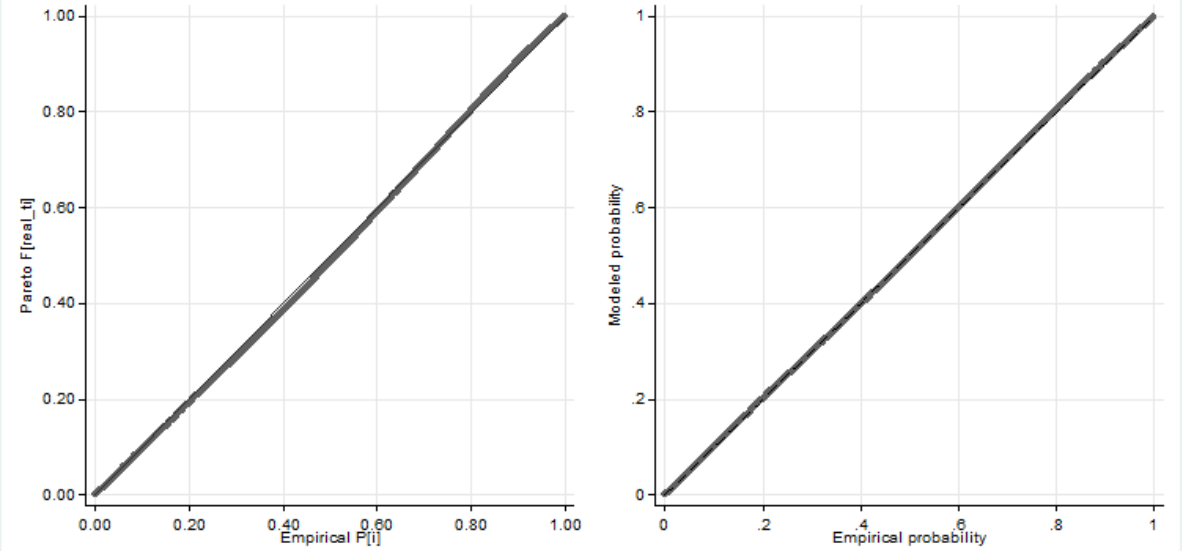


2002, threshold = £120,000

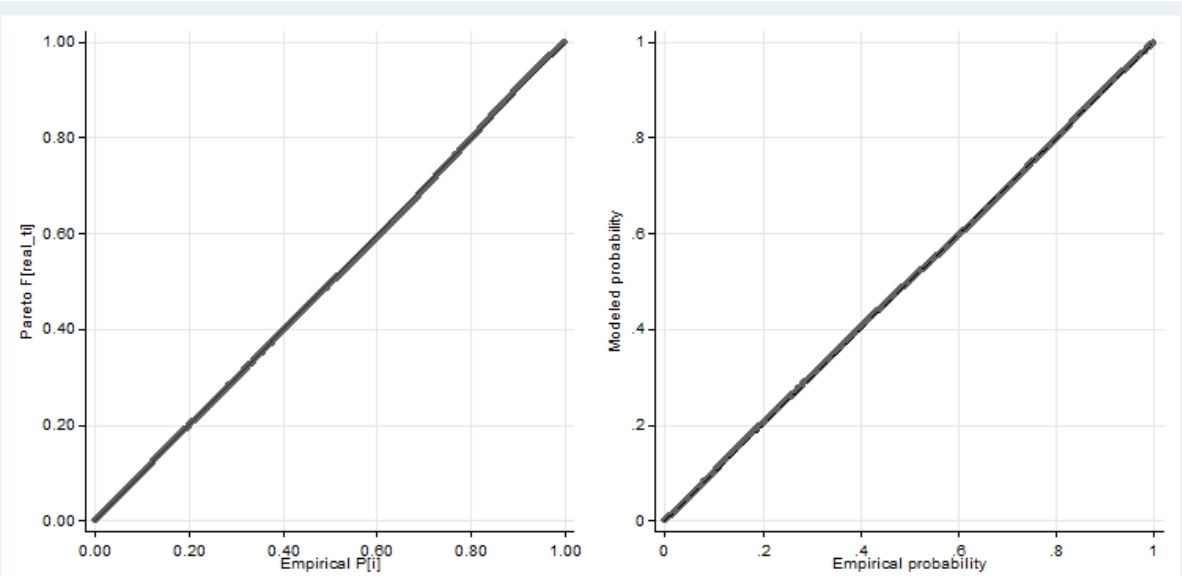


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2003, threshold = £120,000

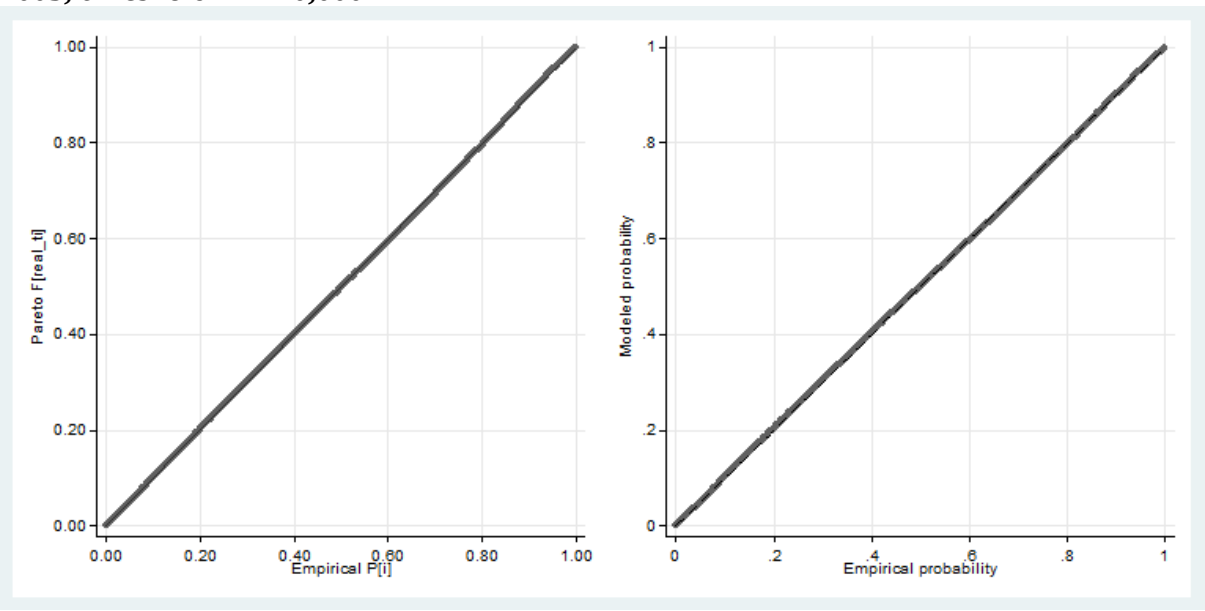


2004, threshold = £120,000

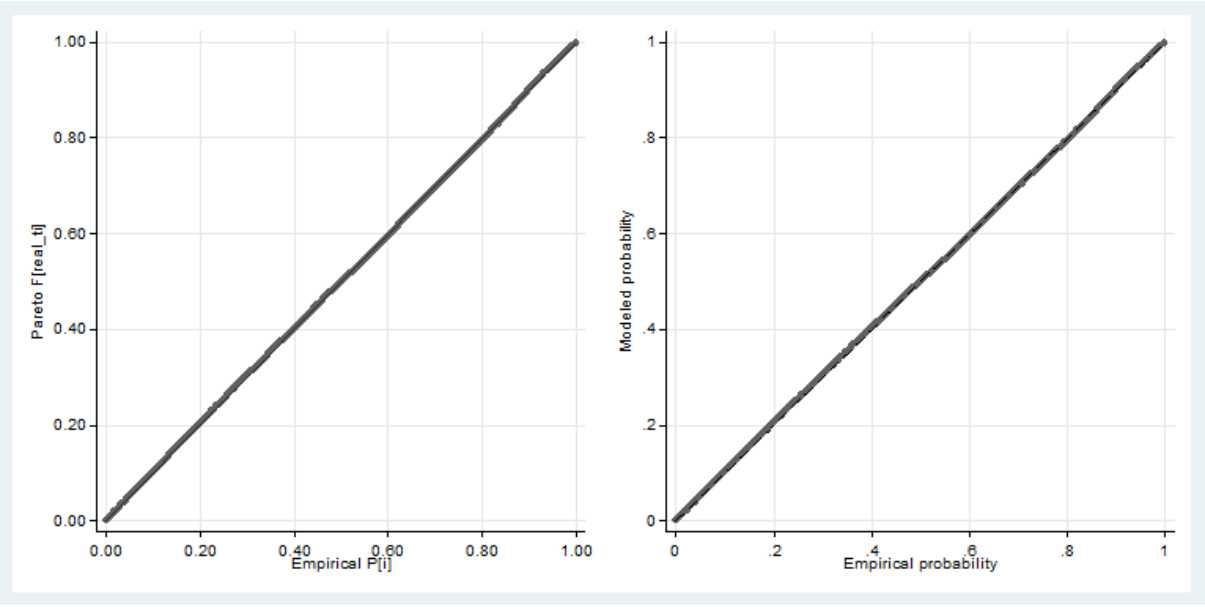


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2005, threshold = £120,000

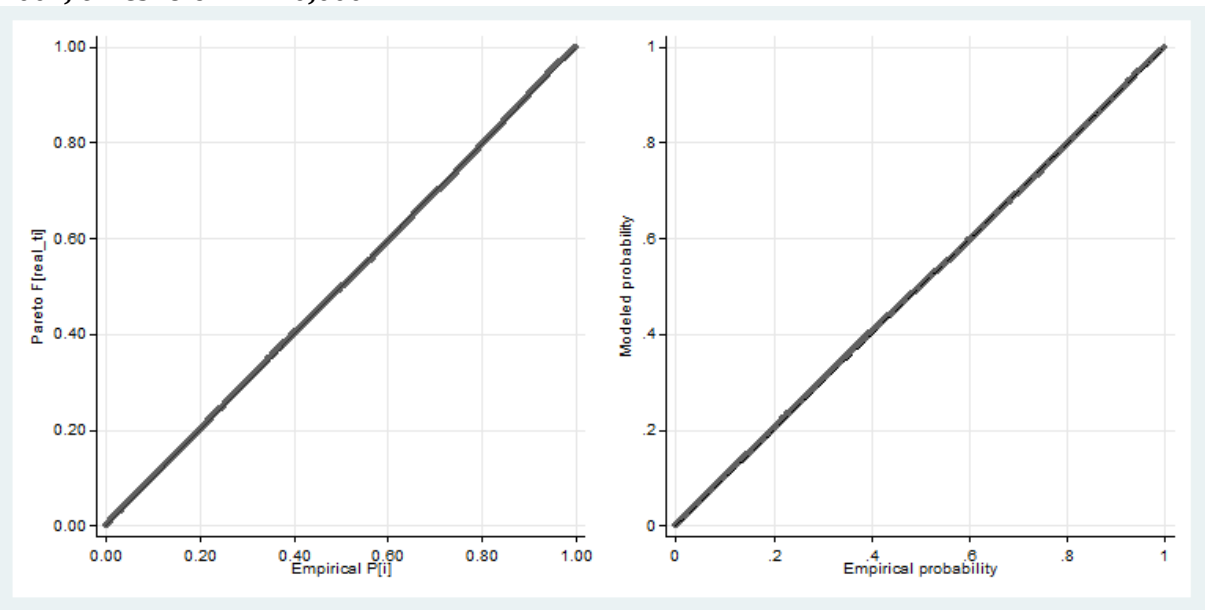


2006, threshold = £120,000

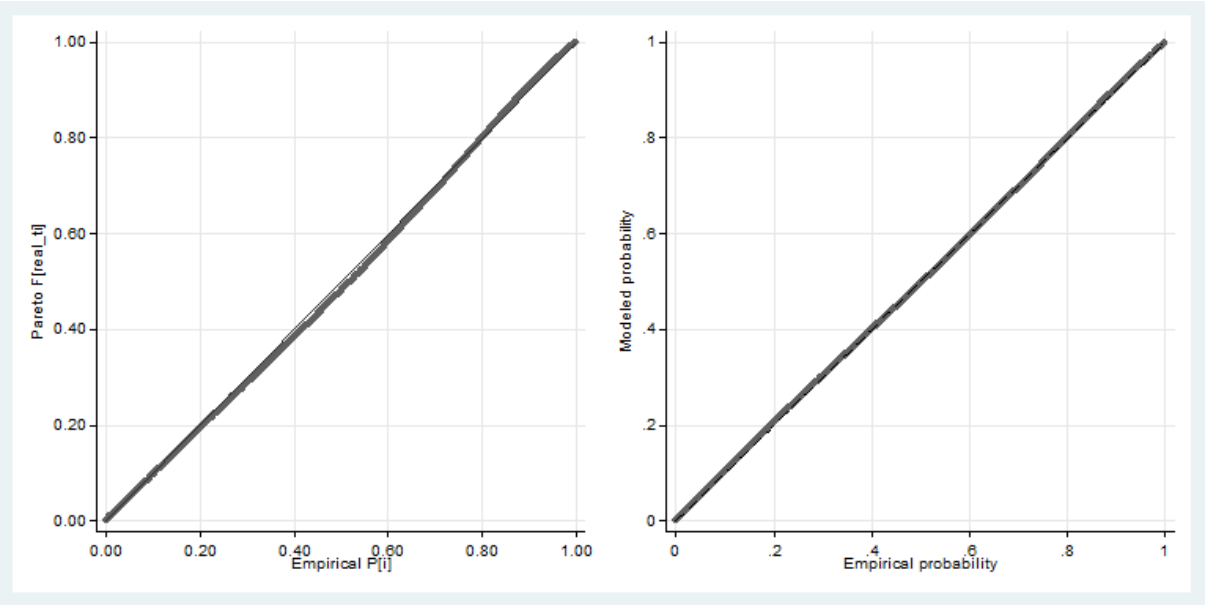


PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

2007, threshold = £120,000

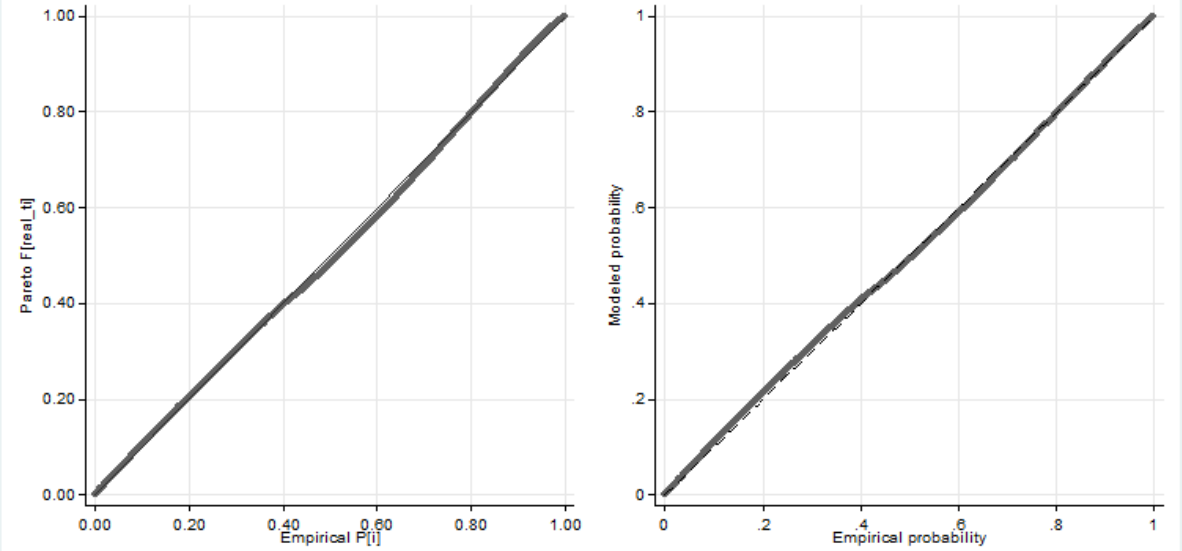


2009, threshold = £120,000



PP plots by year and threshold: Pareto I on left; Pareto II on right (pareto07)

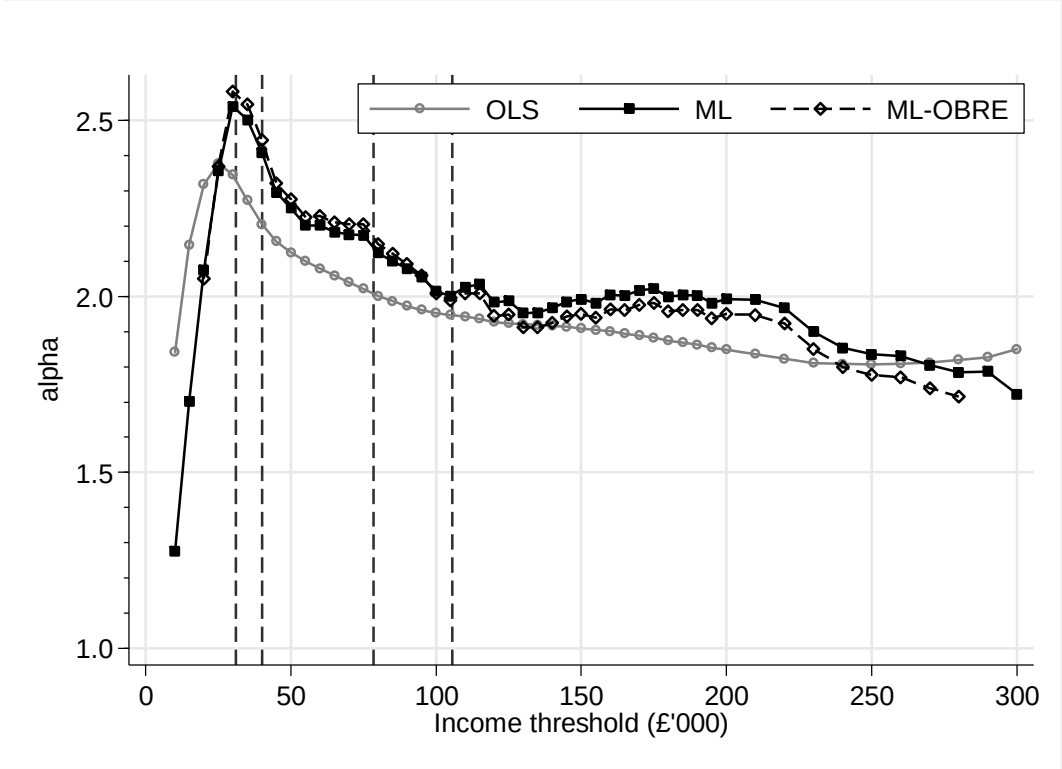
2010, threshold = £120,000



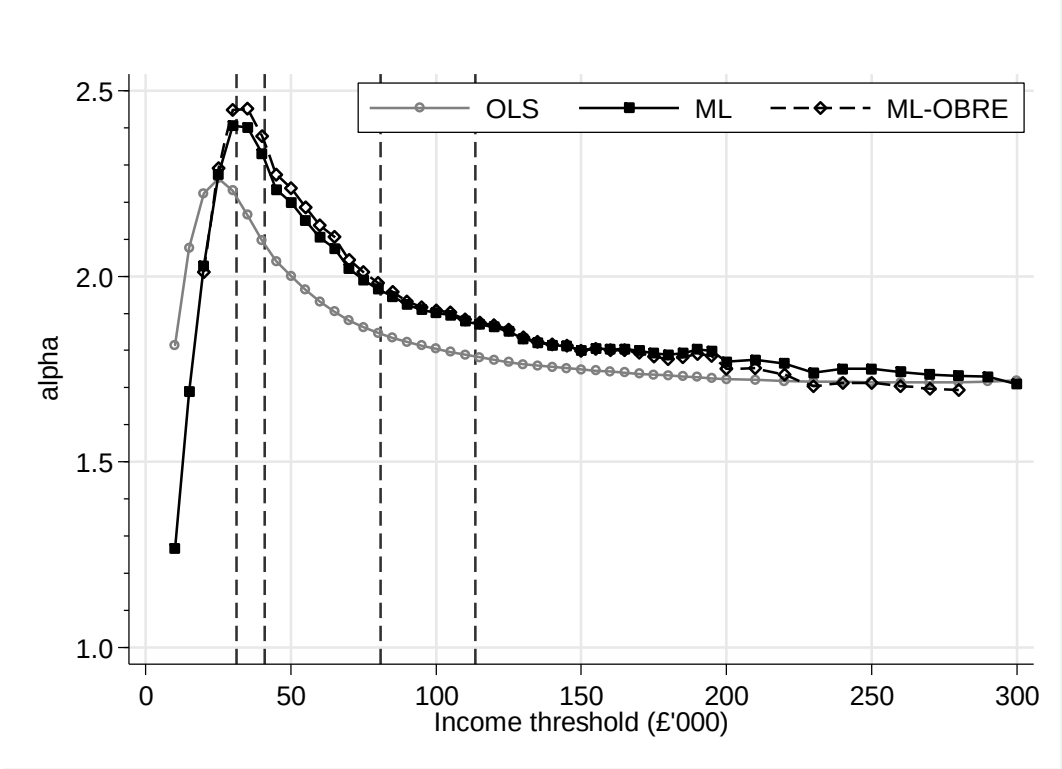
Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

