Measurement error in models of welfare participation

Monica Hernandez

Department of Economics
University of Sheffield

Stephen Pudney

Institute for Social and Economic Research
University of Essex

ISER Working Paper
2006-29

Acknowledgement:

We are grateful to Jean-Yves Duclos, Stephen Jenkins and participants in seminars at the universities of Keele, Southampton and York for helpful comments. Ruth Hancock and Geraldine Barker assisted with data preparation. The Economic and Social Research Council provided initial funding for the research under grant R000239105 and further support through the Research Centre on Micro-Social Change (grant RES518285001). Material from the Family Resources Survey, made available by the Department for Work and Pensions via the UK Data Archive, has been used with permission. All responsibility for the analysis and interpretation of the data presented here lies with the authors.

Readers wishing to cite this document are asked to use the following form of words:

Hernandez, M. and Pudney, S.E. (June 2006) 'Measurement error in models of welfare participation', ISER Working Paper 2006-29. Colchester: University of Essex.

The on-line version of this working paper can be found at http://www.iser.essex.ac.uk/pubs/workpaps/

The Institute for Social and Economic Research (ISER) specialises in the production and analysis of longitudinal data. ISER incorporates

- MISOC (the ESRC Research Centre on Micro-social Change), an international centre for research into the lifecourse, and
- ULSC (the ESRC UK Longitudinal Studies Centre), a national resource centre to promote longitudinal surveys and longitudinal research.

The support of both the Economic and Social Research Council (ESRC) and the University of Essex is gratefully acknowledged. The work reported in this paper is part of the scientific programme of the Institute for Social and Economic Research.

Institute for Social and Economic Research, University of Essex, Wivenhoe Park,
Colchester. Essex CO4 3SQ UK
Telephone: +44 (0) 1206 872957 Fax: +44 (0) 1206 873151 E-mail: iser@essex.ac.uk
Website: http://www.iser.essex.ac.uk

ABSTRACT

We consider the problem of modelling welfare participation when measurement error may affect simulated welfare entitlement. We identify a flaw in past implementations of the ML approach and develop a more appropriate ML approach. A model of welfare participation is estimated for British pensioners, linking the probability of participation to the value of benefit entitlement, incorporating the nonlinear rule relating entitlement to the household's income and financial assets. The model is used to evaluate the claim costs incurred by participants. When we allow for measurement errors in income and assets, estimated claim costs are substantially reduced.

NON-TECHNICAL SUMMARY

It is believed that a substantial proportion of individuals entitled to receive means-tested welfare benefits do not claim them, thus reducing the effectiveness of government programmes designed to reduce poverty. Existing qualitative research on welfare participation emphasises claim costs arising from the difficulty and hassle of making a claim and other intangible costs such as distaste for welfare participation and social stigma associated with dependence on benefits. In Britain, non-take-up is particularly serious for pensioners. Official estimates report that, although approximately 2 million pensioners were living in low income households in 2000-01, between a third and a quarter of them did not claim the Income Support / Minimum Income Guarantee payments to which they were entitled

One of the most serious difficulties faced by researchers in trying to understand non-take-up behaviour is that we cannot observe directly the level of entitlement that a person would be judged to have if they were to apply for benefit. Researchers attempt to overcome this problem by using household survey data giving details of the income, assets and other circumstances of representative individuals, then to use the known rules of the benefit system to simulate the entitlement that would result from a successful claim. However, this simulation process is imperfect because individuals' responses to survey questions may be subject to reporting error.

The existence of measurement error in income and simulated benefit entitlement causes systematic distortion of the results produced by statistical analysis of take-up behaviour. Moreover, this bias is technically very complicated and difficult to remove. We point out a technical flaw in previous influential analyses of benefit take-up in the presence of measurement error and indicate how it can be corrected.

We apply the method to a model of the take-up of Income Support by older British pensioners during the period 1997-2000 and investigate the effect of measurement error bias by estimating a generalised model with explicit allowance for the distortionary effect of measurement error. We find that a failure to take account of measurement error would cause the researcher to over-estimate the extent to which potential claimants respond to financial incentives and, consequently, the magnitude of the claim costs (arising from "stigma" or "hassle") borne by claimants. These findings should be seen as exploratory rather than definitive but they indicate the very important role that measurement error plays in studies of take-up behaviour.

1 Introduction

Means-testing is an obvious way of focusing welfare spending on those most in need and limiting the burden on public finances. The drawback of means-testing is that people who are entitled to receive welfare benefit may not come forward to claim it and there is evidence from several countries that non-participation in welfare programmes is widespread. This issue has seen much applied research; studies by Ashenfelter (1983), Moffitt (1983), Blundell, Fry and Walker (1988), Duclos (1995), Keane and Moffitt (1998) and Hernandez *et. al.* (2006) use a static discrete choice approach, whilst Blank and Ruggles (1996) and Anderson and Meyer (1997) estimate dynamic models of movement in and out of welfare participation.