Pareto I

1995

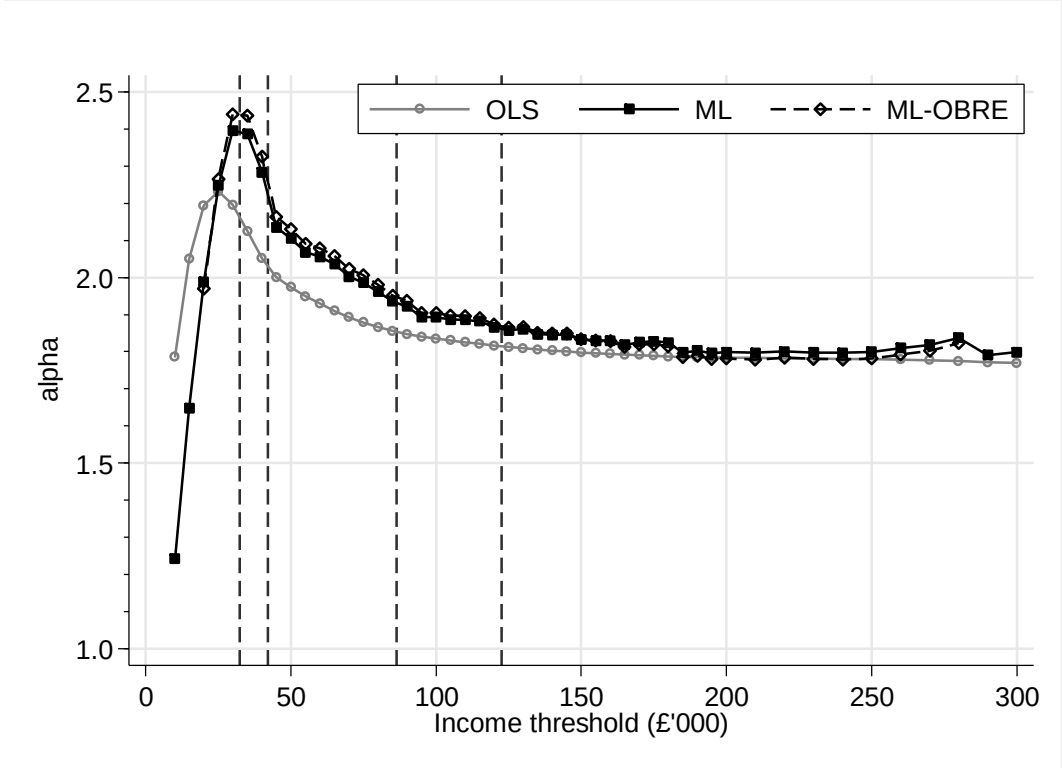


1996

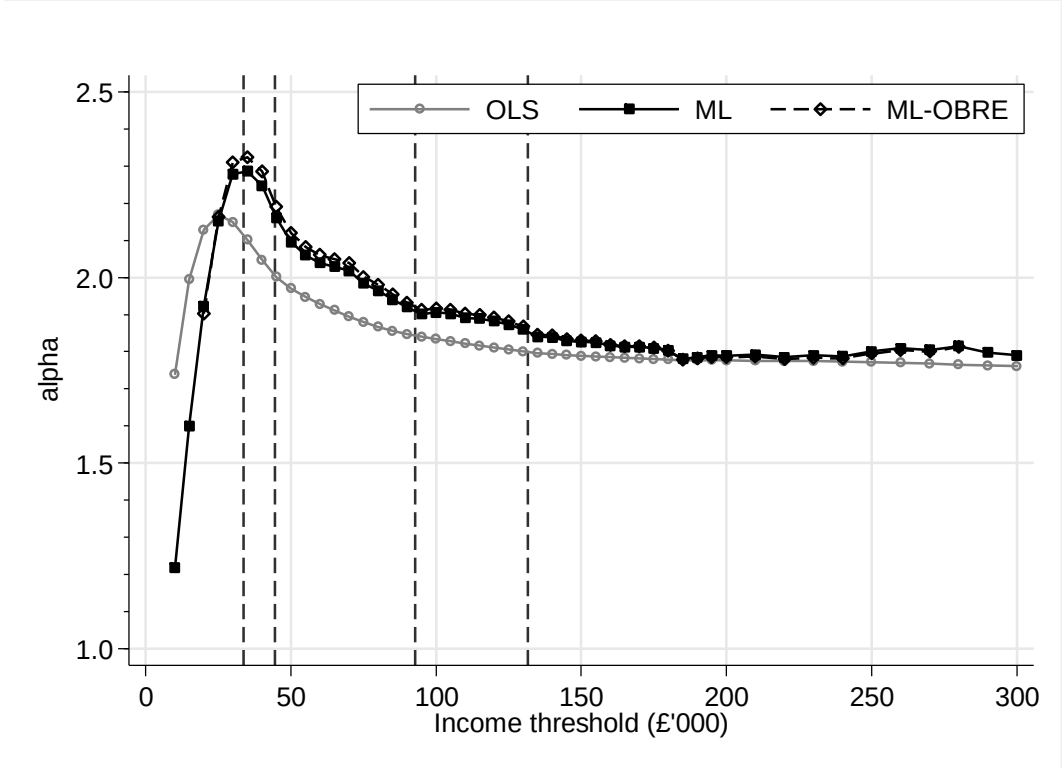


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

1997

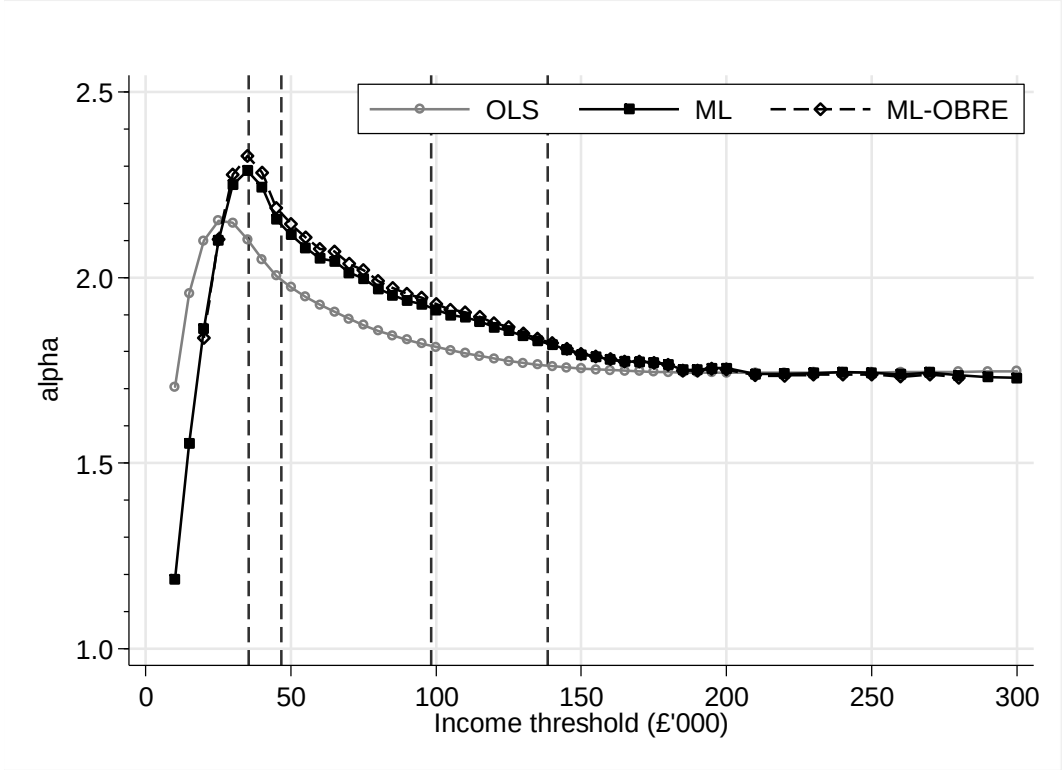


1998

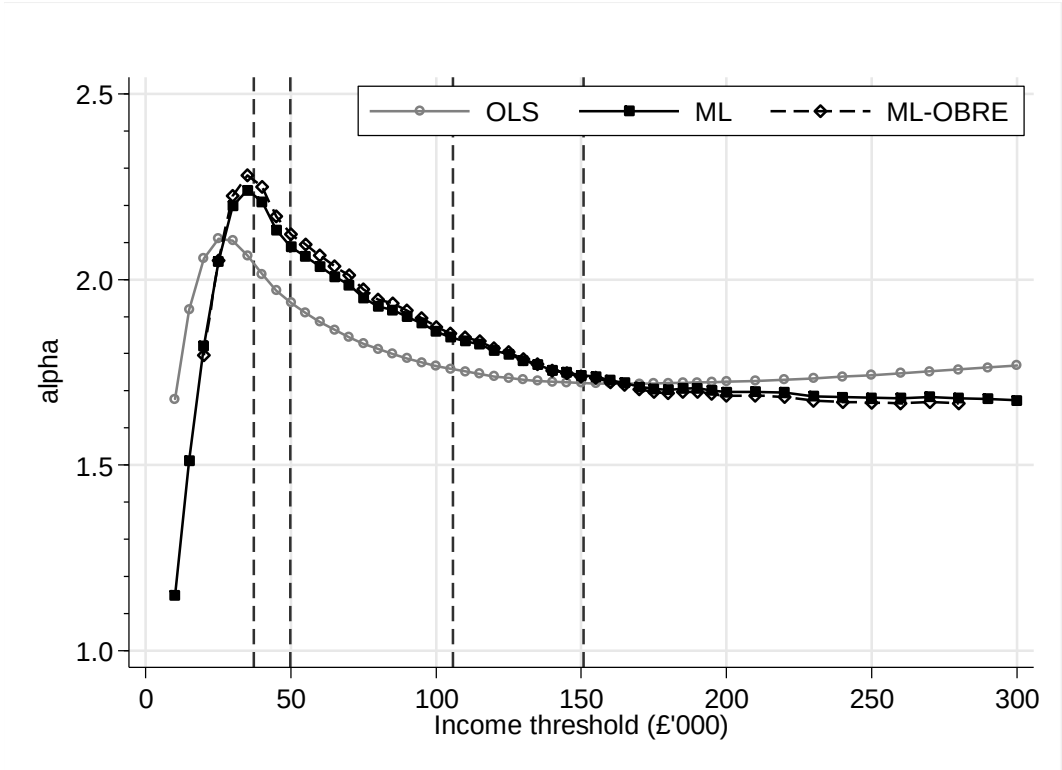


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

1999

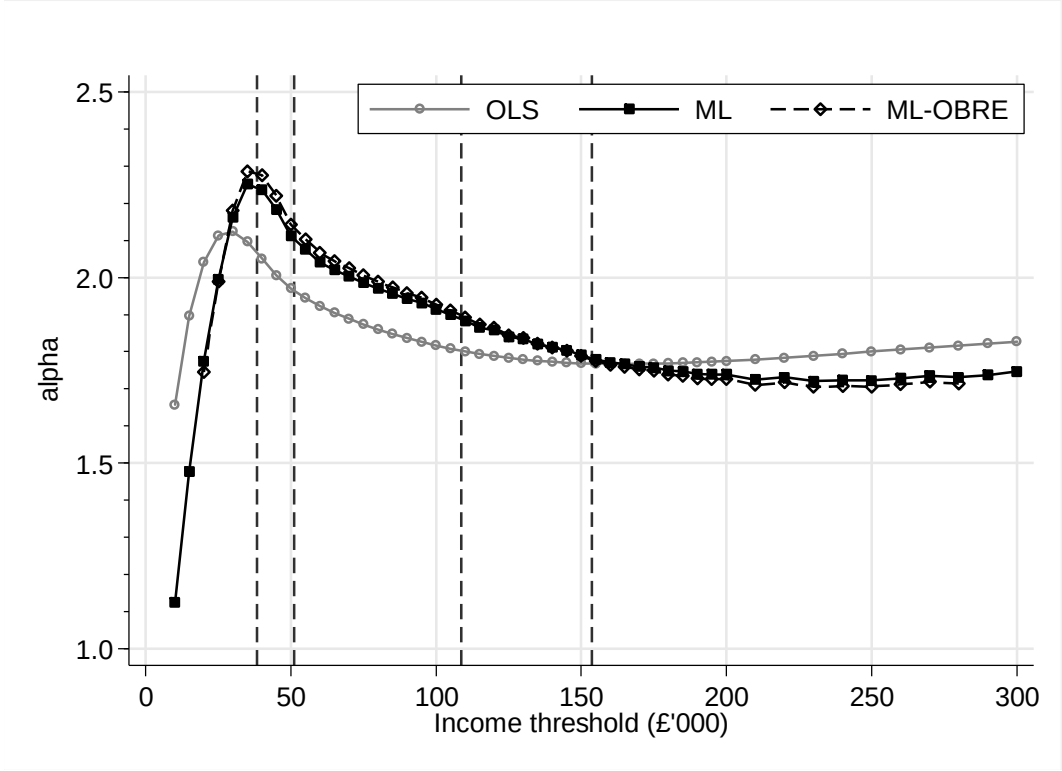


2000

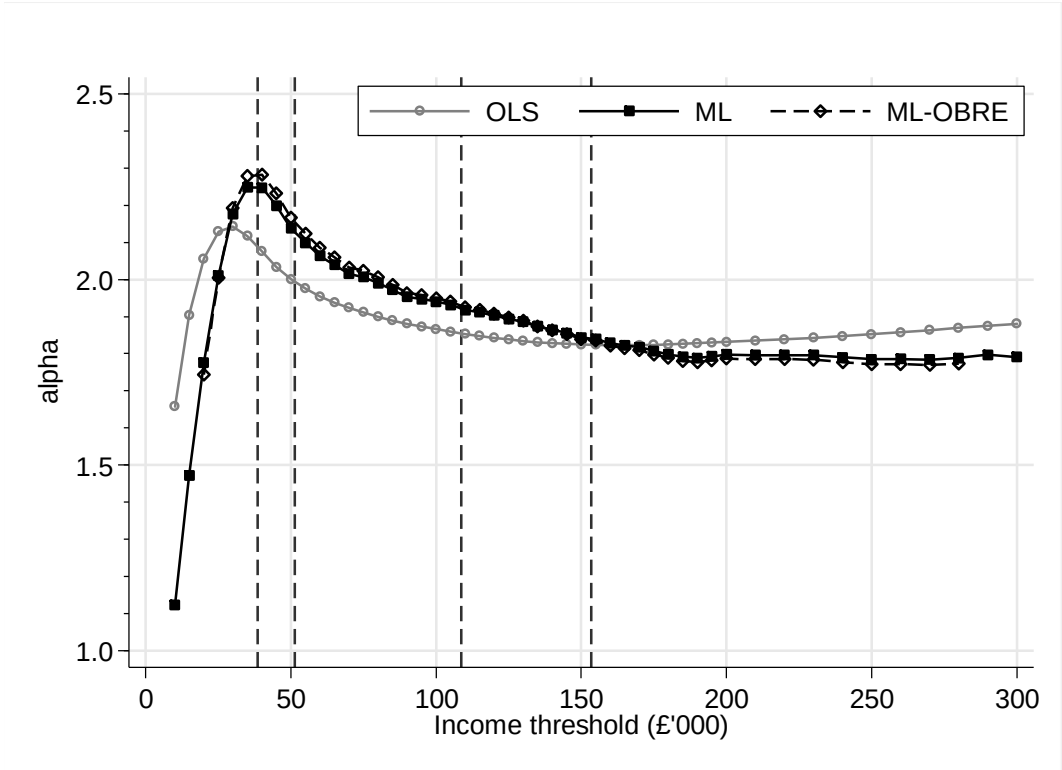


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

2001

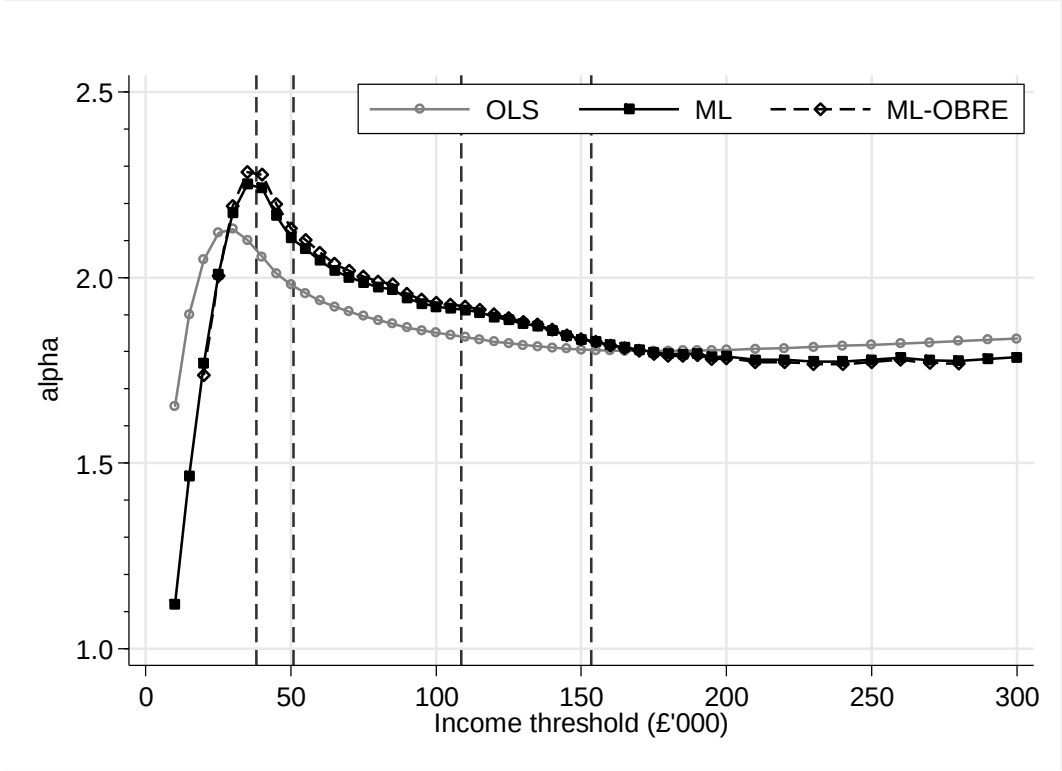


2002

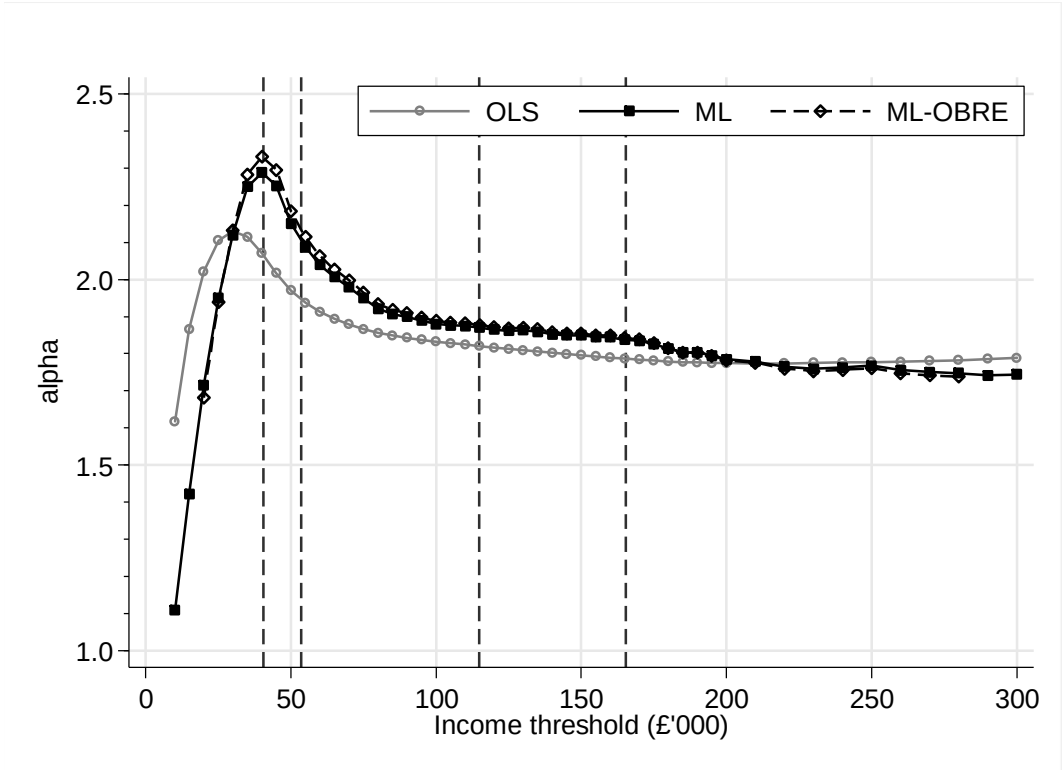


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

2003

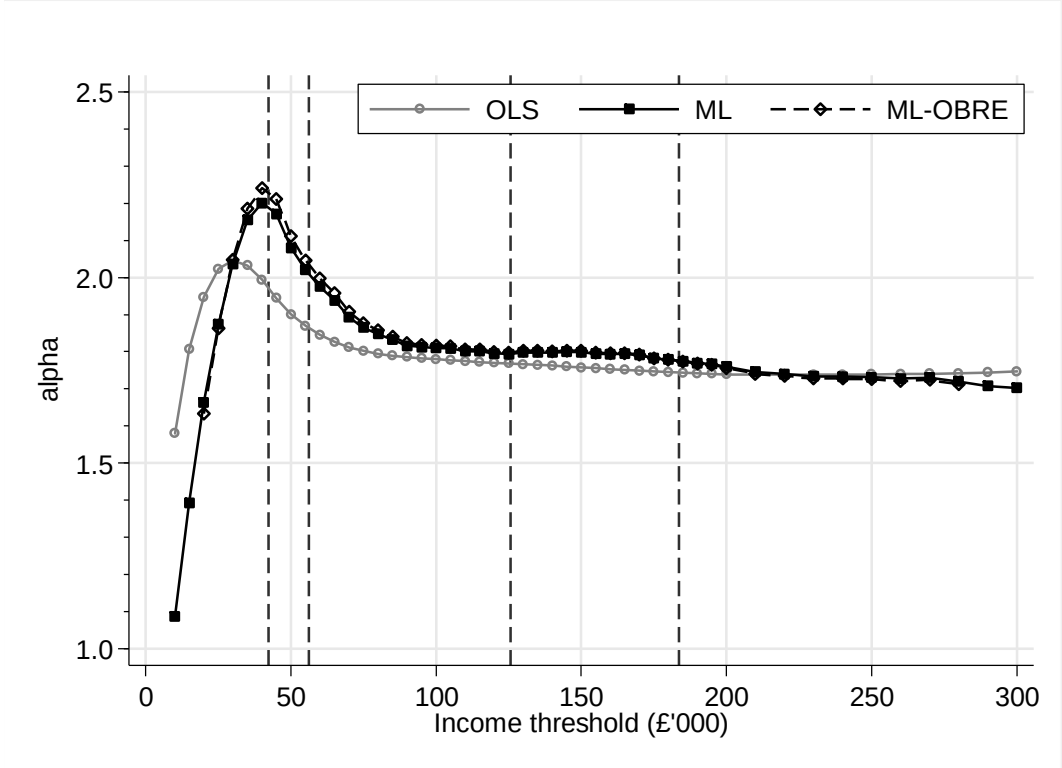


2004

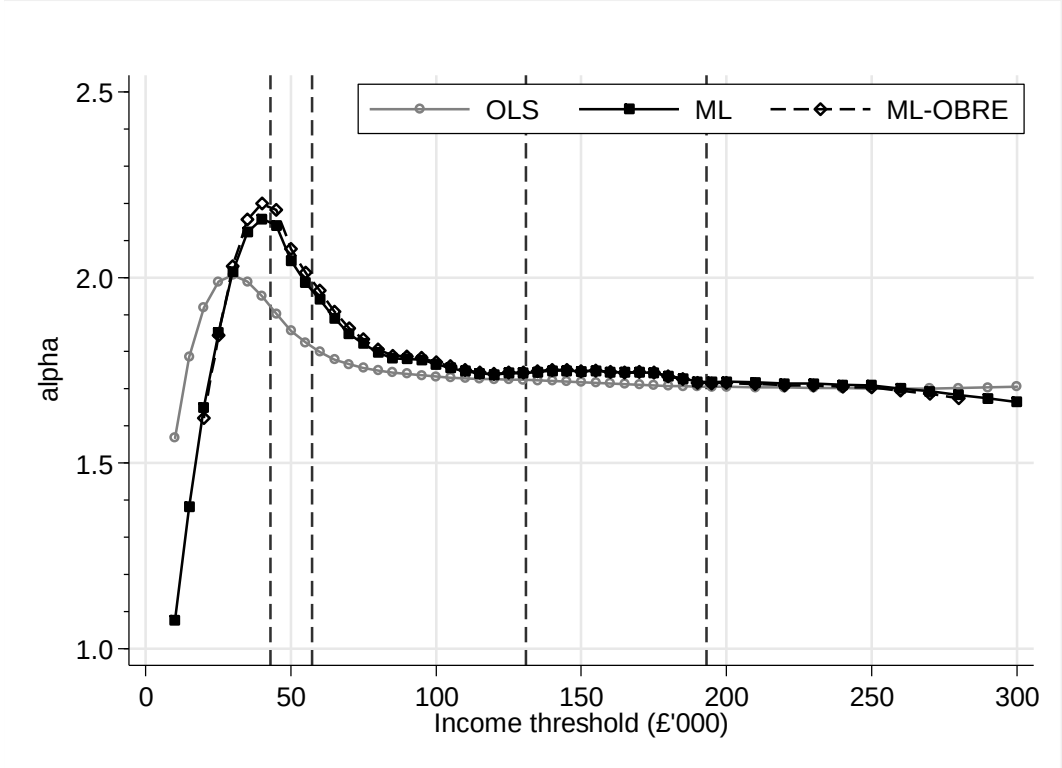


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

2005

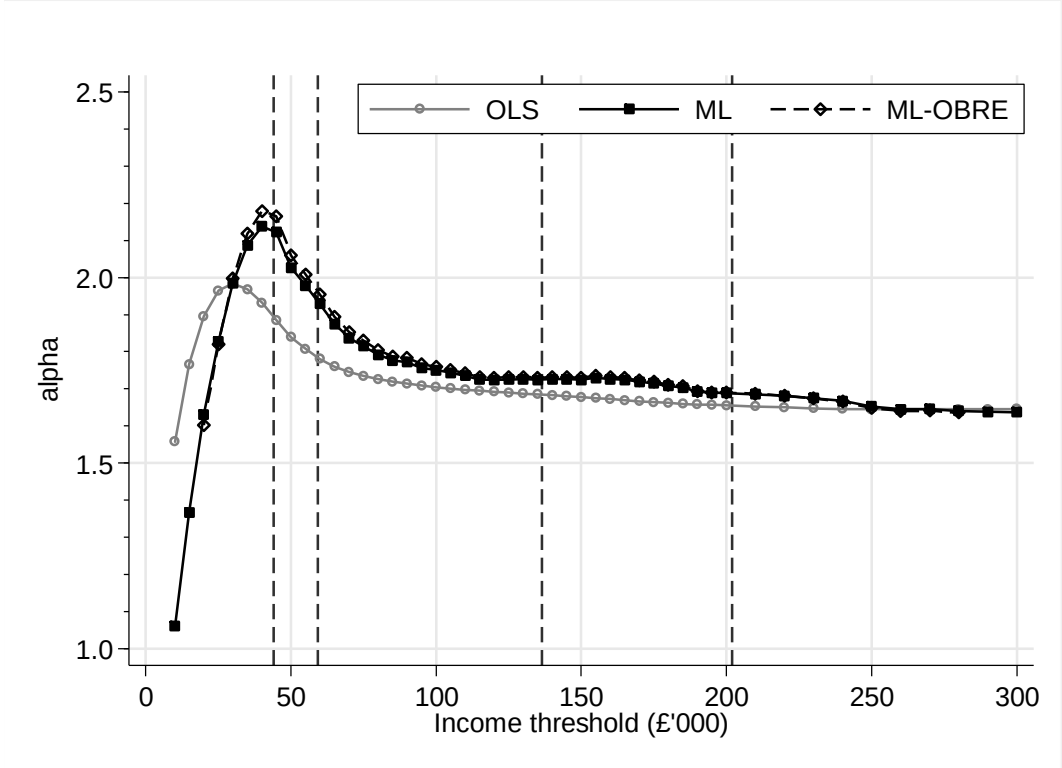


2006

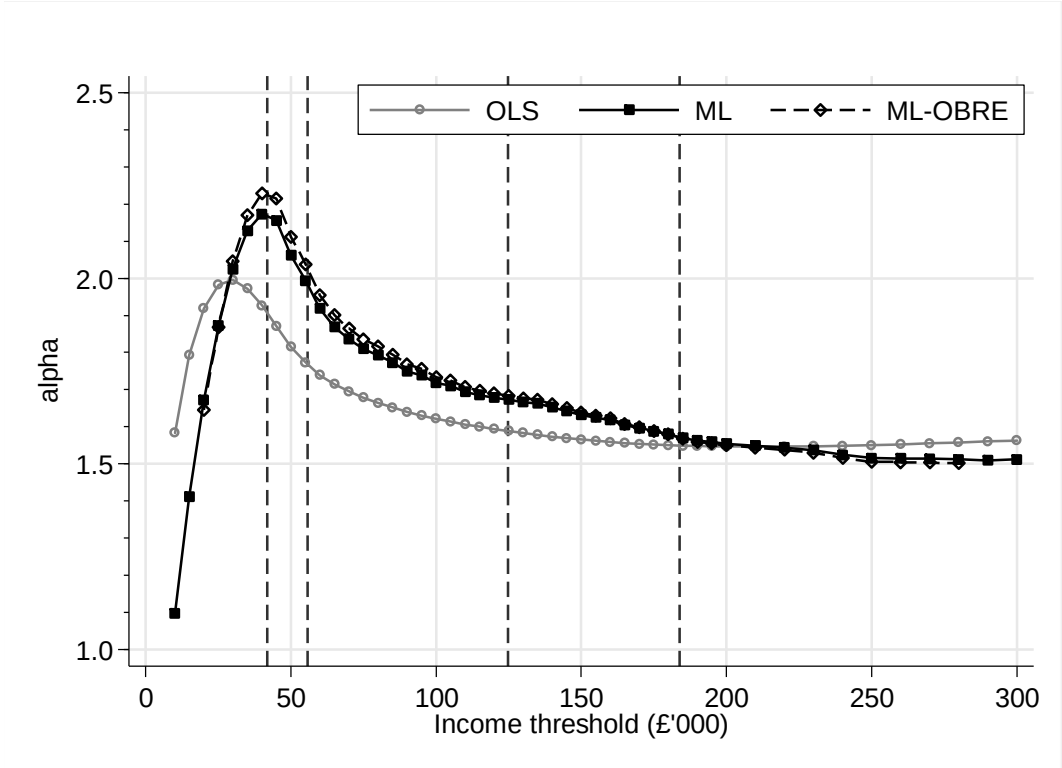


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

2007

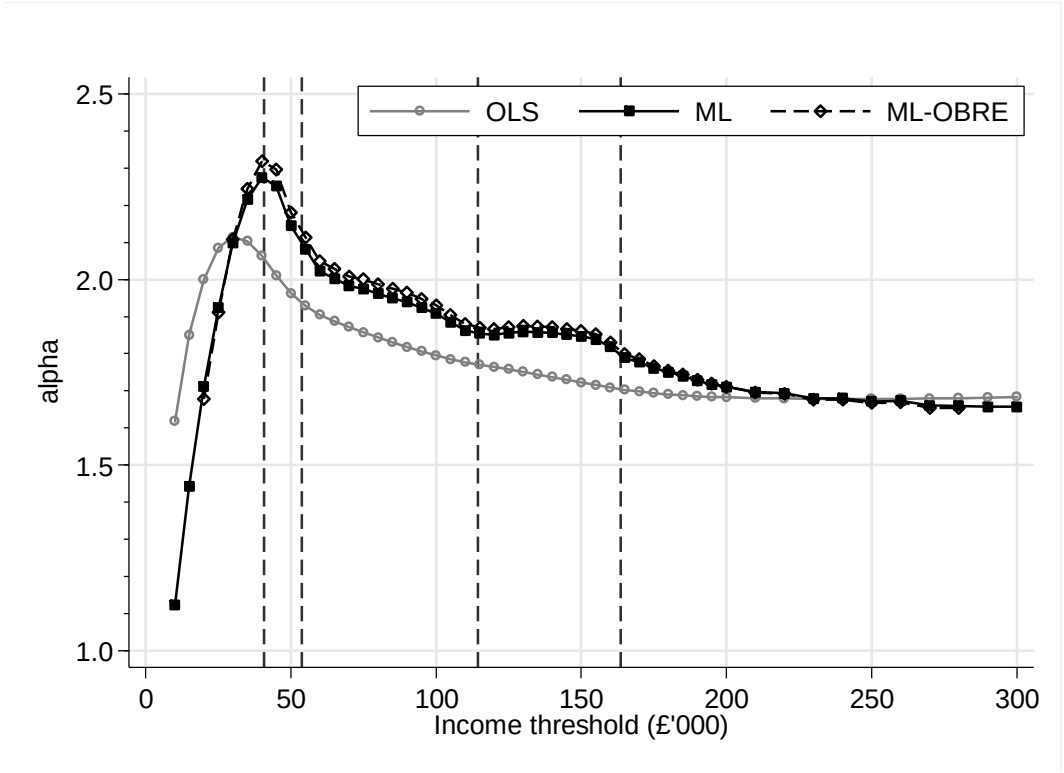


2009



Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

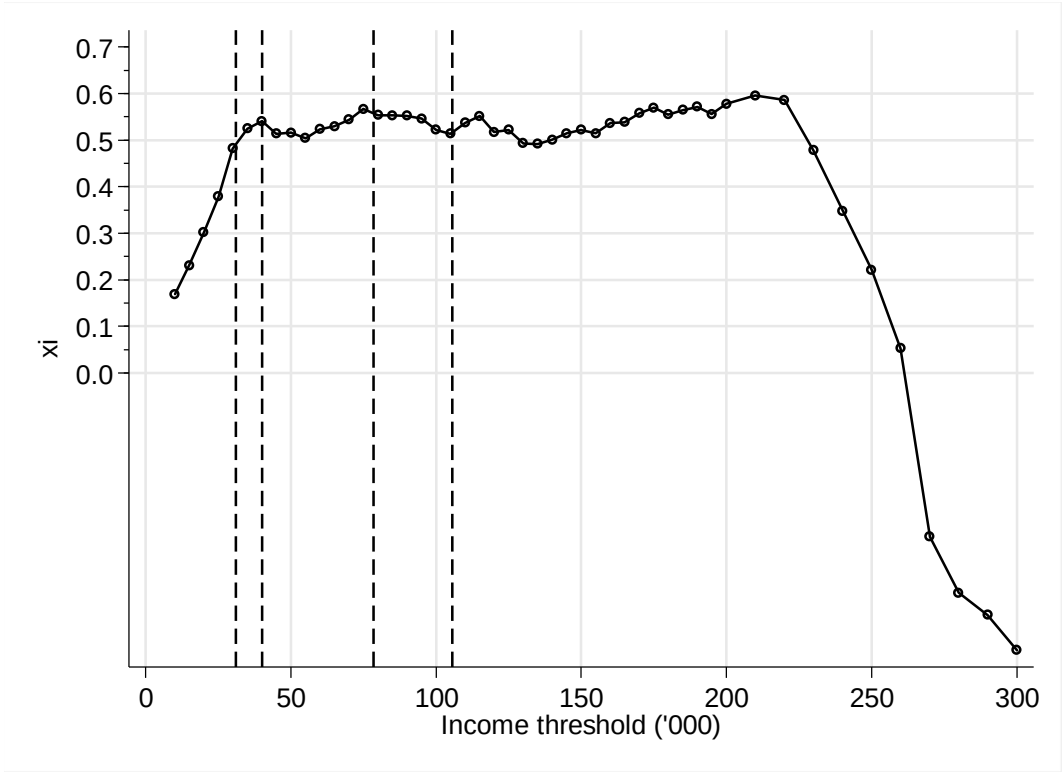
2010

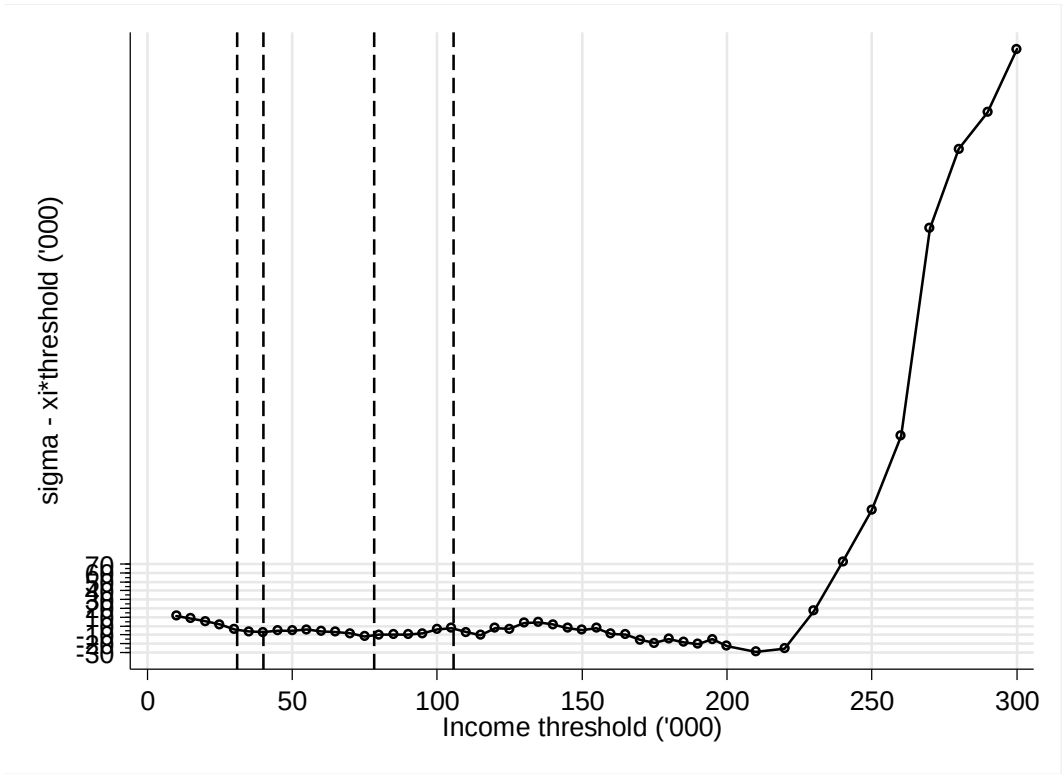


Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

Pareto II (Generalized Pareto)

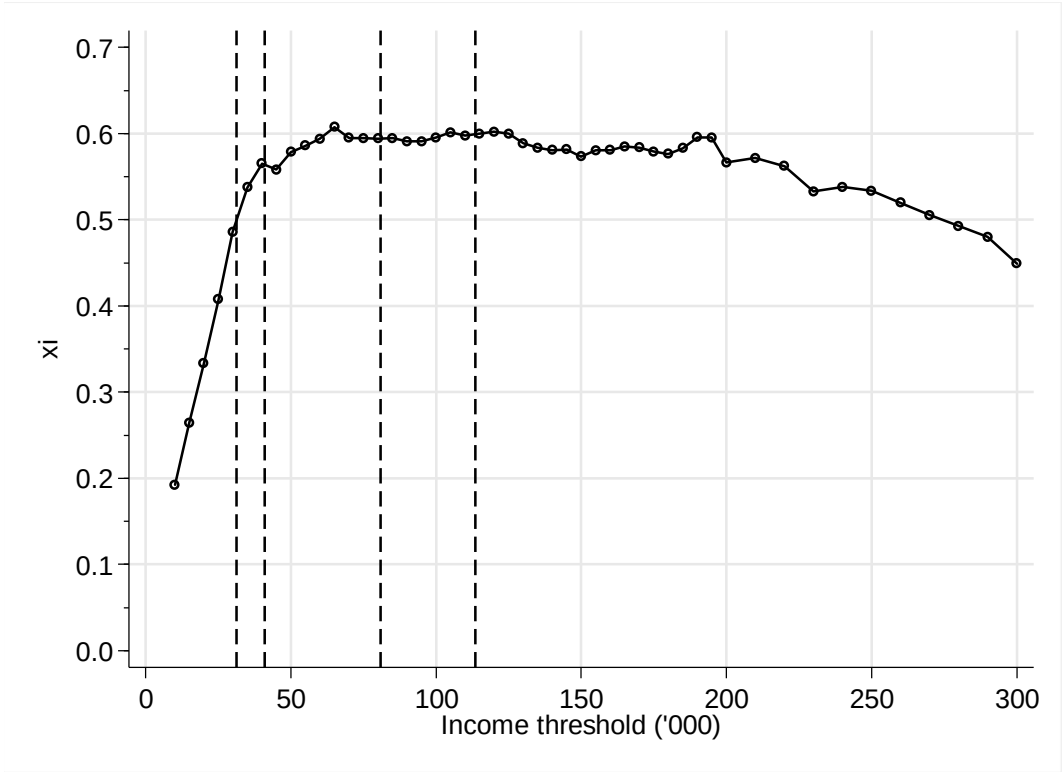
1995

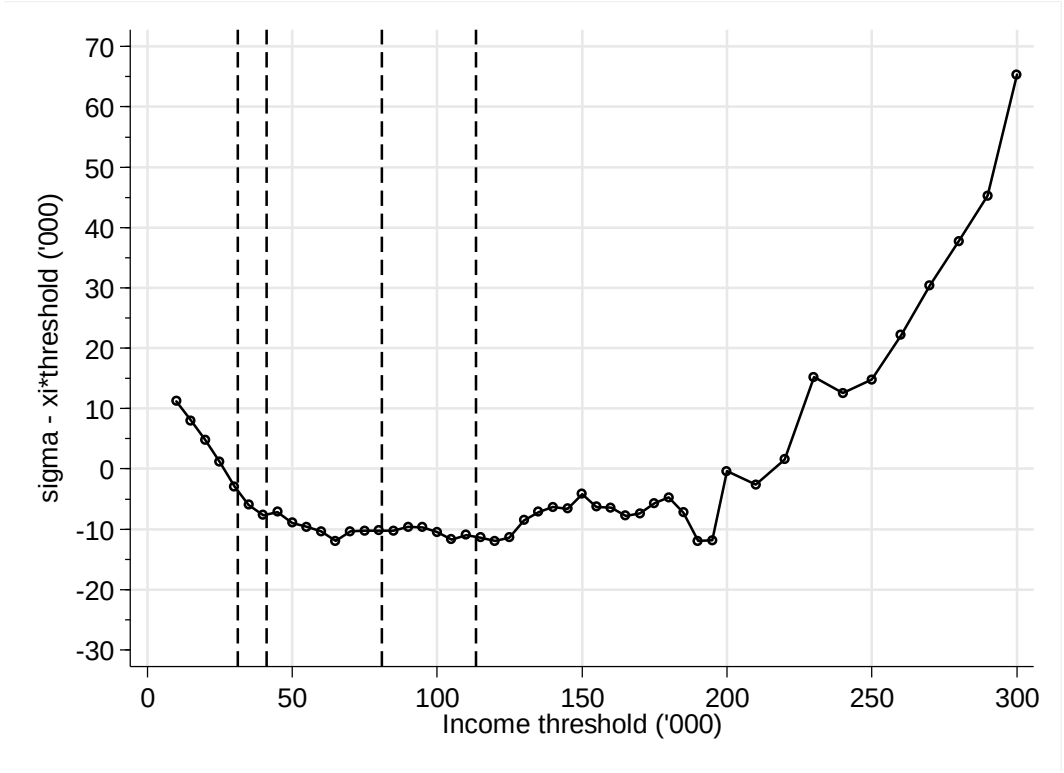




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

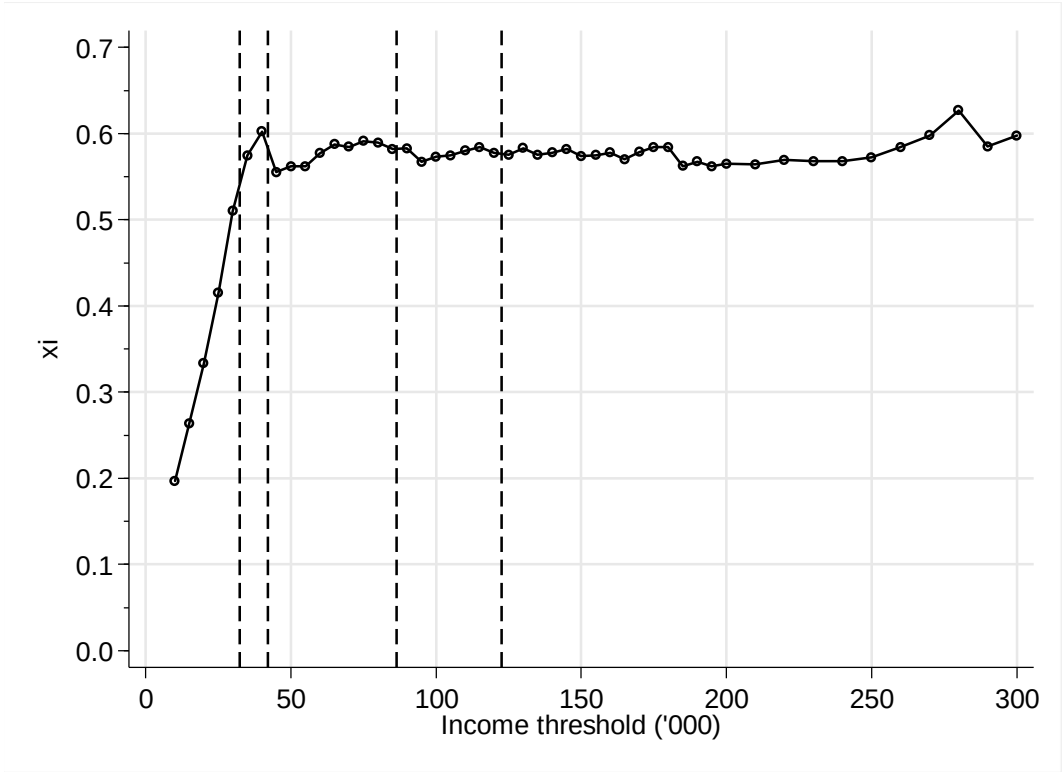
1996

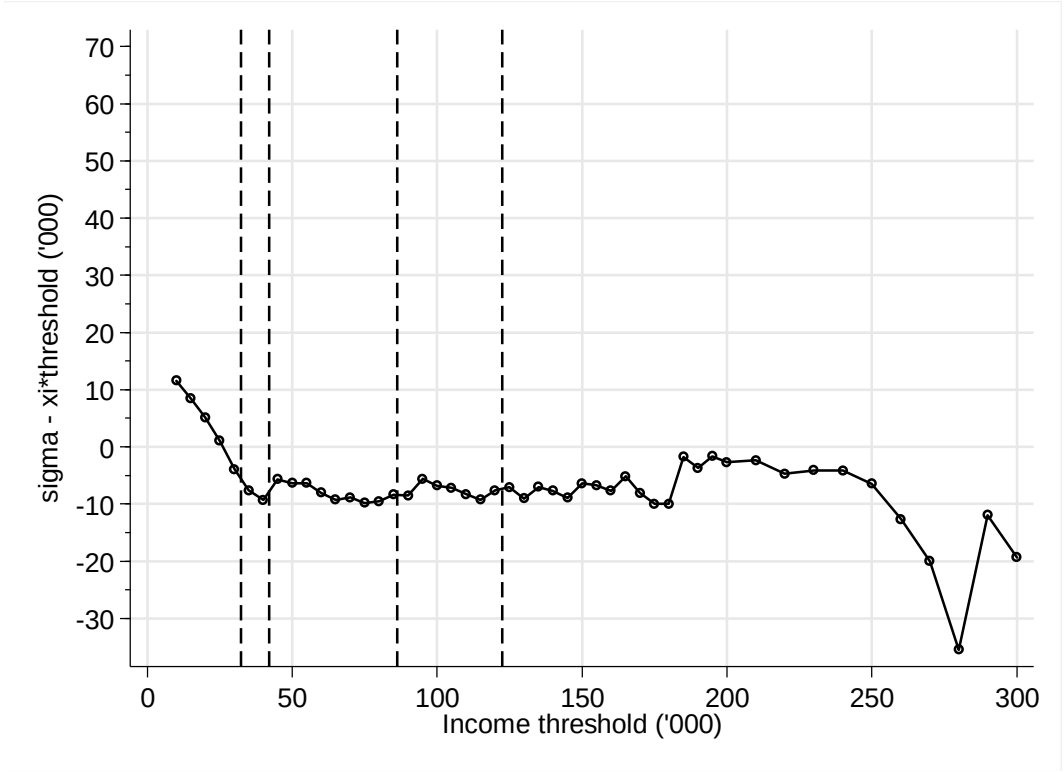




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

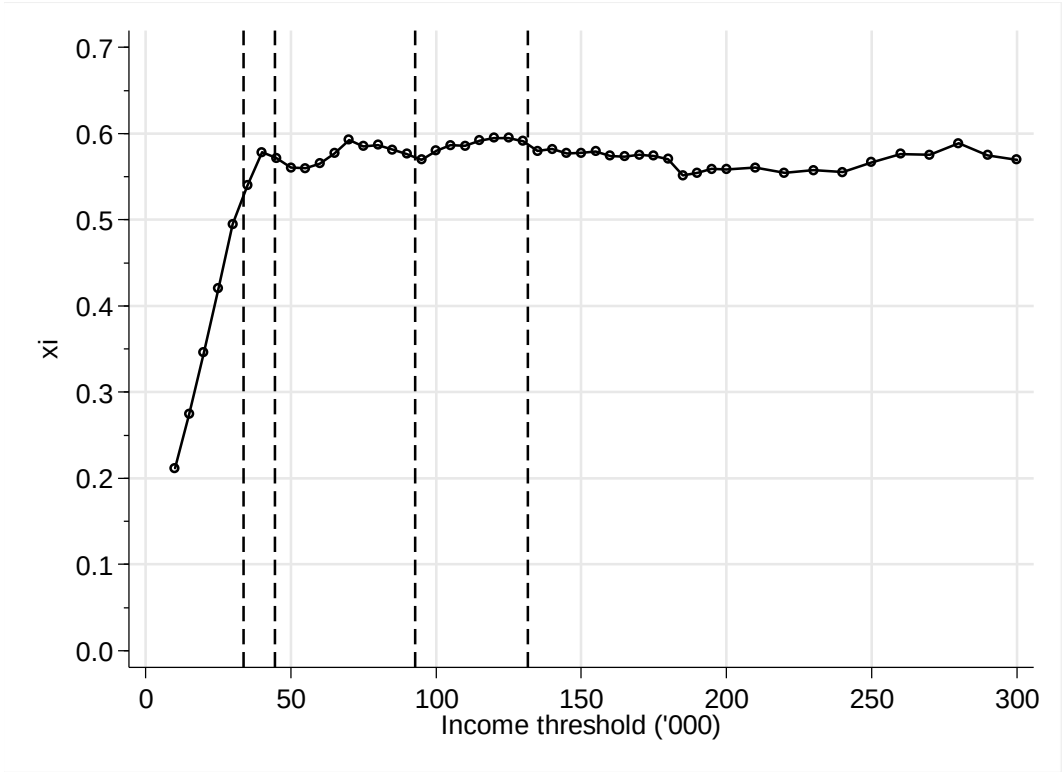
1997

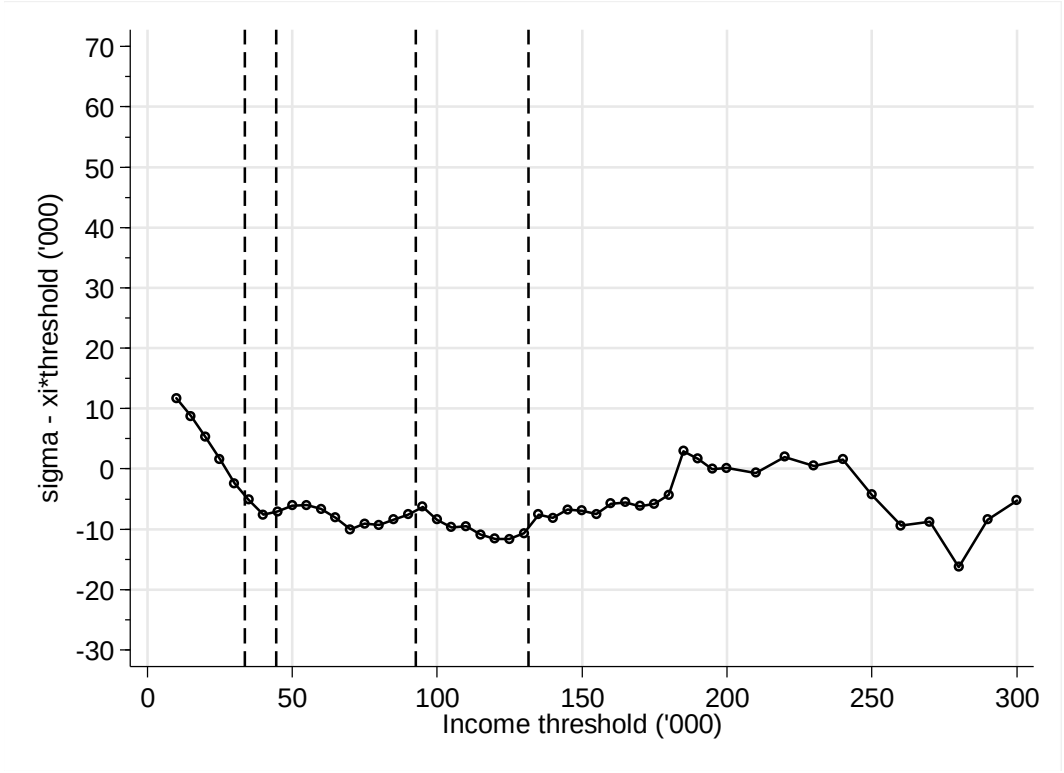




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

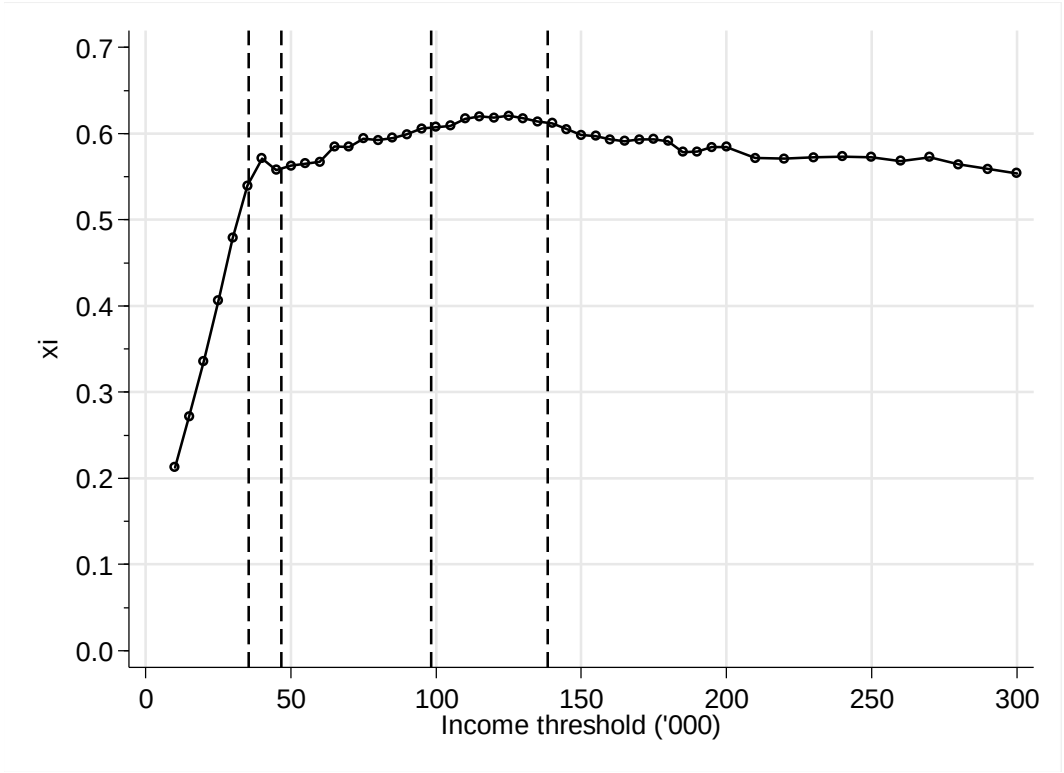
1998

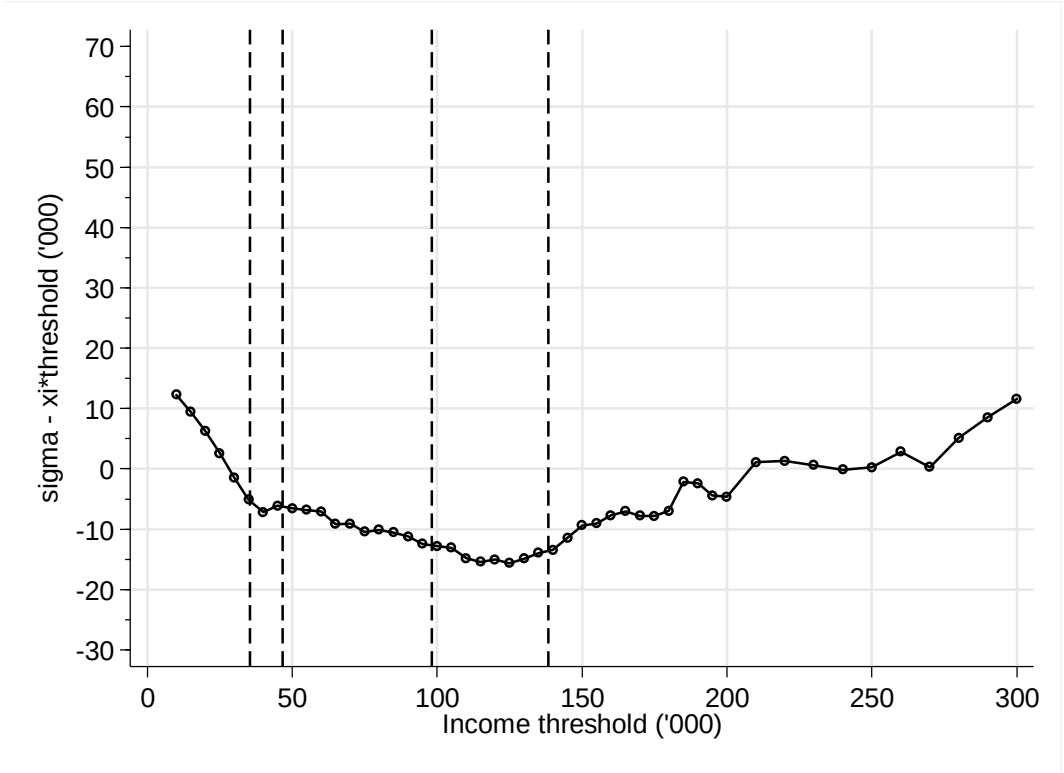




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

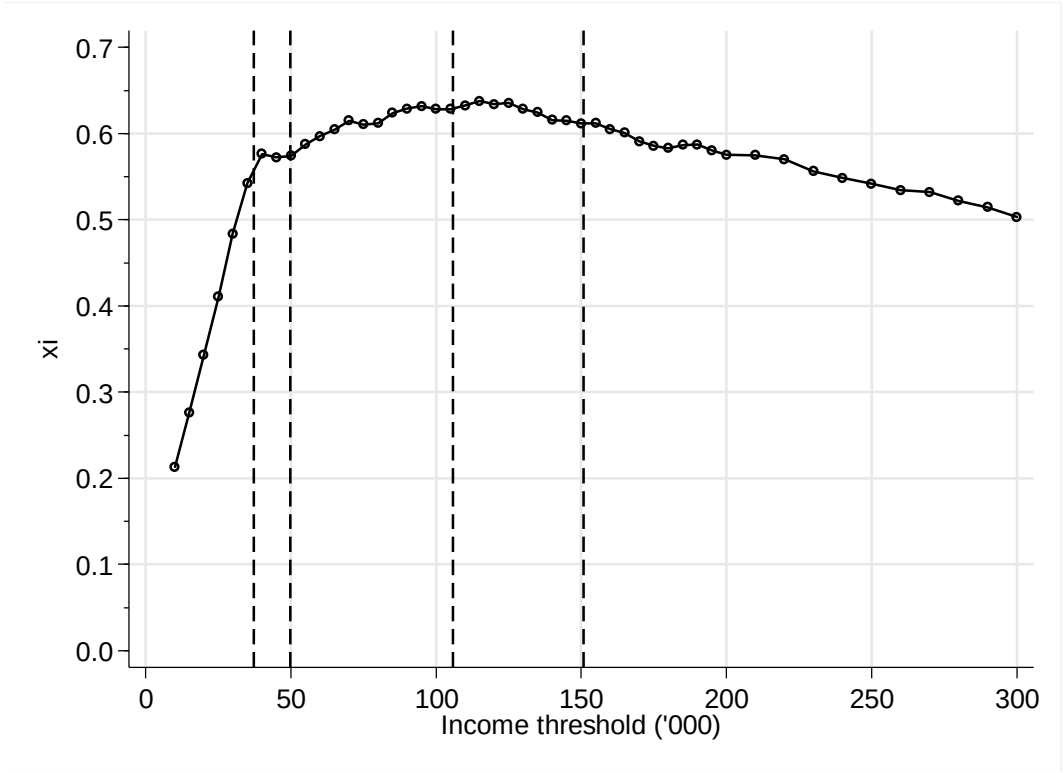
1999

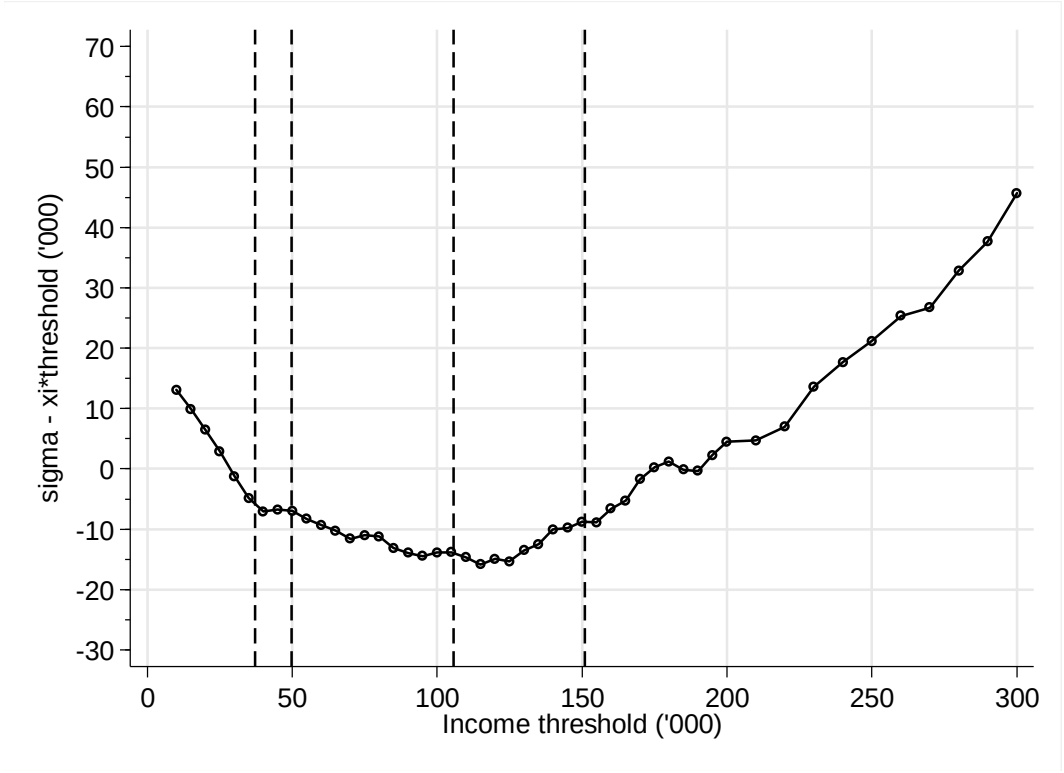




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

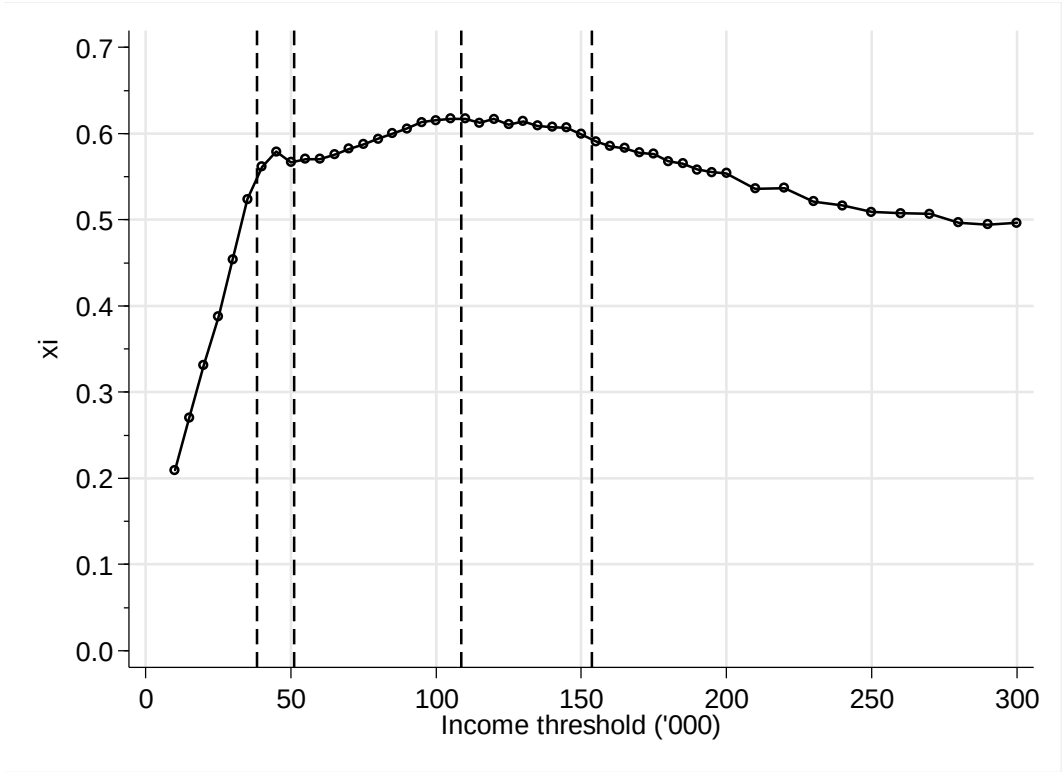
2000

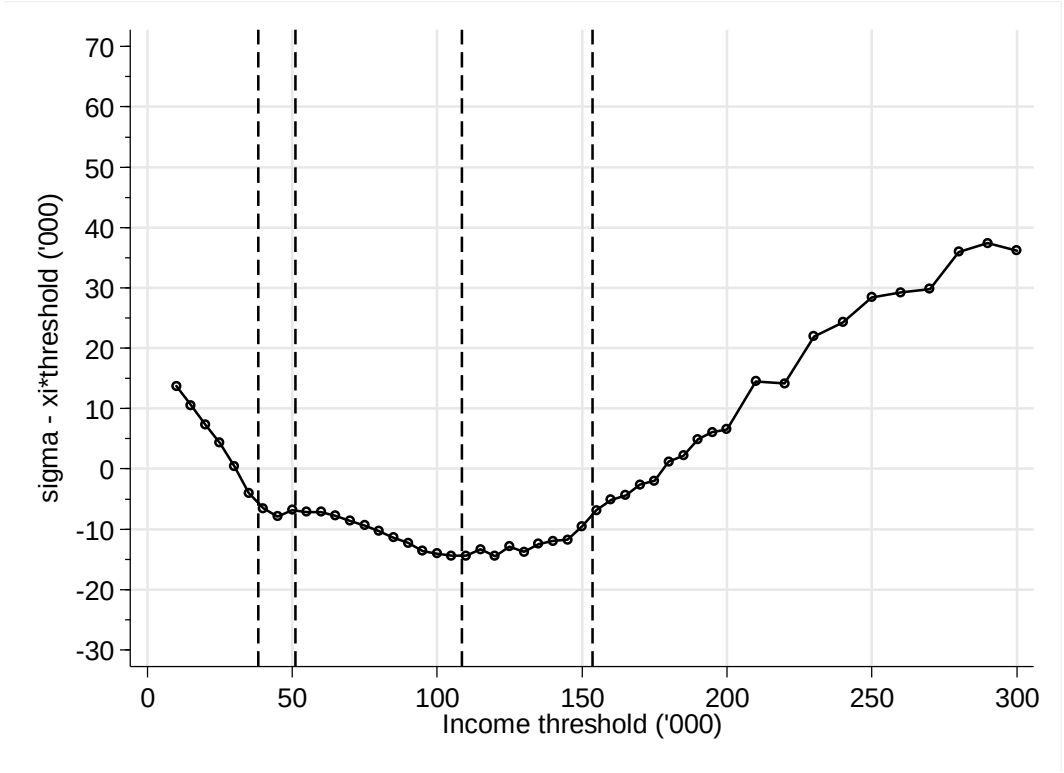




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

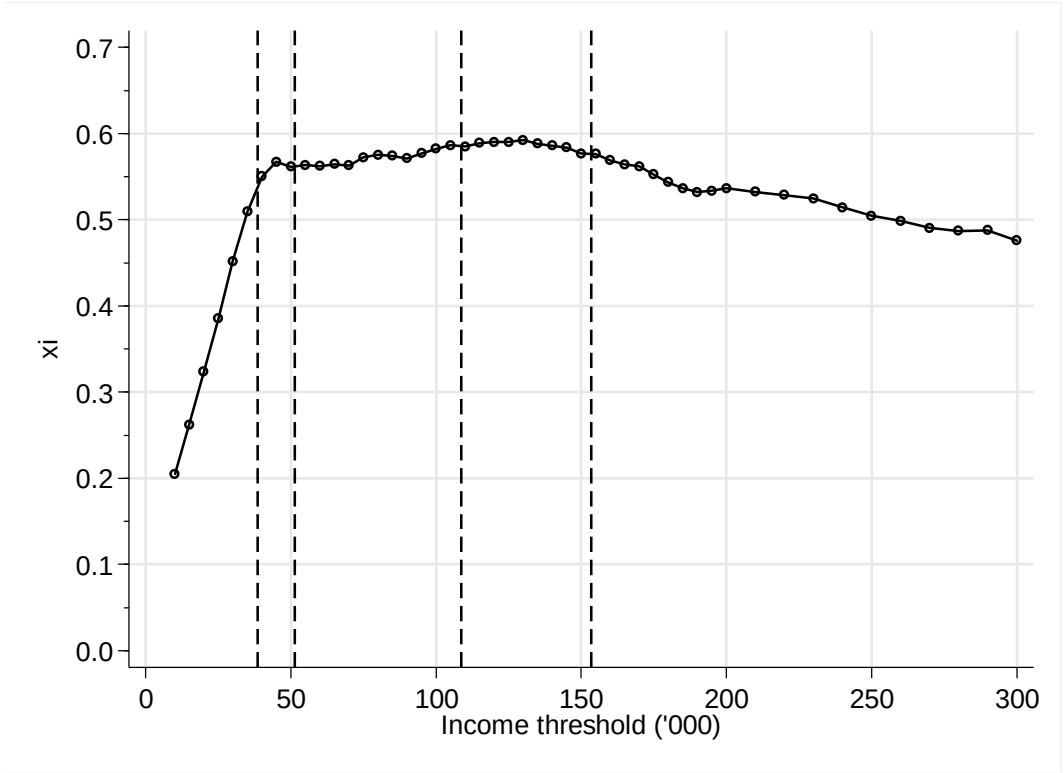
2001

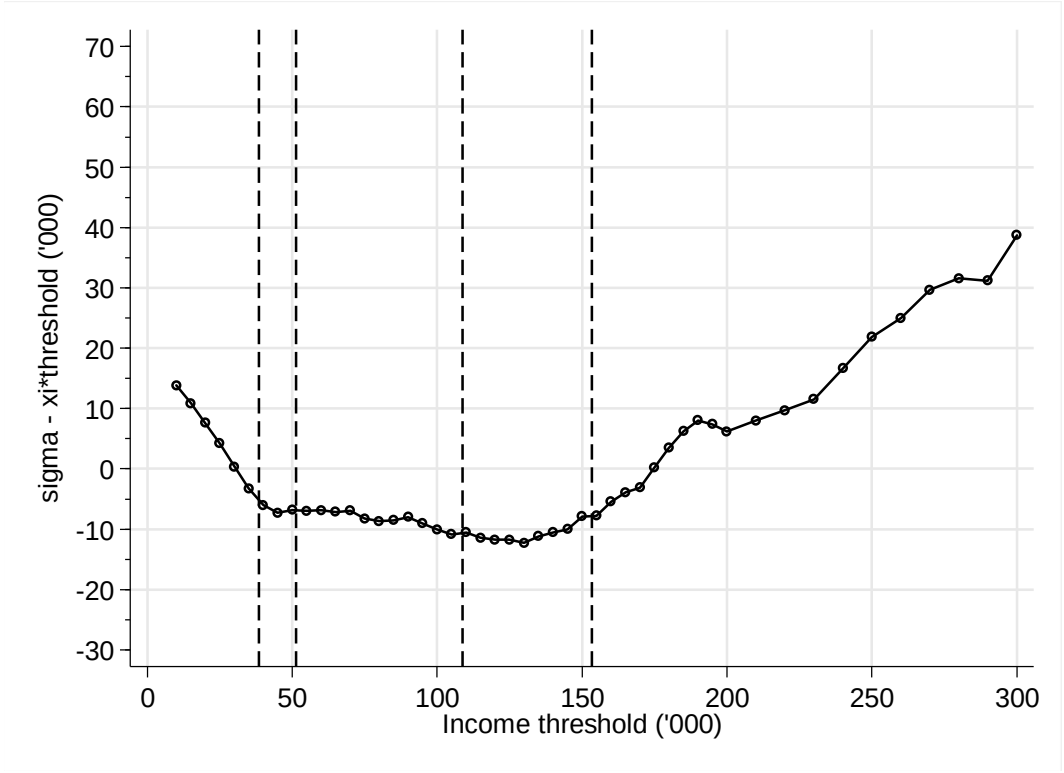




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

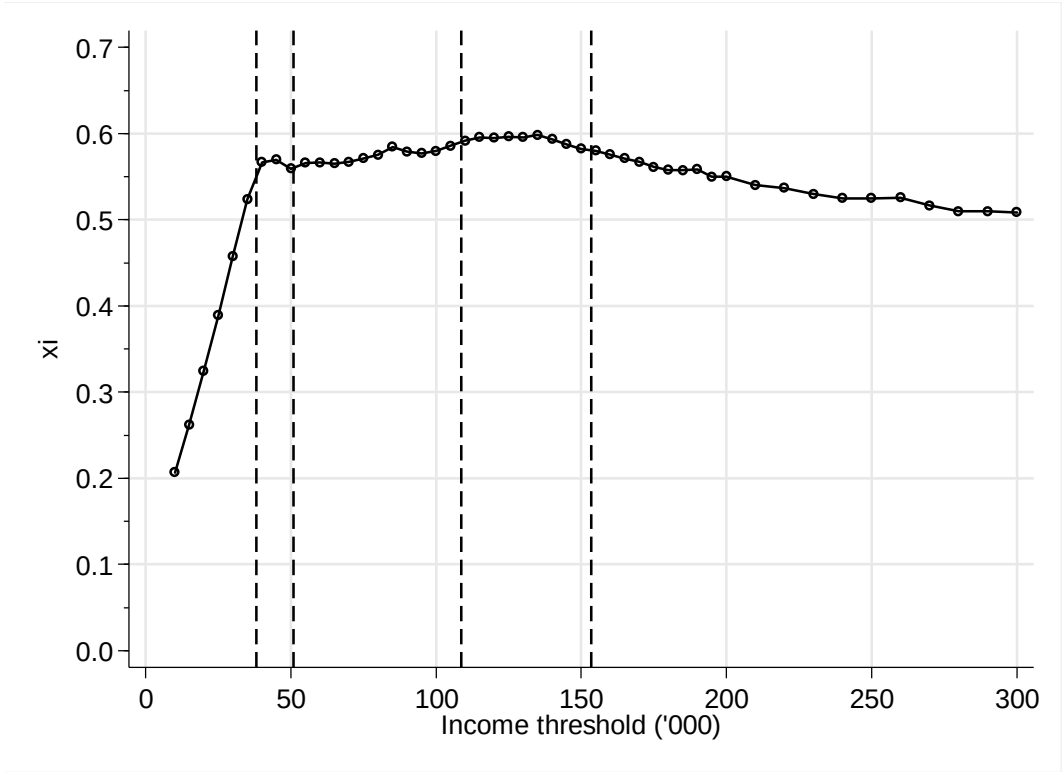
2002

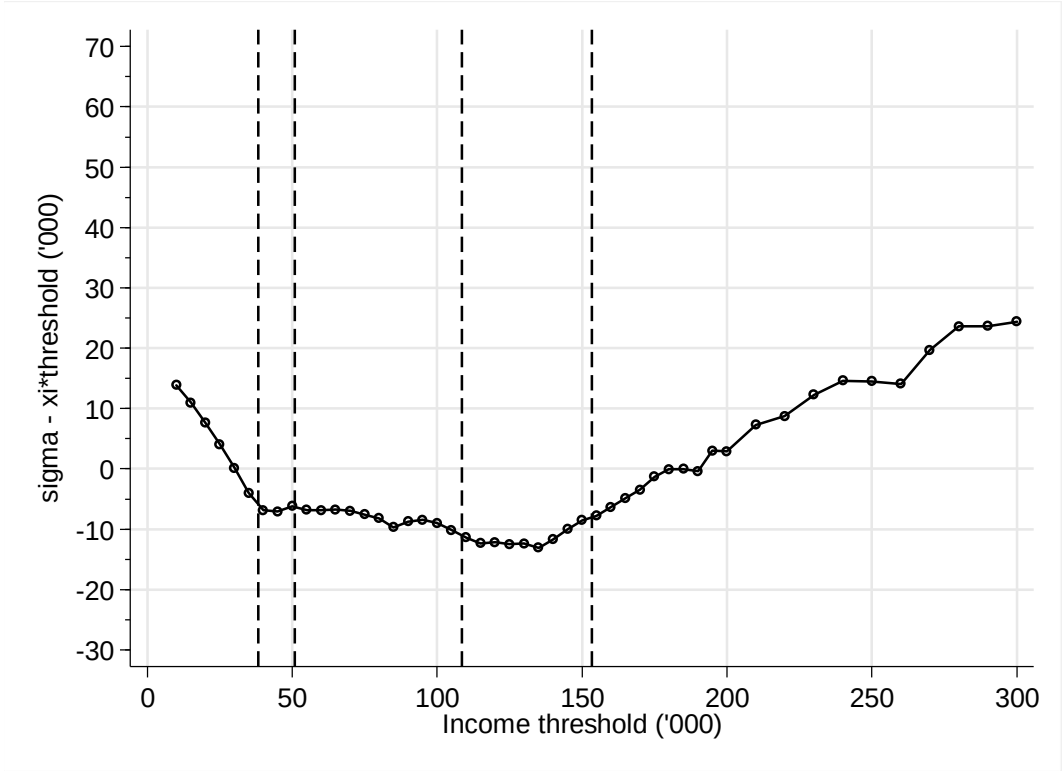




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

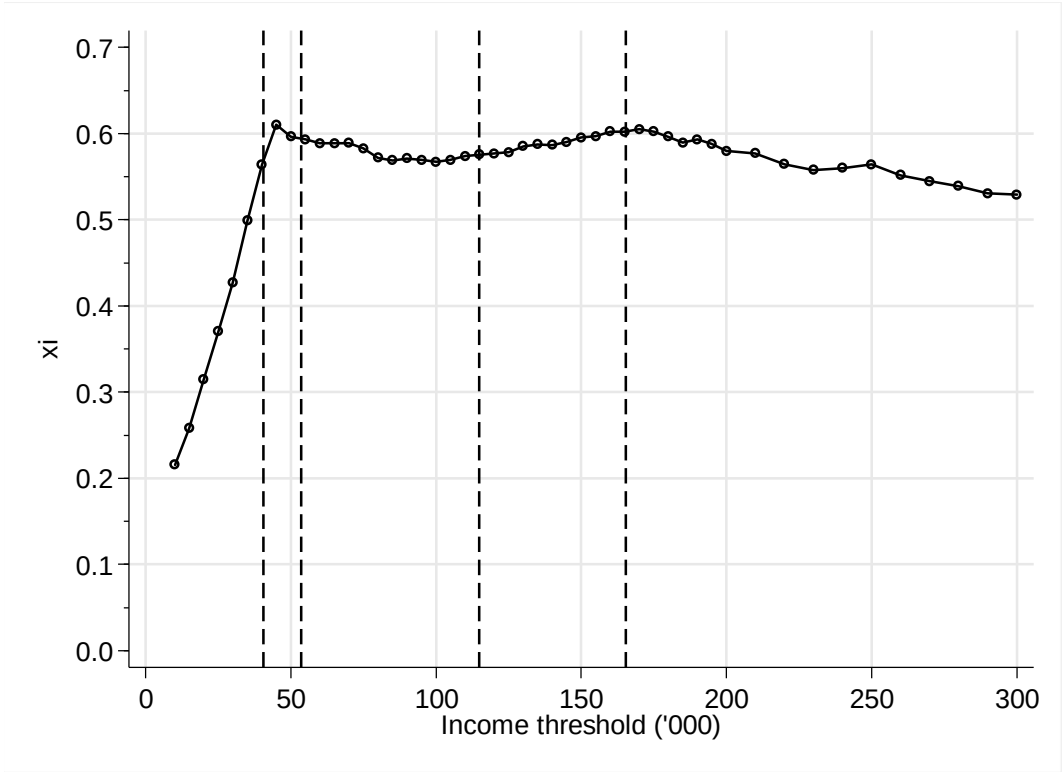
2003

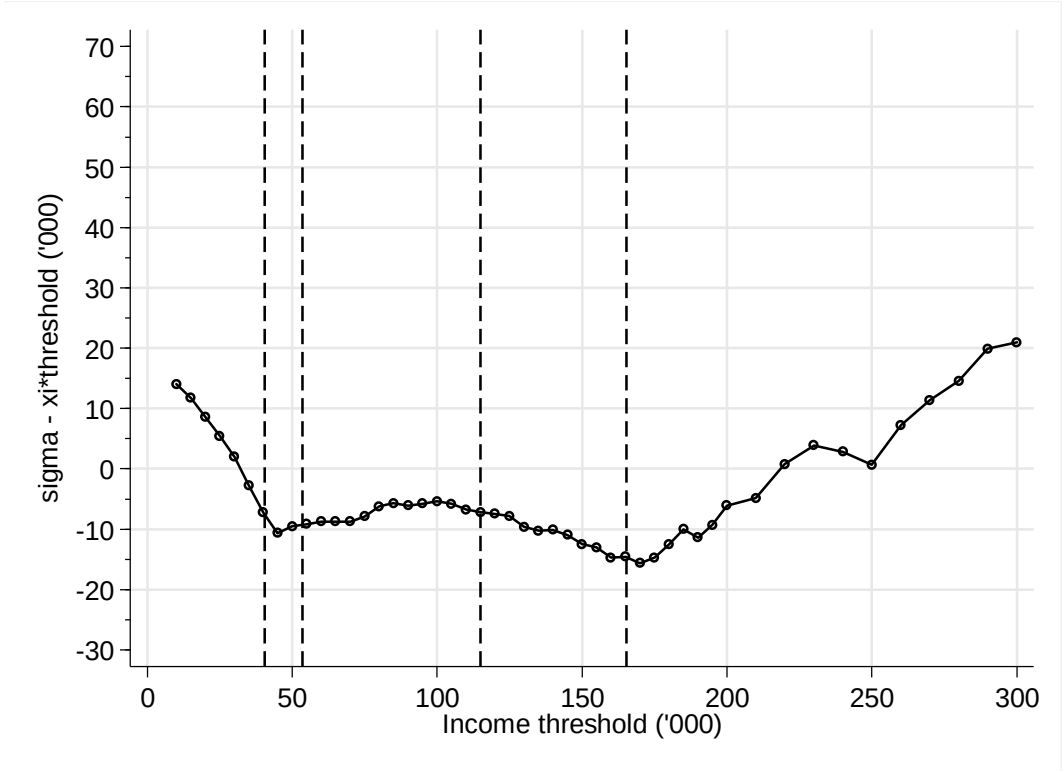




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

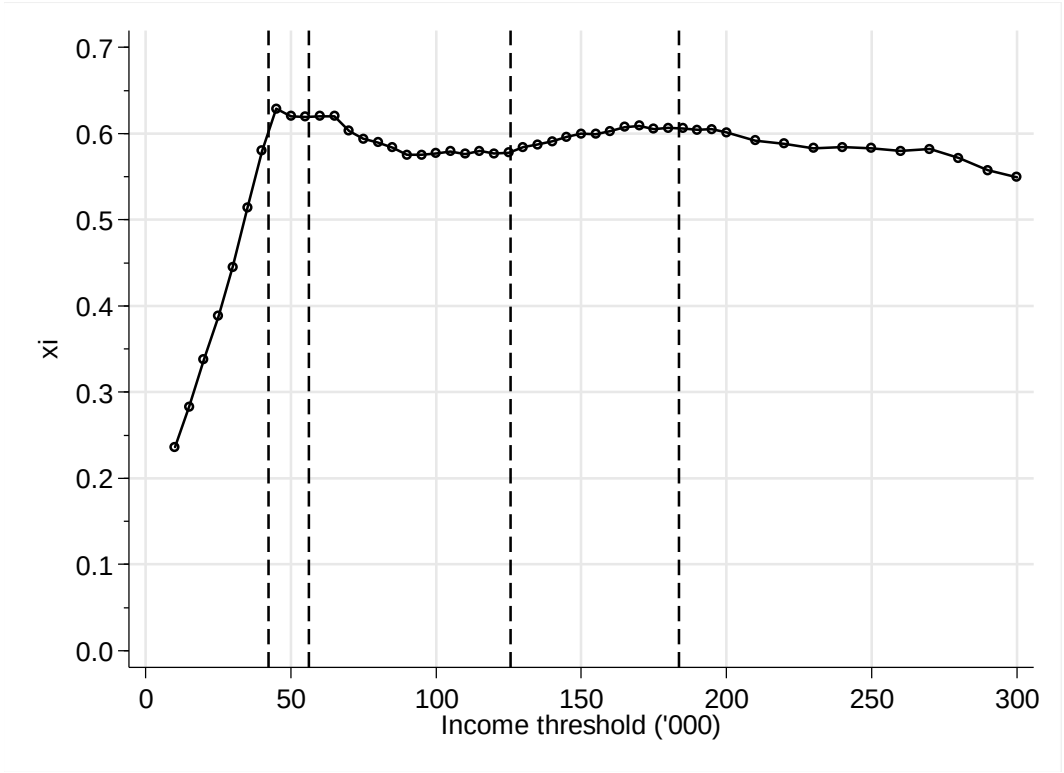
2004

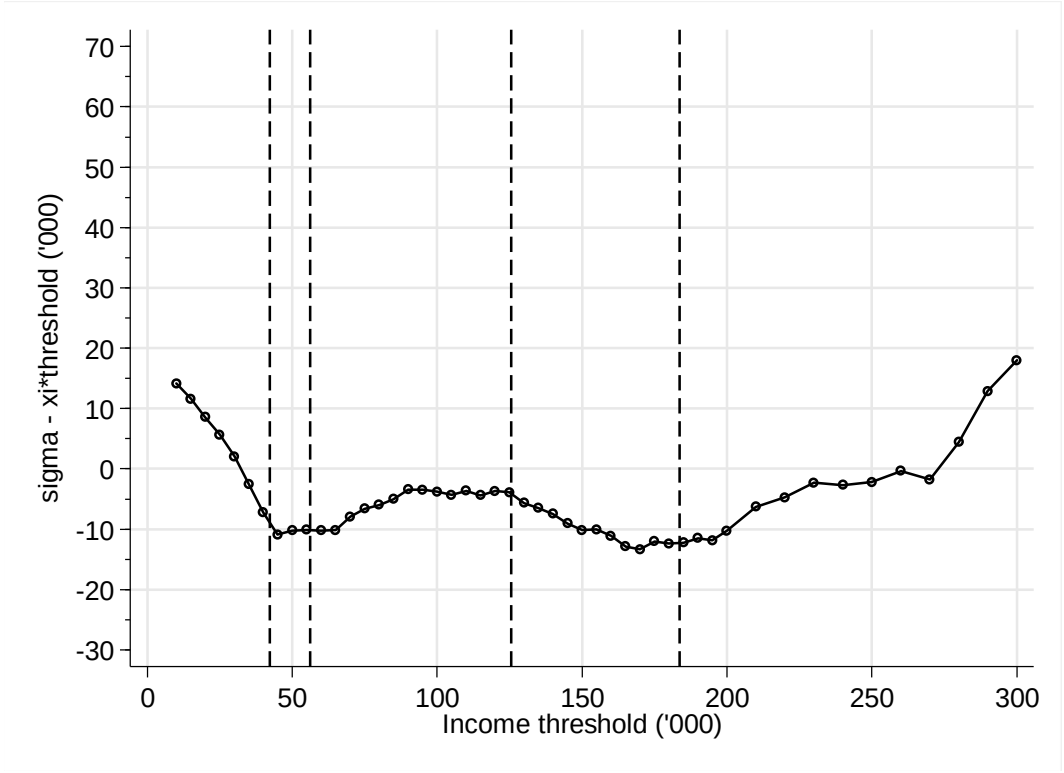




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

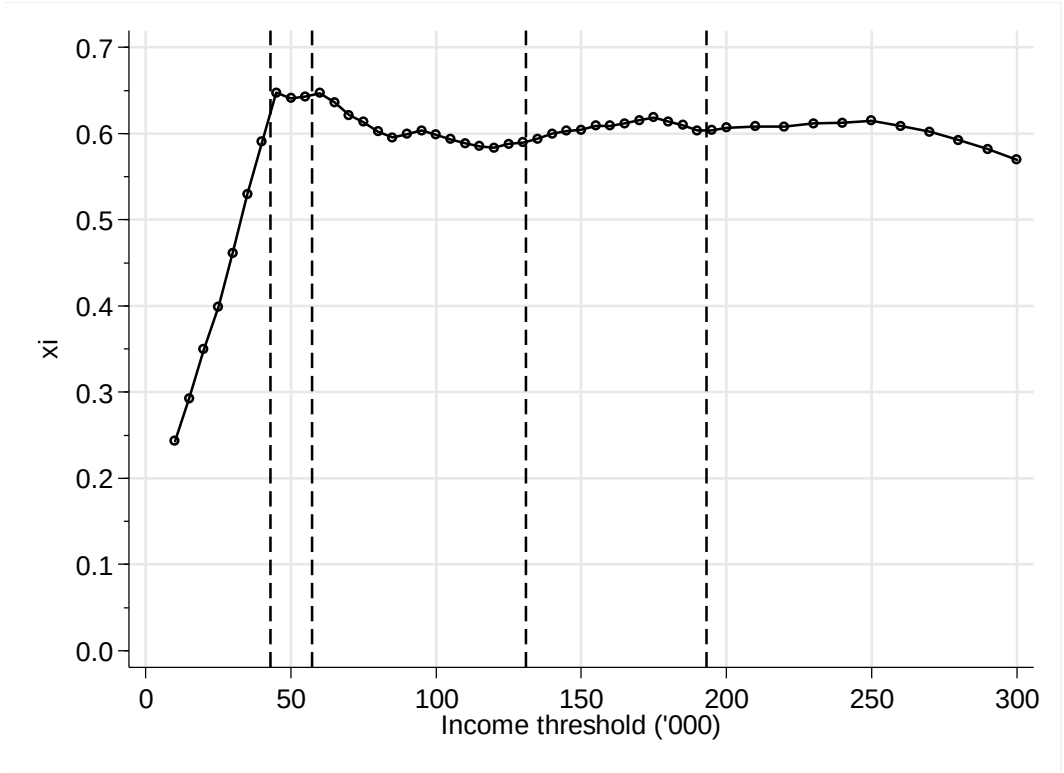
2005

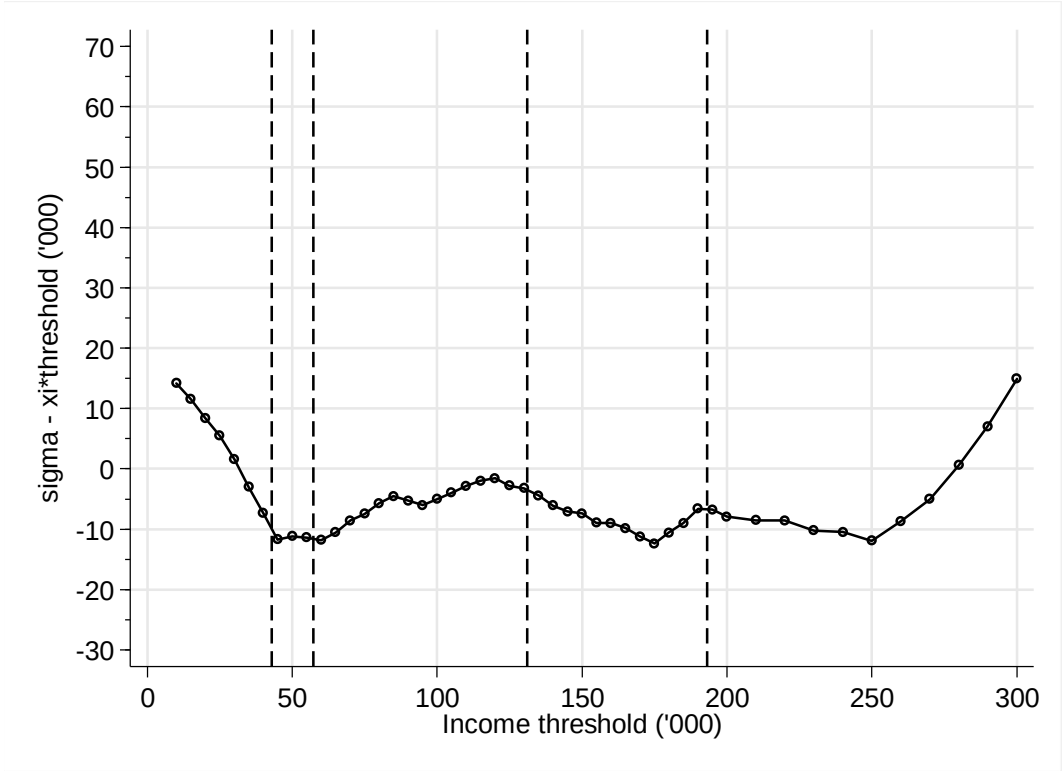




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

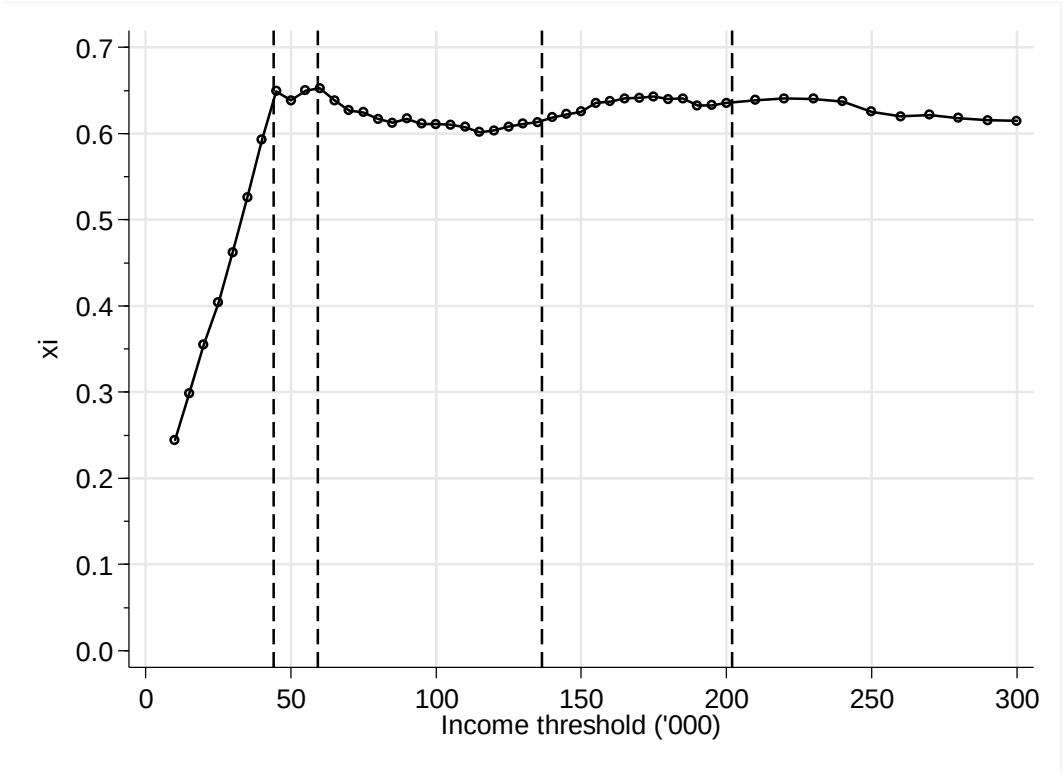
2006

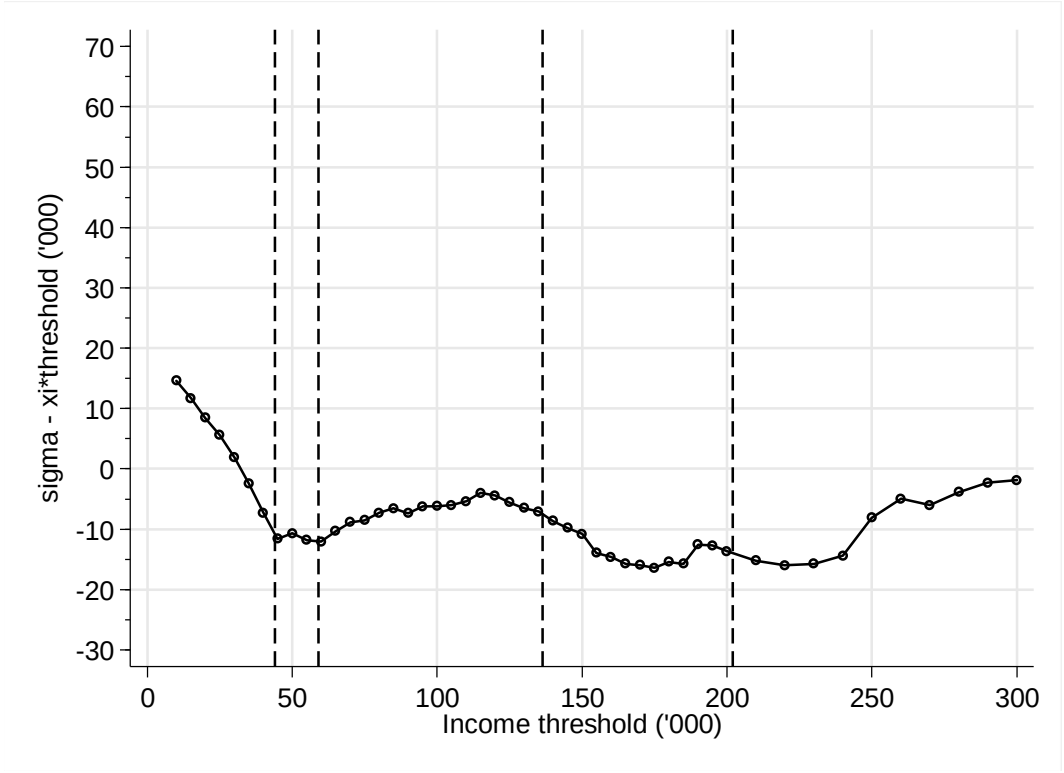




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

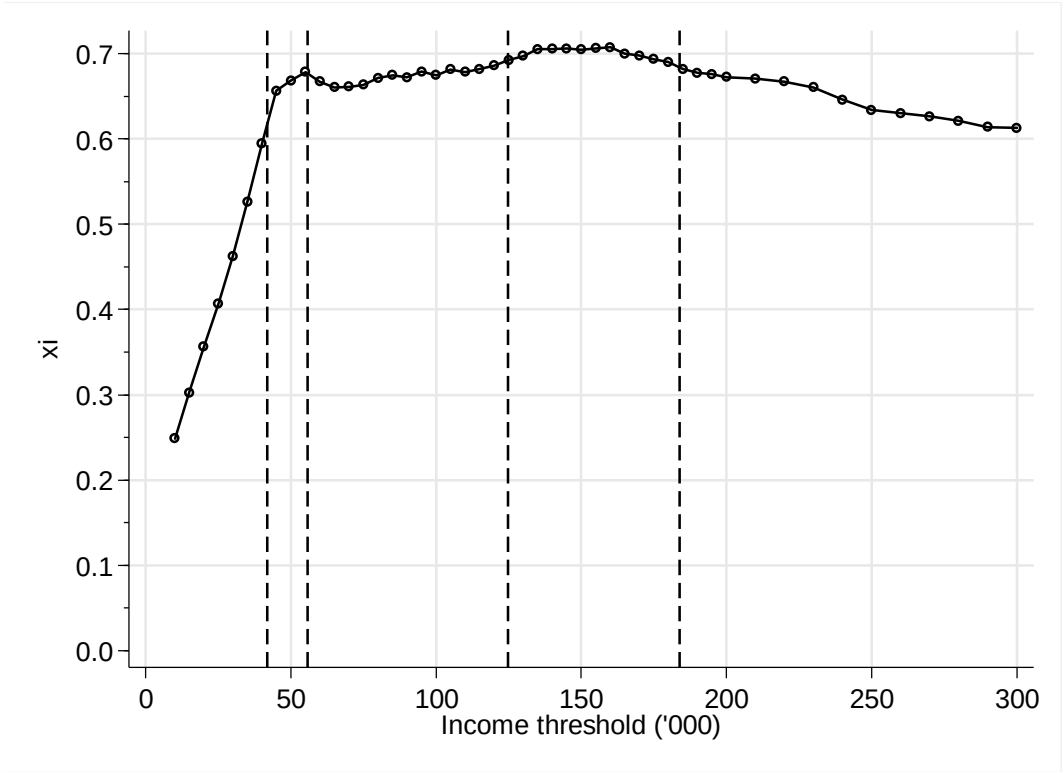
2007

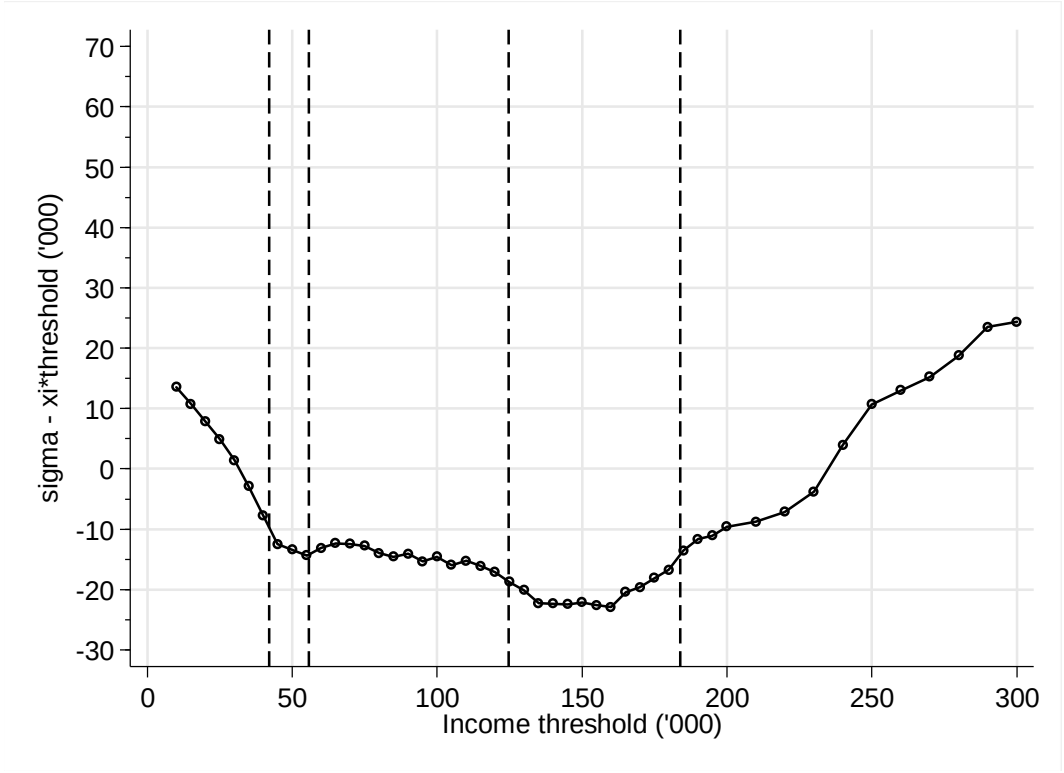




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

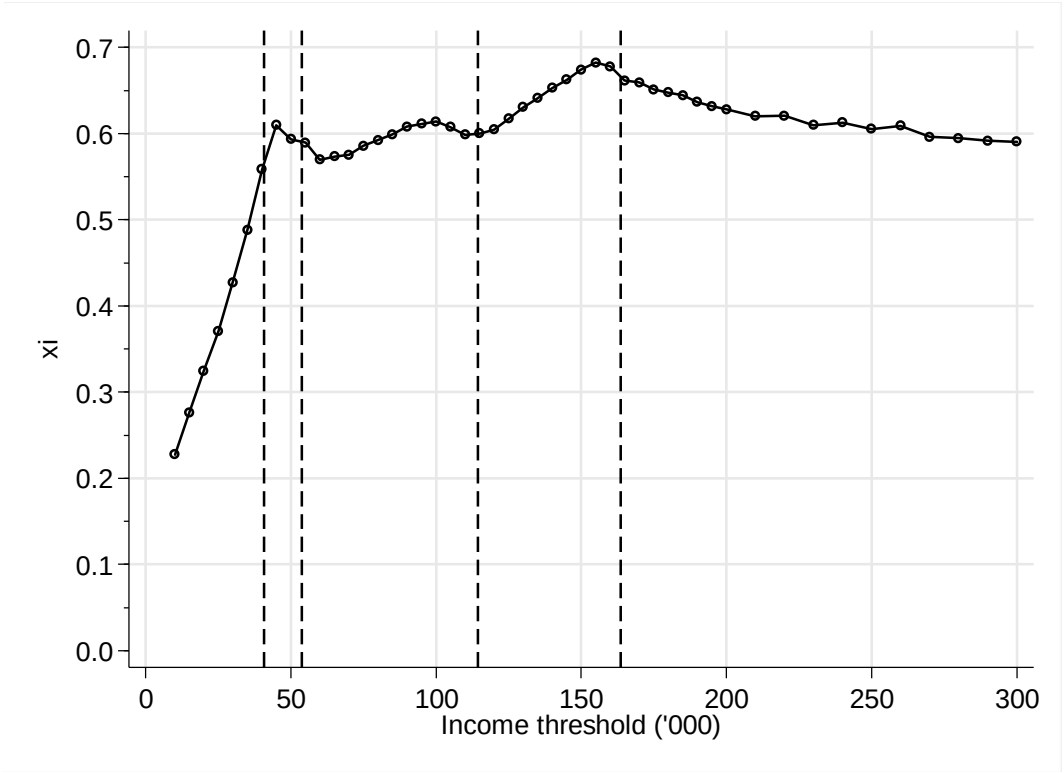
2009

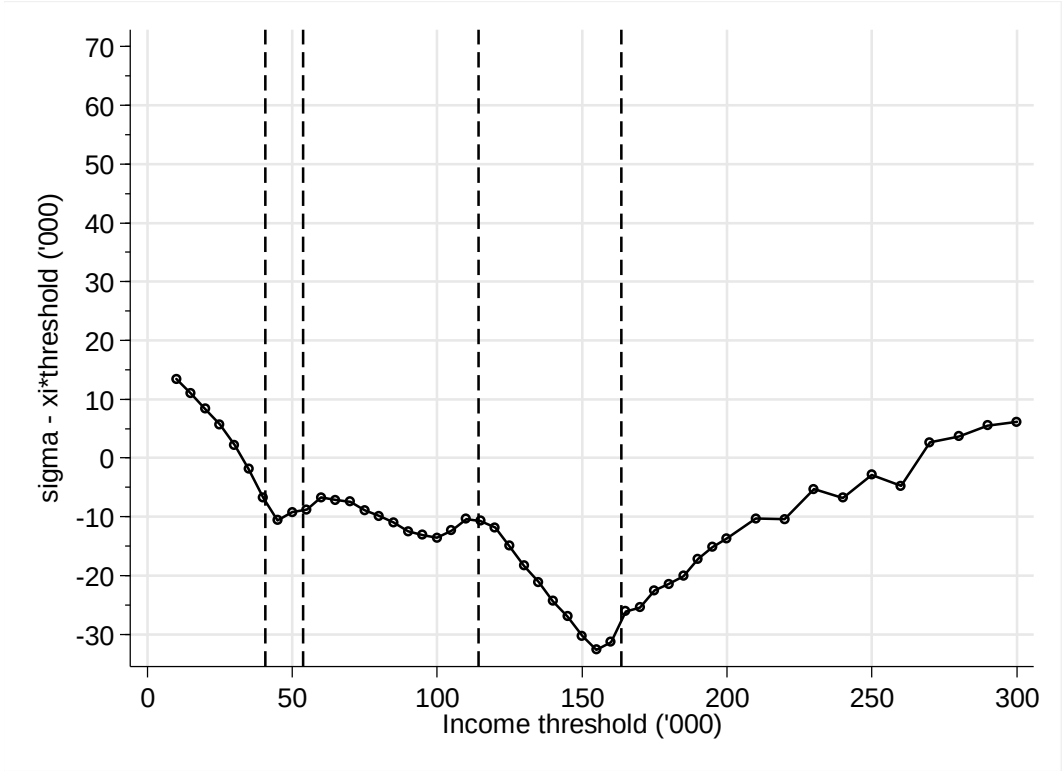




Estimates of Pareto I and II parameters, by estimator, threshold, and year
Vertical dashed lines show (from left to right): *p*90, *p*95, *p*99, and *p*99.5 (pareto10, 12, 13)

2010





Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

Here follow:
(1) Tax data estimates
(2) Survey data estimates
(3) Combined data estimates
(3a) Combined data estimates: figures (in same format as Figure 10)

Statistics derived from tax return data (SPI) can be identified by their suffixes:
"ml": Pareto I, ML estimator
"mlo": Pareto I, ML-OBRE estimator
"gpd": Pareto II, ML estimator
"np": non-parametric estimator

Estimates derived from the survey data (HBAI, non-SPI-adjusted), all non-parametric, can be identified by their suffixes:
"100": poorest 100%
"99": poorest 99%
"95": poorest 95%
"90": poorest 90%

TAX DATA ESTIMATES

Tax data: Pareto I and Pareto II models: estimates of parameter and SEs (threshold = *p*99 in the survey)

year	P_R	alpha_ml	se_alph~l	alpha	se_alpha	xi	se_xi	sig	se_sig
1995	.0085792	2.114052	.1610938	2.13713	.0302296	.5578981	.1210882	36159.08	1580.968
1996	.008958	1.941077	.1106777	1.953106	.0237158	.5911263	.0878998	40597.05	1630.992
1997	.0092068	1.922345	.0644815	1.937097	.0144499	.5825368	.0542783	43944.39	1339.272
1998	.0093161	1.901082	.0444537	1.911827	.01212	.5701846	.0426144	48218.54	962.4487
1999	.0104736	1.926562	.0385267	1.94353	.010403	.6076952	.0352471	45540.21	707.3513
2000	.0098838	1.841782	.0368198	1.851902	.0101842	.6300222	.033269	52864.86	826.1099
2001	.0105668	1.895889	.0327147	1.906846	.0096336	.6161801	.0292421	50943.27	757.7493
2002	.0110572	1.934508	.032749	1.943498	.0097998	.5852064	.0293267	49929.41	664.6404
2003	.011684	1.919675	.0308324	1.929457	.0093381	.5791096	.0284916	49280.36	638.3967
2004	.0113858	1.877548	.0352881	1.885559	.0088971	.5731952	.0337069	54887.98	737.5891
2005	.0131511	1.806126	.0332827	1.811509	.0083703	.5794456	.0327035	58364.29	747.9828
2006	.0136039	1.747885	.0292037	1.751142	.0077817	.588789	.0292206	61837.9	722.4963
2007	.0142837	1.733676	.0283335	1.740903	.0073744	.6068791	.0275526	62273.52	689.8105
2009	.0118367	1.687349	.0262883	1.699297	.0074722	.6794235	.0278293	61522.91	673.331
2010	.0114712	1.874162	.0096601	1.892035	.0079147	.6023272	.0087243	53142	425.3513

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

Tax data: Pareto I and Pareto II models: estimates of parameter and SEs (threshold = *p*95 in the survey)

year	P_R	alpha_ml	se_alph~l	alpha	se_alpha	xi	se_xi	sig	se_sig
1995	.0349138	2.279984	.0662971	2.304698	.0192333	.5070741	.0567215	18653.35	562.1927
1996	.0341588	2.212221	.0543265	2.251051	.016354	.5660636	.044089	19138.83	385.5171
1997	.0359085	2.111353	.0303254	2.136568	.0117075	.5546948	.0263121	20964.23	358.2605
1998	.0370005	2.092855	.0208853	2.117316	.0095908	.5605036	.0192076	22168.79	321.0485
1999	.0408321	2.105451	.0185803	2.134624	.0083306	.5618327	.0165027	22191.23	288.6397
2000	.0434905	2.076405	.0172942	2.109816	.0079736	.5853383	.0151656	22965.56	269.4211
2001	.0428005	2.079124	.0145701	2.1075	.0069765	.5715815	.0135066	23987.99	222.5532
2002	.0436872	2.101361	.0141596	2.12654	.0068907	.5617445	.0134007	23712.9	195.8426
2003	.0424477	2.078119	.013984	2.102025	.0067782	.5645915	.0135475	24147.96	197.6735
2004	.045103	2.078425	.01557	2.104867	.0059277	.5925306	.0149922	23962.17	196.6042
2005	.0500392	2.010589	.0154738	2.035572	.0055749	.6206832	.0151286	24598.67	198.8592
2006	.0525945	1.975508	.0140596	2.00178	.0051725	.6439652	.0136899	24629.75	184.4339
2007	.0564673	1.968754	.0136294	1.997438	.0049032	.6514286	.0131492	24522.2	175.9459
2009	.0477972	1.967872	.0131282	2.009235	.0056849	.6783192	.0127761	24192.06	209.7331
2010	.0485559	2.095737	.0061848	2.128459	.0057145	.5953258	.0046749	22974	139.5351

Tax data: Pareto I and Pareto II models: estimates of parameter and SEs (threshold = *p*90 in the survey)

year	P_R	alpha_ml	se_alph~l	alpha	se_alpha	xi	se_xi	sig	se_sig
1995	.0691512	2.485901	.0476557	2.528681	.0177262	.5234771	.0362594	12615.95	302.6853
1996	.0656919	2.389046	.0402604	2.440527	.0152448	.5588147	.0303443	13397.08	278.5533
1997	.0704028	2.361521	.0267146	2.410876	.0114455	.6046395	.0186198	13070.18	259.8923
1998	.0736046	2.280277	.0186757	2.320734	.0092436	.5776901	.0134352	14533.85	236.7898
1999	.0784177	2.262013	.0165632	2.301965	.0080251	.5720381	.0121477	15101.62	225.8878
2000	.0825183	2.205624	.0144234	2.246422	.0076205	.577918	.0111061	16094.19	203.4729
2001	.0813434	2.223621	.0125648	2.262344	.0064534	.5728109	.0095682	16575.5	174.8562
2002	.0842385	2.235875	.0108814	2.270374	.0061525	.5555899	.0089394	16546.56	131.9748
2003	.0798965	2.221256	.0108645	2.256219	.0060503	.5750315	.0091686	16594.54	132.5893
2004	.0898392	2.278487	.0116492	2.322265	.0052906	.5893388	.0094923	15934.14	115.237
2005	.0972577	2.196848	.0115848	2.238672	.0049804	.6120697	.0098175	16503.35	117.6843
2006	.0975634	2.152272	.0108552	2.194793	.0046516	.6301144	.0092303	16940.74	114.365
2007	.1064382	2.131012	.0102924	2.173568	.0044181	.6218958	.0088186	17108.16	109.6708
2009	.0966601	2.179464	.0096267	2.238696	.0050993	.6327745	.0080159	16274.68	105.4642
2010	.0979533	2.2774	.0054083	2.321593	.0050355	.5717461	.003275	15703.49	77.03366

Tax data: Pareto model and non-parametric estimates of mean among the richest 100*P_R % and their income share (threshold = *p*99 in the survey) [NB income shares of Rich depend on P_R and survey estimates of the mean of non-Rich, shown later in appendix]

year	P_R	mean_mlo	mean_gpd	mean_np	se_mean~o	se_mean~d	se_mean~p	S_R_mlo	S_R_gpd	S_R_np
1995	.0085792	156670.8	165150.9	158323.1	1948.854	21064.17	13025.7	7.607906	7.986809	7.681977

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

1996	.008958	174471.5	184431.2	179533.8	2222.78	20465.06	14000.19	8.404487	8.841833	8.627304
1997	.0092068	186030.3	195260	190278.6	1480.853	12709.63	9183.695	9.008408	9.413278	9.19521
1998	.0093161	200975.8	208037.8	207181	1397.287	10000.22	9597.822	9.48213	9.782721	9.746362
1999	.0104736	197855.3	212137	207898.1	1122.426	9530.946	8189.784	10.07361	10.72278	10.53109
2000	.0098838	231325.4	249299.6	237453.9	1493.285	12061.17	8091.73	10.61583	11.34709	10.86651
2001	.0105668	222344.8	238468.4	228594.3	1238.702	9327.533	6884.204	10.61446	11.29723	10.88035
2002	.0110572	213263.7	223903.5	217793.3	1139.751	7795.354	6100.091	10.70578	11.18018	10.90836
2003	.011684	208325	217440.2	213571	1084.766	7202.349	6287.896	10.85227	11.27357	11.09523
2004	.0113858	228608.5	235968.9	233074.8	1218.097	9291.77	8131.193	11.30617	11.62786	11.50165
2005	.0131511	241493.3	246962.1	243684	1375.028	10001.14	8211.785	13.46539	13.72846	13.57096
2006	.013604	256232.9	260289.5	256858.2	1515.886	10018.38	8300.686	14.53834	14.73459	14.56865
2007	.0142837	261478.8	269689.7	266924.2	1494.96	10569.23	8455.183	15.45302	15.86131	15.72423
2009	.0118367	275611.5	305333.4	289375.5	1733.068	15692.08	10813.05	13.62552	14.8763	14.20928
2010	.0114712	226552.6	240444.9	237673.2	1062.407	2490.087	3508.254	11.55242	12.17457	12.05115

Tax data: Pareto model and non-parametric estimates of mean among the richest 100*P_R % and their income share (threshold = *p*95 in the survey) [NB income shares of Rich depend on P_R and survey estimates of the mean of non-Rich, shown later in appendix]

year	P_R	mean_mlo	mean_gpd	mean_np	se_mean~o	se_mean~d	se_mean~p	S_R_mlo	S_R_gpd	S_R_np
1995	.0349137	80554.7	83444.33	82856.27	515.2545	3609.17	3581.2	16.58459	17.07791	16.97799
1996	.0341588	85688.55	91727.57	91568.84	497.6064	3932.708	4006.27	16.52408	17.48502	17.46004
1997	.0359085	90390.54	95162.36	95004.66	435.7874	2365.043	2557.139	17.83467	18.60106	18.57596
1998	.0370005	95669.81	100926.6	101244.9	387.8547	1800.121	2541.855	18.71842	19.54591	19.59548
1999	.0408321	96070.52	101710.2	102429.3	330.4416	1559.793	2213.287	19.74078	20.66021	20.77593
2000	.0434905	100847.3	108431.9	108048.9	343.4195	1694.147	1978.992	21.05144	22.28192	22.22069
2001	.0428005	104130.1	110712.6	110239.5	311.2445	1470.08	1805.728	20.81193	21.84023	21.76721
2002	.0436872	102690.8	108508.1	107818.8	295.374	1377.423	1643.956	20.99965	21.92839	21.81948
2003	.0424476	104229.2	110104.4	109622	304.9838	1438.175	1821.406	20.4253	21.33105	21.25746
2004	.045103	106393.3	114654.1	112948	271.1832	1838.786	2170.315	21.50003	22.78896	22.52624
2005	.0500392	110387.9	121008.3	117731.8	291.9376	2242.777	2291.413	24.0441	25.76145	25.23997
2006	.0525945	112101.7	125278.5	120702.1	289.148	2336.351	2275.275	25.20766	27.35999	26.62666
2007	.0564673	112157.6	126357.3	122341.9	276.0276	2344.964	2267.789	26.76992	29.17049	28.50776
2009	.0477972	113158.8	132044.5	127767	317.2386	2597.254	2797.313	23.48839	26.37457	25.7401
2010	.0485559	102564.4	111148.8	110504	244.0201	485.5498	856.3663	22.71786	24.1599	24.05346

Tax data: Pareto model and non-parametric estimates of mean among the richest 100*P_R % and their income share (threshold = *p*90 in the survey) [NB income shares of Rich depend on P_R and survey estimates of the mean of non-Rich, shown later in appendix]

year	P_R	mean_mlo	mean_gpd	mean_np	se_mean~o	se_mean~d	se_mean~p	S_R_mlo	S_R_gpd	S_R_np
1995	.0691512	58806.33	62025.57	61553.76	269.6673	1622.162	1868.256	24.89174	25.90144	25.75516
1996	.0656919	62732.61	67394.13	67497.27	272.0257	1732.144	2140.528	24.25465	25.59565	25.62478
1997	.0704028	63766.82	70376.04	68854.54	214.5684	1223.284	1347.415	25.69912	27.62687	27.192
1998	.0736046	67273.27	72700.43	72395.7	202.8829	808.5192	1306.276	27.12538	28.68594	28.60009

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

1999	.0784177	69087.77	74362.52	74414.87	184.9931	727.389	1178.527	28.15536	29.66726	29.68194
2000	.0825183	72761.04	78501.73	78481.57	198.0265	756.7242	1068.388	29.65939	31.26777	31.26225
2001	.0813434	75009.84	80655.31	80370.66	169.5012	664.2412	974.1547	29.38112	30.90894	30.83349
2002	.0842385	73916.98	78592.4	78457.22	157.6766	590.5917	871.2386	29.93278	31.23472	31.19776
2003	.0798965	75311.94	80981.04	80350.22	160.7672	668.1057	985.3011	28.69084	30.19834	30.03375
2004	.0898392	74571.41	81261.06	80611.07	128.4845	734.7617	1108.295	30.74842	32.60737	32.43114
2005	.0972577	77156.92	85233.45	84008.57	138.577	900.1124	1199.984	33.39111	35.64064	35.3093
2006	.0975634	79604.52	89134.7	87512.29	141.2073	968.2331	1246.013	34.05813	36.64152	36.21612
2007	.1064382	79149.23	87981.98	87586.04	137.0898	894.8082	1222.138	36.25751	38.73636	38.62938
2009	.0966601	76925.5	86881.66	87701.7	141.4567	806.3739	1397.995	33.01443	35.75938	35.97547
2010	.0979533	72086.18	77704.49	78298.05	118.3061	200.5599	429.7935	32.74108	34.41469	34.58665

Tax data: Pareto model and non-parametric estimates: Gini among the richest 100*P_R %, SE(Gini); tax data (threshold = *p*99 in the survey)

year	gini_mlo	gini_gpd	gini_np	se_gini~o	se_gini~d	se_gini~p
1995	.3054124	.3434142	.3144873	.0056394	.0732738	.0427343
1996	.3440906	.3821193	.3650996	.0056158	.0599495	.0411213
1997	.3479235	.3803295	.3640029	.0034983	.0354961	.0261172
1998	.354151	.3771464	.3745846	.0030403	.0266204	.0262927
1999	.3463731	.3930254	.380599	.0024962	.0244879	.0221673
2000	.3698494	.4183659	.3891325	.0027862	.0251481	.018145
2001	.3554049	.4022064	.3761665	.0024337	.0209443	.0166525
2002	.3463809	.3799885	.3624798	.0023516	.0191831	.0157493
2003	.3497832	.3789697	.3676875	.002285	.0182886	.0167222
2004	.3608651	.3819696	.3742975	.0023172	.0215185	.0195203
2005	.3812402	.3955823	.3874516	.0024331	.0215328	.0182169
2006	.399635	.4093939	.4015009	.0024856	.0199333	.0171775
2007	.4029322	.4216229	.4156713	.0023945	.0198886	.0164527
2009	.4169108	.4759566	.4469829	.0025976	.0244471	.018722
2010	.3591863	.3976407	.3906763	.0020422	.0056809	.0086046

Tax data: Pareto model and non-parametric estimates: Gini among the richest 100*P_R %, SE(Gini); tax data (threshold = *p*95 in the survey)

year	gini_mlo	gini_gpd	gini_np	se_gini~o	se_gini~d	se_gini~p
1995	.2770547	.3037658	.2987593	.0029527	.0272153	.0271069
1996	.2855428	.3353189	.3342152	.0026668	.0257469	.0267482
1997	.3055175	.3422907	.3412394	.0021856	.0147889	.0163554
1998	.3091542	.3471917	.3492747	.0018333	.0107062	.015546
1999	.3058807	.3462314	.3508723	.0015589	.0092058	.0133336
2000	.3105944	.3610539	.3588559	.0015384	.0091707	.0110062
2001	.3110419	.3540559	.3513153	.0013499	.0078739	.0100201

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2002	.3074009	.3467033	.3425396	.0013023	.0076048	.0094274
2003	.312105	.3509152	.348074	.0013205	.0077804	.010253
2004	.3115523	.3644193	.3547632	.0011507	.0093954	.0117457
2005	.3256116	.3885343	.3713869	.0011821	.0104633	.0115079
2006	.3329382	.4072108	.384547	.0011467	.0102371	.0109423
2007	.3339036	.4128501	.3934216	.0010933	.010094	.0106184
2009	.3312938	.430923	.4117908	.0012479	.0105271	.0123294
2010	.3070388	.3636219	.3598456	.0010774	.0026348	.0048076

Tax data: Pareto model and non-parametric estimates: Gini among the richest 100*P_R %, SE(Gini); tax data (threshold = *p*90 in the survey)

year	gini_mlo	gini_gpd	gini_np	se_gini~o	se_gini~d	se_gini~p
1995	.2464656	.2890843	.2836135	.0021536	.0170911	.0201293
1996	.257662	.3126413	.313723	.0020242	.0162325	.0204607
1997	.2616601	.3366484	.3218432	.0015673	.0108702	.0124256
1998	.2746145	.3328262	.3299924	.0013942	.0070237	.0115752
1999	.2774748	.3323129	.3327698	.0012357	.0062016	.0101002
2000	.2862997	.3415607	.3414138	.0012493	.0059803	.0084948
2001	.283713	.3370788	.3347417	.0010389	.0051123	.0076662
2002	.2824262	.3279836	.3268512	.0009815	.0046838	.0071255
2003	.2847025	.3383912	.3331644	.0009808	.0050997	.0078365
2004	.2743838	.3384849	.3331642	.0007966	.0055726	.0088229
2005	.2875758	.3596169	.3502815	.0008238	.0063067	.0088905
2006	.2950213	.375088	.3635042	.0008097	.0063374	.0086909
2007	.2987629	.3731775	.3704197	.0007887	.0059282	.0084355
2009	.2875718	.3730876	.379119	.0008434	.005465	.0096246
2010	.274485	.3304022	.3355382	.0007588	.0016115	.0035431

Tax data: Pareto model and non-parametric estimates: MLD among the richest 100*P_R %, SE(MLD); tax data (threshold = *p*99 in the survey)

year	ge0_mlo	ge0_gpd	ge0_np	se_ge0_~o	se_ge0_~d	se_ge0_np
1995	.1630389	.217282	.1684216	.0058205	.	.0469774
1996	.2054452	.2691835	.2308747	.006523	.	.0528787
1997	.2099224	.266396	.2285402	.0041094	.	.0337731
1998	.2173044	.2614269	.2447563	.0036366	.	.036755
1999	.2081053	.2893426	.253087	.0029189	.	.0313797
2000	.2365117	.3329697	.2596926	.0034858	.	.0253388
2001	.2188069	.3065211	.2434956	.0029216	.	.0229737
2002	.2081145	.2697532	.2267408	.0027499	.	.0209576
2003	.2121128	.266479	.2343417	.0026987	.	.0227853
2004	.2254138	.2710199	.2424991	.0028259	.	.0268511

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2005	.2509941	.2907401	.2583794	.0031432	.	.0255687
2006	.2753729	.3127077	.2767462	.0033784	.	.0251991
2007	.2798748	.3350168	.2980914	.0032841	.	.0249359
2009	.2994156	.4419066	.3439819	.0037004	.	.0307121
2010	.2233713	.297654	.2662505	.0024785	.	.0132066

Tax data: Pareto model and non-parametric estimates: MLD among the richest 100*P_R %, SE(MLD); tax data (threshold = *p*95 in the survey)

year	ge0_mlo	ge0_gpd	ge0_np	se_ge0_~o	se_ge0_~d	se_ge0_np
1995	.1350818	.166626	.1585496	.0027753	.	.0311939
1996	.1431763	.2058809	.2017504	.0025798	.	.034325
1997	.1631473	.2114191	.2073442	.0022565	.	.0214905
1998	.1669239	.2177577	.2180434	.0019147	.	.0217018
1999	.1635226	.2169992	.2211212	.0016113	.	.0186683
2000	.1684314	.2374165	.2297816	.001614	.	.0153269
2001	.1689013	.2274825	.2194398	.0014183	.	.0138488
2002	.1650977	.217839	.2081922	.0013526	.	.0127265
2003	.1700201	.2228814	.2149935	.001392	.	.0141118
2004	.1694379	.2428998	.2231789	.0012109	.	.0163757
2005	.1845605	.2769366	.2428646	.0012992	.	.0164365
2006	.1927029	.3055029	.2599788	.0012885	.	.0161618
2007	.1937893	.3145581	.2734098	.0012321	.	.015991
2009	.1908597	.3457932	.3018139	.0013953	.	.0195336
2010	.1647218	.2422952	.2319466	.0011178	.	.0071567

Tax data: Pareto model and non-parametric estimates: MLD among the richest 100*P_R %, SE(MLD); tax data (threshold = *p*90 in the survey)

year	ge0_mlo	ge0_gpd	ge0_np	se_ge0_~o	se_ge0_~d	se_ge0_np
1995	.1078295	.1549367	.1466852	.0018135	.	.023404
1996	.1174575	.1814751	.1818338	.0017768	.	.0260218
1997	.1209924	.2128299	.1890867	.0013957	.	.016185
1998	.1327977	.2044127	.1985364	.0012995	.	.0159264
1999	.1354769	.2030479	.2020825	.0011632	.	.0139498
2000	.1439094	.213646	.2113586	.0012115	.	.01171
2001	.1414117	.2082812	.2027446	.0009988	.	.010496
2002	.1401772	.1963152	.192992	.0009396	.	.0095672
2003	.1423646	.2100056	.2001443	.0009461	.	.0107198
2004	.1325828	.2125305	.2021878	.0007419	.	.012105
2005	.1451496	.2393892	.2217228	.0008023	.	.0124923
2006	.1524909	.2606301	.2381978	.0008082	.	.0125754
2007	.1562483	.2566667	.2483471	.0007969	.	.0123932

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2009	.1451457	.2587453	.2641096	.0008214	.	.0146127
2010	.132677	.2012096	.2069788	.0007069	.	.0051116

Tax data: Pareto model and non-parametric estimates: Theil among the richest 100*P_R %, SE(Theil); tax data (threshold = *p*99 in the survey)

year	ge1_mlo	ge1_gpd	ge1_np	se_ge1_~o	se_ge1_~d	se_ge1_np
1995	.2484504	.	.2239827	.0109391	.	.0644242
1996	.3317514	.	.3732583	.0133669	.	.0899333
1997	.3409663	.	.357114	.0084946	.	.0632853
1998	.3563347	.	.415646	.0076248	.	.0905943
1999	.3372167	.	.4308738	.0060125	.	.0778595
2000	.3973471	.	.4109592	.0075776	.	.0543522
2001	.3594896	.	.3871077	.0061434	.	.0548116
2002	.3372355	.	.3577004	.0056644	.	.0505689
2003	.3455037	.	.3762938	.0056023	.	.0527927
2004	.3734696	.	.3923422	.0060169	.	.0605882
2005	.4292523	.	.4197822	.0070164	.	.0586319
2006	.4848777	.	.4596834	.0078761	.	.0660984
2007	.4954144	.	.5117994	.0077167	.	.0677345
2009	.5421128	.	.600757	.008992	.	.0874639
2010	.3691292	.	.4449106	.005257	.	.0414407

Tax data: Pareto model and non-parametric estimates: Theil among the richest 100*P_R %, SE(Theil); tax data (threshold = *p*95 in the survey)

year	ge1_mlo	ge1_gpd	ge1_np	se_ge1_~o	se_ge1_~d	se_ge1_np
1995	.197483	.	.2302815	.0049025	.	.0562968
1996	.2119147	.	.3416048	.0046418	.	.0735728
1997	.2486543	.	.3412075	.0042419	.	.0492886
1998	.2557822	.	.3805847	.0036284	.	.0632589
1999	.24936	.	.3900273	.0030315	.	.0547237
2000	.2586436	.	.3904677	.0030684	.	.0399822
2001	.2595372	.	.3681855	.0026989	.	.0386248
2002	.2523283	.	.3431315	.0025533	.	.035173
2003	.2616688	.	.3591827	.0026552	.	.0384055
2004	.2605589	.	.376606	.002307	.	.0442051
2005	.289827	.	.4126986	.0025538	.	.0442555
2006	.3059646	.	.4518899	.0025748	.	.0484498
2007	.308138	.	.4923144	.0024674	.	.0496636
2009	.3022883	.	.5673997	.0027778	.	.0648812
2010	.2516191	.	.4079681	.0021084	.	.0268149

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

Tax data: Pareto models and non-parametric: estimates: Theil among the richest 100*P_R %, SE(Theil); tax data (threshold = *p*90 in the survey)

year	ge1_mlo	ge1_gpd	ge1_np	se_ge1_~o	se_ge1_~d	se_ge1_np
1995	.1508662	.	.2197679	.0029998	.	.0473592
1996	.1669855	.	.3129517	.0030102	.	.0618038
1997	.1730002	.	.3181325	.002385	.	.0408789
1998	.1934589	.	.3506664	.0022834	.	.0504378
1999	.1981813	.	.3602051	.0020566	.	.0442391
2000	.2132349	.	.3665609	.0021835	.	.0332759
2001	.2087459	.	.3465109	.0017901	.	.031576
2002	.2065366	.	.3236651	.0016792	.	.0284392
2003	.2104556	.	.3397191	.0016993	.	.0315792
2004	.1930813	.	.3492759	.001303	.	.035752
2005	.2154732	.	.3871073	.00145	.	.0366512
2006	.2288503	.	.4253711	.0014847	.	.0404455
2007	.2357809	.	.4590642	.0014759	.	.0413343
2009	.2154662	.	.5150243	.0014845	.	.0528829
2010	.1932468	.	.3717209	.0012418	.	.0207152

Tax data: non-parametric estimates: HSCV among the richest 100*P_R %, SE(SCV); (threshold = *p*99 in the survey)

year	ge2_np	se_ge2_np
1995	.3703653	.1119796
1996	2.553078	1.068631
1997	1.609383	.5912639
1998	2.457971	1.326763
1999	2.450336	1.14099
2000	1.545045	.5096064
2001	1.662092	.7344644
2002	1.46042	.6613069
2003	1.522263	.5258301
2004	1.610249	.5806384
2005	1.802406	.6680321
2006	2.406135	1.107318
2007	3.023111	1.326159
2009	4.192604	2.127515
2010	2.026619	.6967632

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

Tax data: non-parametric estimates: HSCV among the richest 100*P_R %, SE(SCV); (threshold = *p*95 in the survey)

year	ge2_np	se_ge2_np
1995	.4732059	.154803
1996	2.74252	1.096368
1997	1.833767	.6364387
1998	2.780726	1.462337
1999	2.777043	1.260713
2000	1.912189	.5951119
2001	1.958515	.8098394
2002	1.69013	.7083473
2003	1.765383	.5785629
2004	1.927185	.6621051
2005	2.23813	.7891974
2006	3.045173	1.345713
2007	3.881392	1.656142
2009	5.593761	2.805344
2010	2.425711	.7907994

Tax data: non-parametric estimates: HSCV among the richest 100*P_R %, SE(SCV); (threshold = *p*90 in the survey)

year	ge2_np	se_ge2_np
1995	.4943735	.1599499
1996	2.69334	1.064673
1997	1.855051	.62982
1998	2.811559	1.45893
1999	2.815917	1.262346
2000	1.988952	.6061137
2001	2.016029	.8118414
2002	1.731156	.7023242
2003	1.821363	.5827562
2004	1.980654	.6665972
2005	2.347095	.8118706
2006	3.207164	1.397407
2007	4.106353	1.735639
2009	5.97191	2.985444
2010	2.478532	.7903417

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

SURVEY DATA ESTIMATES

Survey data: Mean incomes (and SEs), poorest 100% 99% 95% 90% in survey (HBAI no_spi)

year	mean_100	mean_99	mean_95	mean_90	se_me~100	se_mea~99	se_mea~95	se_mea~90
1994	17568.84	16247.06	14424.05	12935.52	108.7958	66.63673	52.30388	44.79353
1995	17710.32	16464.41	14657.58	13181.87	95.9942	65.27045	51.71943	44.63707
1996	18538.32	17187.34	15309.6	13774.5	121.3486	68.33523	54.44384	47.08815
1997	18923.65	17460.8	15510.47	13962.58	113.5129	72.93839	57.1982	49.29964
1998	19664.83	18041.53	15961.72	14359.92	146.5929	78.164	60.08782	51.64522
1999	20279.88	18694.7	16627.54	15000.79	129.3617	75.48351	58.4411	50.38337
2000	21758.89	19443.25	17196.16	15520.13	246.9471	82.2681	62.35143	53.89151
2001	22240.14	19996.4	17716.19	15963.92	368.3678	81.9846	62.33885	53.3261
2002	21568.29	19888.21	17648.29	15916.24	129.1392	78.04877	60.23559	51.80199
2003	21966.77	20231.44	18000.59	16253.87	137.6665	78.61457	60.87997	52.18654
2004	22426.24	20654.14	18348.18	16577.8	133.8744	81.61338	62.44443	53.7698
2005	22970.71	20681.77	18368.74	16581.99	414.9853	80.98448	62.77187	54.05857
2006	22716.45	20773.34	18464.66	16662.82	155.5964	87.59673	67.0558	57.34881
2007	22617.67	20730.44	18361.62	16574.89	154.3692	89.82967	68.16478	58.42841
2008	23517.89	21061.79	18678.15	16861.51	606.0826	92.54469	70.56518	60.24641
2009	23017.63	20928.21	18502.72	16701.05	180.0382	91.6692	68.81789	58.71284
2010	22138.77	20128.06	17806.01	16080.47	165.6817	86.42778	65.01446	55.71654
2011	21301.97	19622.54	17351.86	15658.85	157.8365	102.2405	72.4127	60.25822
2012	21129.72	19420.89	17237.61	15613.27	175.2743	95.18062	72.70496	63.28411

Survey data: Gini coefficient (and SEs), poorest 100% 99% 95% 90% in survey (HBAI no_spi)

year	gini_100	gini_99	gini_95	gini_90	se_gi~100	se_gin~99	se_gin~95	se_gin~90
1994	.472323	.4391356	.4065409	.3857156	.0025595	.0012603	.0011326	.0011489
1995	.463222	.4321555	.3997137	.3789609	.0021399	.001236	.0011115	.0011279
1996	.4630767	.4304282	.398096	.3774397	.0028988	.0012436	.0011264	.0011422
1997	.4652113	.4300135	.3958455	.3749504	.0024285	.0013241	.0011741	.0011849
1998	.4699149	.4318592	.3953609	.3737798	.0033113	.0013765	.001193	.001201
1999	.4563874	.4197488	.3844032	.362718	.002792	.0012888	.0011154	.0011172
2000	.4764894	.4237607	.3856301	.3645655	.0055104	.0013732	.0011598	.0011666
2001	.4726044	.4229882	.3857013	.3634131	.0084824	.0013266	.0011368	.0011387
2002	.4556557	.4191605	.3823379	.3599685	.0025654	.0012871	.0011283	.001134
2003	.4522397	.4146416	.3786189	.3563949	.0027911	.0012633	.0011049	.0011105
2004	.451385	.4137261	.3763697	.3540265	.0025624	.001308	.0011285	.001134
2005	.4657283	.4160367	.3791531	.3569149	.0094399	.0012942	.0011388	.0011494
2006	.4558289	.4143978	.3776759	.3549412	.0030107	.00139	.0012054	.0012152
2007	.4569121	.4169575	.3783844	.3559613	.0029592	.0014498	.001243	.0012518

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2008	.4679517	.4153965	.3774598	.355032	.0135159	.0014692	.0012728	.0012836
2009	.4590352	.414584	.3743414	.351203	.0035715	.0014857	.0012647	.001269
2010	.4580632	.4134066	.3734187	.3503199	.0034063	.0014563	.0012406	.0012503
2011	.4469958	.4090377	.3678748	.3431162	.0031719	.0018079	.0014205	.0014182
2012	.4462832	.4069328	.3679185	.34591	.0037592	.0016767	.0014532	.0014674

Survey data: MLD (and SEs), poorest 100% 99% 95% 90% in survey (HBAI no_spi)

year	ge0_100	ge0_99	ge0_95	ge0_90	se_ge0~00	se_ge0_99	se_ge0_95	se_ge0_90
1994	.4969565	.4448276	.4006435	.3745283	.0054016	.0036828	.0035871	.0036475
1995	.4747472	.4272833	.3843256	.3590096	.0046725	.0035971	.0035141	.0035797
1996	.4790595	.42893	.3866238	.3619172	.0059842	.0037961	.003736	.003819
1997	.4774522	.4233427	.3787588	.3539031	.0051463	.0037943	.0036948	.0037654
1998	.4805134	.4207649	.3728894	.347151	.0065848	.0037965	.0036577	.0037194
1999	.4213244	.3655647	.3197096	.2932615	.005058	.0027277	.0025128	.0024972
2000	.4631009	.3778259	.3282733	.3026329	.0102552	.0029838	.0027376	.0027349
2001	.458056	.3781143	.3297572	.3030414	.0159411	.0029147	.0026927	.0026866
2002	.4265866	.3714052	.3242906	.297941	.0046978	.0028221	.002627	.0026202
2003	.4179406	.3612774	.3157693	.2898809	.0050713	.0027819	.0026013	.0026012
2004	.4190383	.3627216	.3159965	.2905196	.0047054	.0029172	.0027272	.0027355
2005	.4488186	.3700412	.3235898	.2980318	.0175593	.0029716	.0028022	.0028142
2006	.4279332	.3651884	.319093	.2931317	.0054905	.0031005	.0029006	.002907
2007	.4344676	.3739877	.3255582	.3001172	.0054694	.0032935	.0030772	.0030906
2008	.4599358	.376112	.3288922	.3038459	.0252607	.0034626	.0032708	.0032958
2009	.4375291	.3694542	.3195554	.2939922	.0065587	.003401	.0031819	.0032025
2010	.429448	.3610908	.3114119	.2856084	.00618	.0031452	.0029056	.0029087
2011	.4079798	.3518671	.3016068	.2749273	.0057902	.0037732	.0034014	.00341
2012	.4111338	.3530922	.3058012	.2818496	.0067128	.0037038	.0034716	.0034913

Survey data: Theil coefficient (and SEs), poorest 100% 99% 95% 90% in survey (HBAI no_spi)

year	ge1_100	ge1_99	ge1_95	ge1_90	se_ge1~00	se_ge1_99	se_ge1_95	se_ge1_90
1994	.4144205	.3202805	.2704902	.2444596	.0107548	.0019112	.0015339	.0014773
1995	.3908314	.3097924	.2611008	.2356814	.0067953	.0018404	.0014762	.0014222
1996	.3993823	.3070809	.2589458	.2337766	.0148014	.0018362	.0014897	.0014352
1997	.4025891	.3074812	.2561327	.2308487	.0074576	.0019656	.0015391	.0014757
1998	.4268303	.3110523	.2553305	.2291471	.0156527	.0020726	.0015629	.0014919
1999	.3956642	.2916681	.2397054	.2140793	.0114663	.0018624	.0014021	.0013268
2000	.4978181	.2992969	.2417646	.2168843	.0322911	.0020311	.001467	.0013976
2001	.5114286	.2980739	.242203	.21585	.073371	.0019468	.0014347	.0013575
2002	.3918648	.2923957	.2382627	.2121447	.0087857	.0018537	.0014115	.0013395
2003	.3930174	.2852631	.2332013	.2075343	.0112301	.0017939	.0013701	.0013013
2004	.3877697	.2851863	.2308694	.2053042	.0085443	.0018679	.0013931	.0013238

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2005	.5024381	.2883504	.2346063	.2089942	.0856763	.0018495	.0014174	.0013532
2006	.4045244	.2860975	.2327275	.2066422	.0108688	.001989	.0014905	.0014187
2007	.401395	.2908952	.2340048	.2082963	.0098258	.0021045	.0015448	.0014721
2008	.5302129	.2888031	.2333733	.2078357	.1269576	.0021162	.0015836	.0015133
2009	.4237711	.2885957	.229237	.2030157	.0145881	.0021497	.0015551	.001475
2010	.4230374	.2865068	.2278205	.2016953	.0138805	.0020971	.0015152	.0014405
2011	.3829484	.281124	.2212614	.1936216	.0105618	.0026761	.0017106	.0016095
2012	.3839085	.2770038	.221145	.1967897	.0138443	.0023497	.0017597	.001685

Survey data: Half Squared Coefficient of Variation, GE(2) (and SEs), poorest 100% 99% 95% 90% in survey (HBAI no_spi)

year	ge2_100	ge2_99	ge2_95	ge2_90	se_ge2~00	se_ge2_99	se_ge2_95	se_ge2_90
1994	.8257974	.354761	.2708966	.2354188	.1253902	.0027133	.0016951	.0015191
1995	.645525	.3425754	.2611488	.2268152	.0473049	.0026017	.0016178	.0014522
1996	.8625702	.3385621	.2586074	.2246478	.2486128	.0025553	.0016292	.0014618
1997	.6974645	.3421382	.2552415	.2208847	.040231	.0028237	.0016835	.0014974
1998	1.034494	.3514183	.2556981	.2199998	.1942446	.0030724	.001719	.0015143
1999	.8393157	.3271122	.2406055	.2064352	.1196628	.0027047	.0015464	.0013577
2000	2.260654	.3405344	.2414693	.2082268	.5673445	.003063	.0016034	.0014183
2001	6.382292	.3379169	.2425138	.2067916	3.753239	.0028972	.0015744	.0013758
2002	.7398049	.3290322	.2379725	.2025414	.0639681	.0026723	.0015332	.0013434
2003	.871011	.3193993	.2331922	.1987943	.1070736	.0025466	.0014815	.0013029
2004	.7469031	.3215204	.2301449	.1956907	.0557357	.0027102	.0014965	.0013117
2005	7.426051	.3239304	.2336284	.1989975	5.621798	.0026434	.0015188	.0013406
2006	.862484	.3219229	.2322933	.1970586	.0811135	.0028968	.0016007	.0014102
2007	.7764937	.3299067	.2325905	.1976771	.0583485	.0031267	.0016513	.0014516
2008	10.10986	.3255871	.2313047	.1965393	8.788145	.0030831	.0016766	.0014753
2009	1.084875	.3300335	.2277553	.1919303	.1527846	.0032134	.0016564	.0014378
2010	1.09438	.3272814	.2268171	.1913613	.1474963	.0031293	.0016146	.0014136
2011	.743455	.3234162	.221305	.1835307	.0746583	.0042953	.0018151	.0015554
2012	.7690785	.3128728	.2185574	.1858407	.108897	.003363	.0018445	.0016299

COMBINED DATA ESTIMATES

Combined data (tax and survey) estimates: Means (threshold = *p*99 in the survey)

year	Mall_mlo	Mall_gpd	Mall_np
1995	17667.27	17740.02	17681.45
1996	18596.3	18685.52	18641.65
1997	19012.79	19097.77	19051.91
1998	19745.78	19811.57	19803.58
1999	20571.16	20720.74	20676.34
2000	21537.45	21715.11	21598.02

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2001	22134.57	22304.95	22200.61
2002	22026.4	22144.05	22076.48
2003	22429.13	22535.63	22490.42
2004	23021.86	23105.67	23072.71
2005	23585.69	23657.61	23614.5
2006	23976.53	24031.71	23985.04
2007	24169.21	24286.49	24246.99
2009	23942.82	24294.63	24105.74
2010	22496	22655.37	22623.57

Combined data (tax and survey) estimates: Means (threshold = *p*95 in the survey)

year	Mall_mlo	Mall_gpd	Mall_np
1995	16958.3	17059.18	17038.65
1996	17713.66	17919.95	17914.53
1997	18199.29	18370.64	18364.98
1998	18910.96	19105.47	19117.25
1999	19871.36	20101.64	20131
2000	20834.19	21164.05	21147.39
2001	21414.75	21696.49	21676.24
2002	21363.55	21617.69	21587.58
2003	21660.79	21910.19	21889.71
2004	22319.27	22691.86	22614.91
2005	22973.3	23504.74	23340.79
2006	23389.46	24082.5	23841.8
2007	23658.03	24459.85	24233.11
2009	23027.02	23929.7	23725.25
2010	21921.53	22338.35	22307.04

Combined data (tax and survey) estimates: Means (threshold = *p*90 in the survey)

year	Mall_mlo	Mall_gpd	Mall_np
1995	16336.85	16559.47	16526.84
1996	16990.65	17296.87	17303.65
1997	17468.94	17934.25	17827.13
1998	18254.58	18654.05	18631.62
1999	19242.16	19655.79	19659.9
2000	20243.56	20717.27	20715.61
2001	20766.91	21226.13	21202.98
2002	20802.14	21195.99	21184.6
2003	20972.4	21425.34	21374.94
2004	21787.9	22388.9	22330.5
2005	22473.37	23258.88	23139.75

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2006	22803.63	23733.43	23575.14
2007	23235.19	24175.34	24133.19
2009	22522.35	23484.71	23563.98
2010	21566.41	22116.74	22174.89

Combined data (tax and survey) estimates: Gini coefficient and between-group Gini (threshold = *p*99 in the survey)

year	G_mlo	G_gpd	G_np	GB_mlo	GB_gpd	GB_np
1995	.4635513	.4657528	.4639826	.0674999	.0712889	.0682406
1996	.4660671	.4686185	.4673679	.0750868	.0794603	.077315
1997	.4688395	.4712043	.4699313	.0808772	.0849259	.0827453
1998	.473086	.4748367	.4746251	.0855052	.0885111	.0881475
1999	.4641395	.4680108	.4668685	.0902625	.0967542	.0948373
2000	.4716938	.4760194	.4731787	.0962745	.1035871	.0987813
2001	.4700716	.4741232	.4716515	.0955779	.1024055	.0982367
2002	.4665582	.4693954	.4677714	.0960006	.1007446	.0980264
2003	.4626068	.465149	.4640739	.0968387	.1010517	.0992683
2004	.4649119	.4668542	.4660928	.1016759	.1048928	.1036307
2005	.477459	.4790488	.4780977	.1215028	.1241335	.1225585
2006	.4819032	.4830937	.4820877	.1317794	.1337419	.1320825
2007	.4886256	.4910962	.4902672	.1402466	.1443294	.1429587
2009	.478947	.4864964	.4824725	.1244185	.1369263	.1302561
2010	.4659827	.469741	.4689959	.104053	.1102745	.1090403

Combined data (tax and survey) estimates: Gini coefficient and between-group Gini (threshold = *p*95 in the survey)

year	G_mlo	G_gpd	G_np	GB_mlo	GB_gpd	GB_np
1995	.4543183	.4575554	.4569014	.1309324	.1358656	.1348664
1996	.4536565	.4599621	.459799	.131082	.1406914	.1404416
1997	.4579634	.4630322	.4628669	.1424383	.1501021	.1498511
1998	.4617901	.4672842	.4676127	.1501837	.1584586	.1589543
1999	.4549628	.4612223	.4620086	.1565757	.16577	.1669272
2000	.4610763	.4694976	.4690802	.1670239	.1793287	.1787164
2001	.4604463	.4674718	.4669746	.1653188	.1756017	.1748715
2002	.4579822	.464375	.4636279	.1663093	.1755968	.1745076
2003	.4530074	.4592525	.4587468	.1618055	.170863	.1701271
2004	.4550429	.4640244	.4622001	.1698973	.1827867	.1801594
2005	.4678978	.4799767	.4763221	.1904018	.2075752	.2023605
2006	.4715123	.4867799	.4815961	.1994822	.2210055	.2136722
2007	.477724	.4949121	.4901837	.211232	.2352376	.2286103
2009	.4635308	.4838179	.4793687	.1870867	.2159485	.2096038
2010	.4565831	.4667588	.4660097	.1786228	.1930432	.1919787

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

Combined data (tax and survey) estimates: Gini coefficient and between-group Gini (threshold = *p*90 in the survey)

year	G_mlo	G_gpd	G_np	GB_mlo	GB_gpd	GB_np
1995	.448957	.4564277	.4553542	.1797662	.1898632	.1884004
1996	.4480722	.4579046	.4581174	.1768547	.1902646	.190556
1997	.4503004	.4646723	.4614529	.1865884	.2058659	.2015172
1998	.4554734	.4672198	.4665779	.1976492	.2132548	.2123962
1999	.4494206	.4610904	.4612026	.203136	.2182549	.2184017
2000	.4563595	.4688694	.4688273	.2140757	.2301596	.2301043
2001	.4550109	.4668828	.4663009	.2124678	.2277461	.2269915
2002	.4531839	.46342	.4631323	.2150893	.2281087	.2277391
2003	.4473753	.4591452	.4578691	.2070119	.222087	.2204411
2004	.4483678	.4633034	.4619003	.217645	.2362345	.2344721
2005	.4606078	.4789814	.4762992	.2366533	.2591487	.2558353
2006	.4640407	.4852055	.4817491	.243018	.2688518	.2645978
2007	.4704146	.491175	.4902895	.2561369	.2809254	.2798555
2009	.4551767	.4776365	.4793996	.2334842	.2609337	.2630946
2010	.4508021	.4645844	.4659903	.2294575	.2461936	.2479132

Combined data (tax and survey) estimates: MLD and between-group MLD (threshold = *p*99 in the survey)

year	L_mlo	L_gpd	L_np	LB_mlo	LB_gpd	LB_np
1995	.4762005	.480323	.4769587	.0511841	.0548413	.0518961
1996	.4849565	.4898164	.4873637	.0580284	.0623173	.0602078
1997	.4847484	.489282	.4867671	.0633707	.0673843	.0652179
1998	.4866759	.4900915	.4895717	.0678064	.070811	.0704465
1999	.4348551	.4422211	.4399079	.0709396	.0774547	.0755212
2000	.4542472	.4626758	.4570264	.077818	.0852932	.0803681
2001	.4525674	.4604222	.4555143	.0761364	.0830644	.0788225
2002	.4454822	.4509524	.447727	.0758826	.0806712	.0779214
2003	.4354116	.4402836	.4381098	.075877	.0801138	.0783155
2004	.4423144	.4461065	.4444951	.0811562	.0844289	.0831423
2005	.467543	.4708159	.4687421	.0990674	.1018176	.1001694
2006	.4731927	.4757859	.4735329	.109226	.1113114	.1095476
2007	.4899141	.4951009	.4930929	.1172707	.1216699	.1201893
2009	.472682	.4877432	.4794142	.1040568	.1174314	.1102614
2010	.4429634	.4501919	.4485602	.0834524	.0898287	.0885573

Combined data (tax and survey) estimates: MLD and between-group MLD (threshold = *p*95 in the survey)

year	L_mlo	L_gpd	L_np	LB_mlo	LB_gpd	LB_np
1995	.4619308	.4677332	.4664939	.0863073	.0910084	.090051

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

1996	.4653344	.4767282	.4763437	.0870265	.0962784	.096035
1997	.4675911	.4768483	.4764532	.0965746	.1040984	.1038497
1998	.4685597	.4786941	.4792045	.1032912	.1115447	.1120445
1999	.4199312	.4313073	.4326475	.106599	.1157916	.1169635
2000	.4362996	.4518546	.450889	.1149779	.1275328	.1268992
2001	.4366668	.4496211	.4485264	.1137944	.1242413	.1234908
2002	.4314469	.4431695	.4416325	.1141109	.1235295	.1224139
2003	.4201358	.4314996	.4304161	.1105532	.1196732	.1189246
2004	.426033	.4425295	.4389193	.1166467	.1298298	.1271091
2005	.4505777	.473473	.4661418	.1339448	.1522176	.1465914
2006	.4540156	.4833028	.4728209	.14157	.1649245	.156837
2007	.4693717	.5027903	.4929771	.1512541	.1778532	.1703636
2009	.4455998	.4840799	.4749712	.1321957	.1632704	.1562638
2010	.4272032	.4459027	.4442801	.1229139	.1378468	.1367267

Combined data (tax and survey) estimates: MLD and between-group MLD (threshold = *p*90 in the survey)

year	L_mlo	L_gpd	L_np	LB_mlo	LB_gpd	LB_np
1995	.4528123	.4659187	.4639039	.111172	.121021	.1195768
1996	.4561094	.4734688	.4737836	.1102512	.1234052	.1236964
1997	.4546175	.4804277	.4743042	.117112	.1364566	.1320046
1998	.4576799	.4788875	.477561	.1263063	.1422427	.1413487
1999	.4101242	.4309219	.4309998	.1292358	.1447347	.1448883
2000	.4277395	.4503586	.4501107	.1382041	.1550688	.1550096
2001	.4270621	.448471	.4472168	.1371682	.1531377	.1523338
2002	.4230101	.4413287	.4406563	.1383589	.1519485	.1515561
2003	.4104653	.4314381	.4289199	.1323704	.1479389	.1462086
2004	.4145304	.441205	.4383857	.1381997	.1576919	.1558018
2005	.4376406	.4714796	.4660342	.1544778	.1791512	.175424
2006	.4405716	.4800545	.4729664	.1611613	.1900937	.1851942
2007	.4561727	.495265	.4931148	.1713686	.1997726	.1985079
2009	.4310059	.4720636	.4750435	.1514012	.1814783	.1839398
2010	.4172048	.4417642	.4442093	.1465766	.164423	.1663029

Combined data (tax and survey) estimates: Theil and between-group Theil (threshold = *p*99 in the survey)

year	T_mlo	T_gpd	T_np	TB_mlo	TB_gpd	TB_np
1995	.406015	.	.4057626	.1008894	.	.1025621
1996	.4251463	.	.4339782	.1159919	.	.1211879
1997	.4384786	.	.4444668	.1279809	.	.1324218
1998	.4536504	.	.4659604	.1383044	.	.1447141
1999	.438273	.	.4592516	.1420164	.	.1529236
2000	.4702946	.	.4782601	.1605889	.	.1668295

Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

2001	.4586726	.	.4682822	.1540798	.	.1605211
2002	.4490657	.	.4562163	.1518696	.	.1566969
2003	.4417379	.	.4509963	.1499374	.	.1556332
2004	.4584501	.	.4655118	.1632824	.	.1680009
2005	.5068599	.	.5083229	.1995365	.	.2021359
2006	.5368554	.	.5340071	.2218586	.	.2226205
2007	.5607189	.	.5707525	.2382193	.	.2451218
2009	.5398196	.	.5648244	.2166809	.	.2318728
2010	.4644972	.	.4862355	.1684455	.	.1806392

Combined data (tax and survey) estimates: Theil and between-group Theil (threshold = *p*95 in the survey)

year	T_mlo	T_gpd	T_np	TB_mlo	TB_gpd	TB_np
1995	.3873488	.	.399426	.1367988	.	.1435578
1996	.3899024	.	.4285385	.1387281	.	.1551604
1997	.4092895	.	.4396783	.1544906	.	.1677421
1998	.4210593	.	.4614742	.1656442	.	.1815996
1999	.4096497	.	.457465	.1680383	.	.1865288
2000	.4257922	.	.4763707	.1804743	.	.2015632
2001	.4248226	.	.4658311	.179012	.	.1962052
2002	.4199902	.	.4545569	.1787738	.	.1934124
2003	.4126235	.	.4484146	.1736076	.	.1884328
2004	.4192195	.	.4640104	.1819669	.	.2003121
2005	.4553958	.	.5089023	.2075122	.	.2293455
2006	.4693946	.	.5354087	.2182057	.	.2443255
2007	.4848503	.	.5708519	.2310001	.	.2632088
2009	.4529904	.	.565032	.2065948	.	.2487516
2010	.4230736	.	.4848856	.1898465	.	.2137334

Combined data (tax and survey) estimates: Theil and between-group Theil (threshold = *p*90 in the survey)

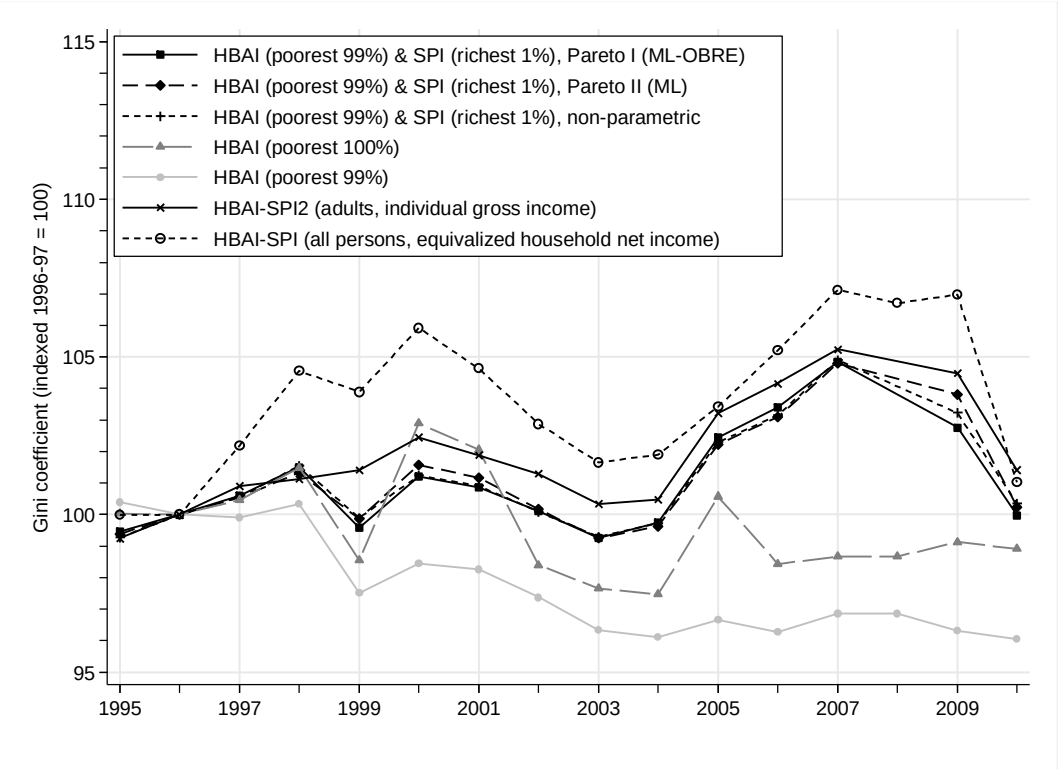
year	T_mlo	T_gpd	T_np	TB_mlo	TB_gpd	TB_np
1995	.3722211	.	.402344	.1576516	.	.1707611
1996	.3754482	.	.4332132	.1578715	.	.1791481
1997	.3822703	.	.4441222	.1662881	.	.1895393
1998	.3983943	.	.4661478	.1789278	.	.202246
1999	.3906119	.	.4623459	.1810087	.	.2048936
2000	.4083539	.	.481603	.192552	.	.2179264
2001	.4053435	.	.470694	.1915808	.	.2145567
2002	.4023989	.	.4586729	.1919328	.	.2117364
2003	.393415	.	.4533047	.1850425	.	.2060703
2004	.3906173	.	.4670285	.1890715	.	.2150327
2005	.42054	.	.5115795	.2093824	.	.2396947

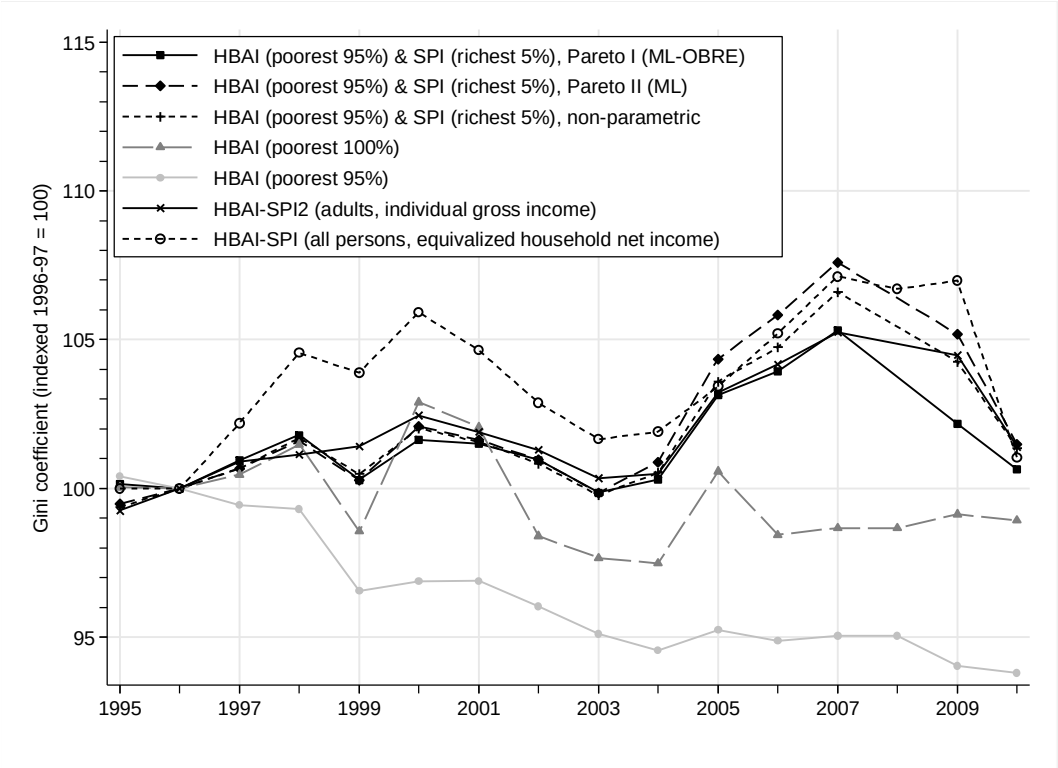
Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

```
     2006   .4330982          .   .5395249   .2188924          .   .2536675
     2007   .4473491          .    .572543   .2290876          .   .2673767
     2009   .4123406          .    .567657   .2052145          .   .2523947
     2010   .3965979          .   .4866246   .1976687          .   .2261232
```

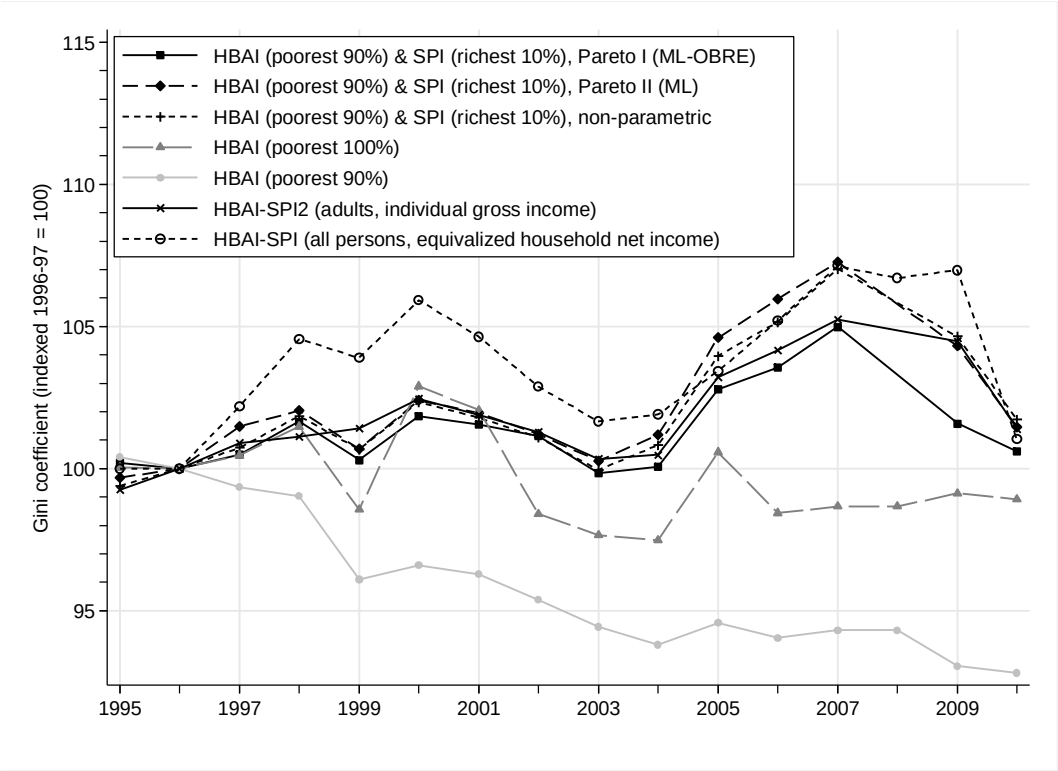
Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

`Gini coefficient (indexed 1996/97 = 100), by threshold`



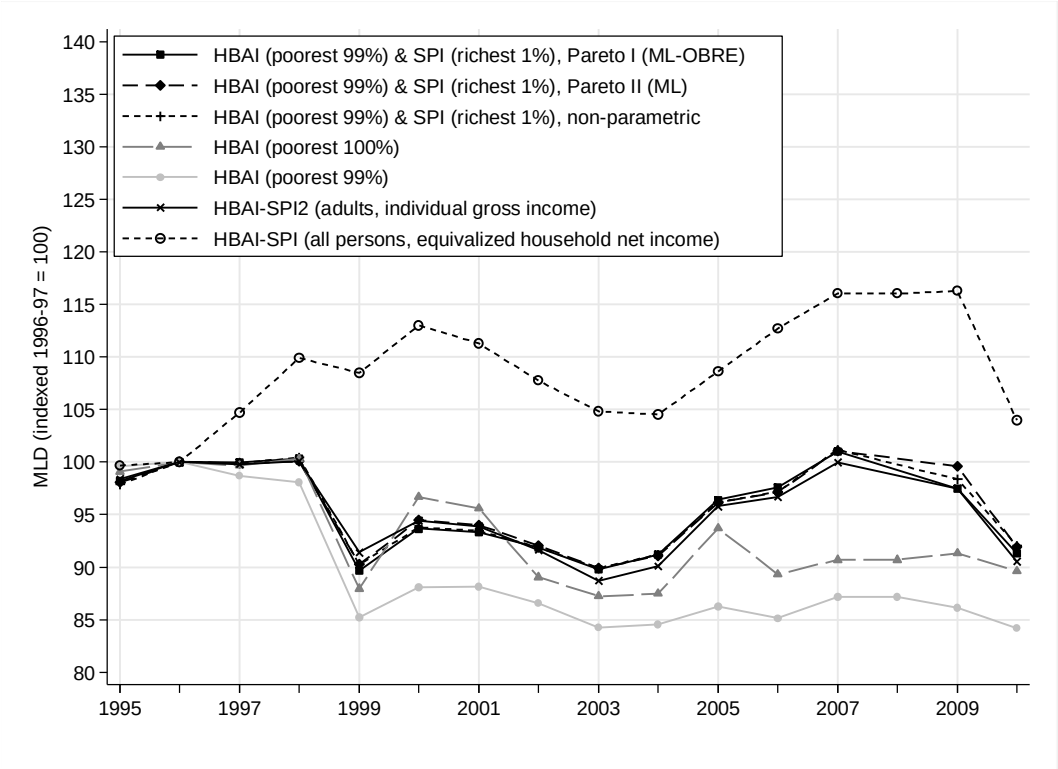


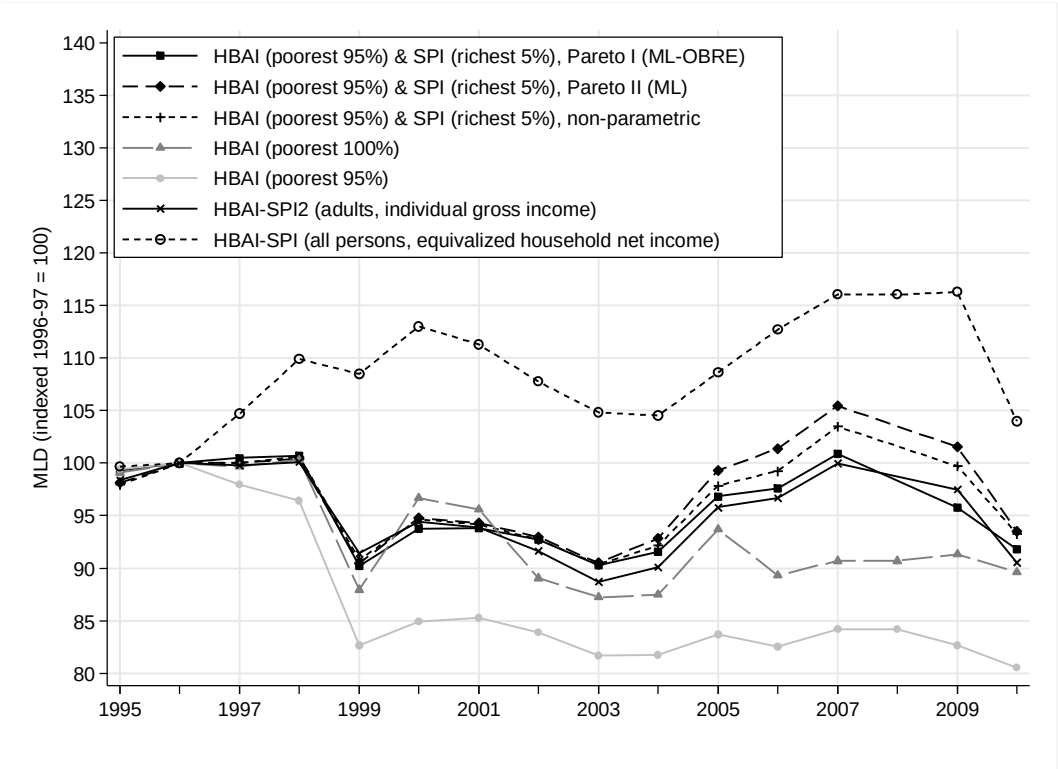
Approach C estimates, by threshold, with graphs of trends at the end (pareto14);



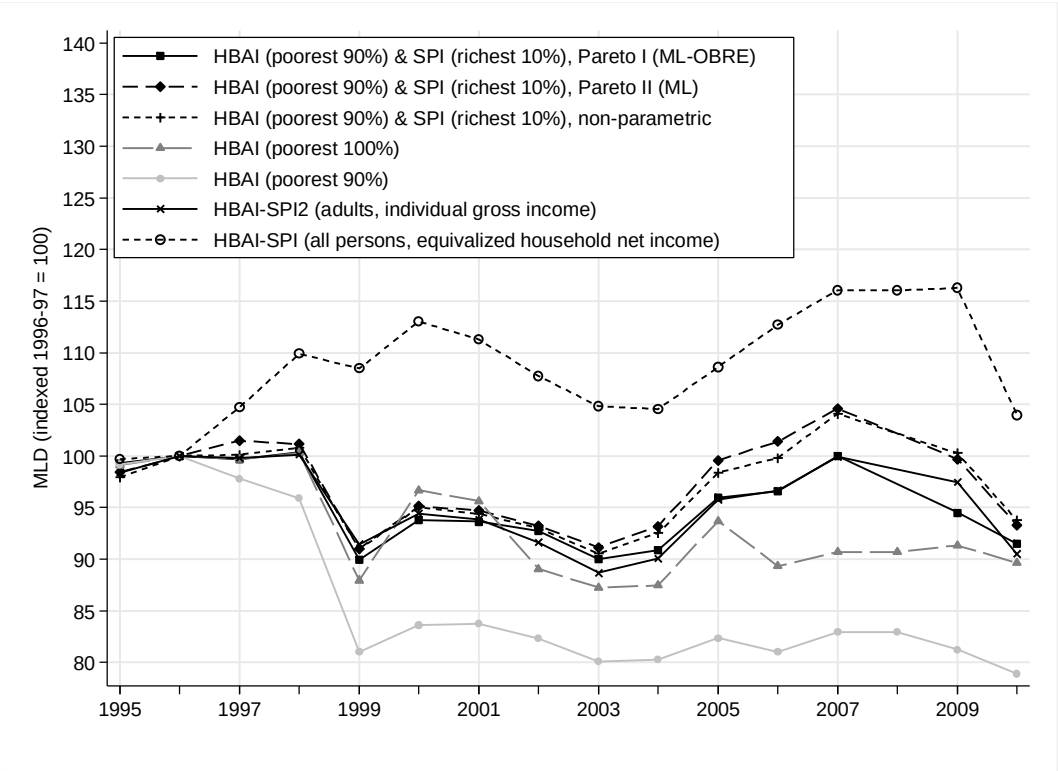
Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

Mean Logarithmic deviation **(indexed 1996/97 = 100), by threshold**



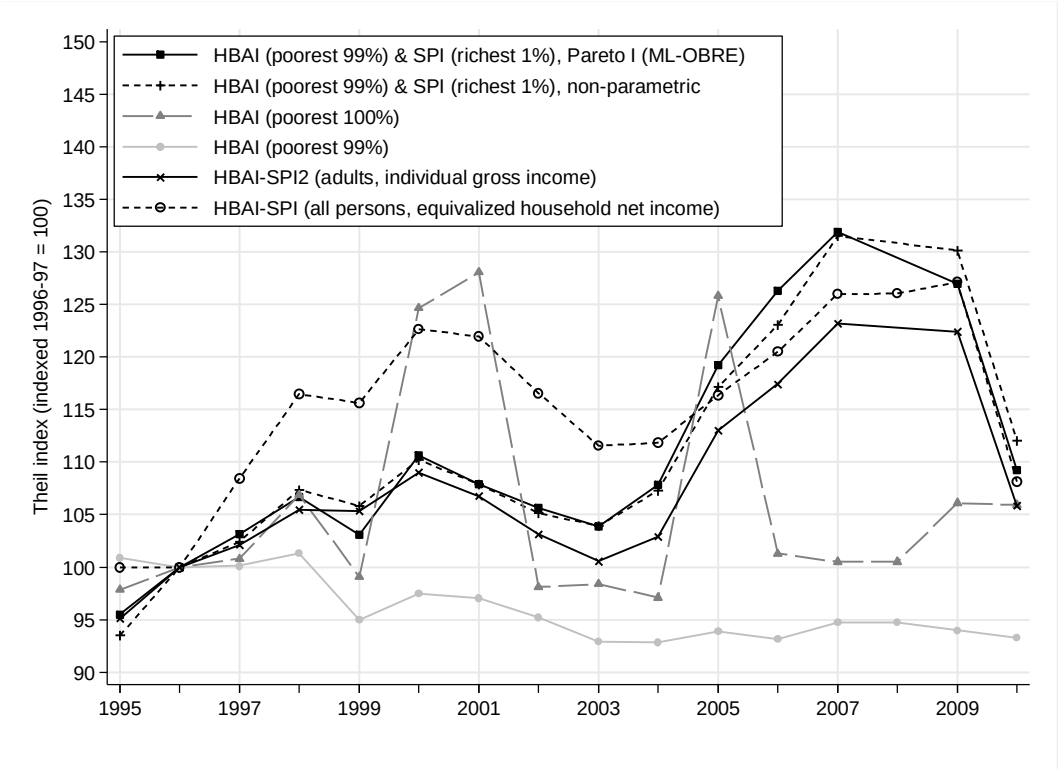


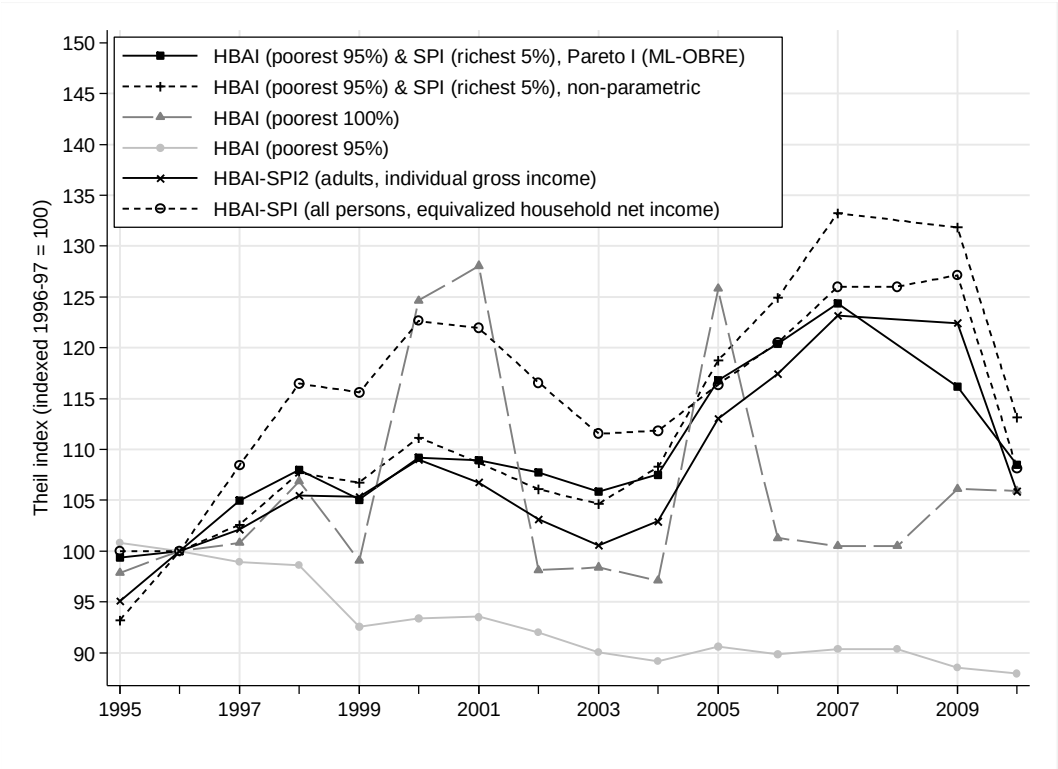
Approach C estimates, by threshold, with graphs of trends at the end (pareto14);



Approach C estimates, by threshold, with graphs of trends at the end (pareto14);

`Theil coefficient`**`(indexed 1996/97 = 100), by threshold`**





Approach C estimates, by threshold, with graphs of trends at the end (pareto14);