One of the main difficulties facing applied researchers is that welfare eligibility is not directly observable but must be inferred from survey evidence on household incomes and other relevant characteristics. This leaves open the possibility that some apparent non-participation is due to measurement error. Because of under-reporting of income or other survey errors, households which appear not to be claiming the welfare benefits to which they appear entitled may in fact not be entitled at all. There has been much recent interest in measurement error problems (see Carroll *et. al.*, 1995, for a survey); however, surprisingly few studies of welfare programme participation have addressed this issue in depth. Notable exceptions include Bollinger and David (1997) who use an external validation sample as direct evidence on measurement error in benefit receipt and Duclos (1995) who estimates a complex structural model of participation in the UK Supplementary Benefit (SB) programme. Duclos' study has been influential because it remains the only model to allow formally for the possibility that assessments of entitlement made by programme administrators and econometric analysts are both subject to error. There is much external evidence to suggest that survey evidence is prone to error (see, for example, Rodgers et. al., 1993; Hancock and Barker, 2002). Consistent with this, Duclos' central finding is that measurement error cannot be ignored if welfare participation behaviour is to be properly understood. Whilst allowing for measurement error, Duclos estimates an econometric model which has important policy implications: notably very high levels of implicit claim costs or social stigma for some groups of potential claimants. Our purpose is to point out a technical flaw in the maximum likelihood measurement error (MLME) approach adopted by Duclos and to apply a corrected MLME estimator to an illustrative benefit takeup model for more recent British welfare participation data. Like Duclos, we find that measurement error has an important impact on the results but, unlike Duclos, we find that correcting for measurement error reduces the estimated size of claim costs incurred by claimants.

2 The Duclos model and ML estimation

In general terms, Duclos' (1995) model is as follows.[1] The Supplementary Benefit (SB) system involves a payment to bring the net income of the benefit unit up to a guaranteed minimum. Write the difference between the guaranteed minimum and

[1] This is a highly simplified description. See Pudney (2003) for a more detailed discussion.

original income, as assessed by programme administrators, as B_g. Entitlement to SB is equal to B_g. when positive, otherwise to zero. Assessment errors are assumed to take the conventional random measurement error form:

$$B_a = B_g + \varepsilon_a \tag{1}$$

where B_a is the analyst's simulation of entitlement, based on household survey information. Conditional on B_g, ε_a is a $N(\mu_a, \sigma_a^2)$ variate.

A vector X contains all the exogenous variables describing the benefit unit and its financial circumstances. Observed receipt and non-receipt of SB are indicated by the events $R = 1$ and $R = 0$. Without going into detail, there is a rational choice assumption (Duclos' equation (16)) which allows the derivation of an explicit form for the conditional participation probability:

$$\Pr(R = 1|X, B_g) = p(X, B_g) \tag{2}$$

However, the government administrator's assessment B_g is not observed, so the probability structure (2) cannot be fitted directly to observed data. Duclos' procedure is to write $B_g = B_a - \varepsilon_a$ and then integrate out the unobservable ε_a:

$$\Pr(R = 1|X, B_a) = \int p(X, B_a - \varepsilon_a)\frac{1}{\sigma_a}\phi\left(\frac{\varepsilon_a - \mu_a}{\sigma_a}\right) d\varepsilon_a \tag{3}$$

However, this assumes that $\varepsilon_a \sim N(\mu_a, \sigma_a^2)$ conditional on X and (implicitly) on B_a. This requires ε_a and B_a to be statistically independent conditional on X, which is the source of the difficulty. In fact, assumption (1) implies that B_a and ε_a are independent only if $corr(B_a, B_g)^2 = var(B_a)/var(B_g)$. This is only sure to be satisfied in the extreme case of perfect measurement. It can never be satisfied if $var(B_a) > var(B_g)$ which would occur under the standard assumption that the measurement error ε_a is independent of the 'true' variable B_g.

What would be required for a correct maximum likelihood treatment of Duclos' model? If we condition on X but not on the observed B_a, the likelihood function is based on the following participation probability:

$$\Pr(R = 1, B_a|X) = \int p(X, B_g) f(B_a|B_g) dF(B_g|X) \tag{4}$$

where f is the normal density of $(B_a|B_g)$ and $F(B_g|X)$ is the distribution function of the unobservable official entitlement B_g, conditional on all relevant observed characteristics. As a (less efficient) alternative to (4), we might use a conditional likelihood, based on the participation probability conditional on X and B_a:

$$\Pr(R = 1|X, B_a) = \frac{\int p(X, B_g) f(B_a|B_g) dF(B_g|X)}{\int f(B_a|B_g) dF(B_g|X)} \tag{5}$$

The important point here is that either approach to likelihood construction requires an explicit assumption about the distribution $F(B_g|X)$, which Duclos' model does not provide. The function maximised by Duclos' estimator is generally not a correct likelihood function. Consequently, his results should be used with caution.

A further critical issue in this area is identification. Secure identification of measurement error models requires further information, such as repeated measures of the underlying 'true' variables (see Carroll *et. al.*, 1995). The Duclos model uses only a simulated value for entitlement to benefit, B_a, and a binary indicator, R, of whether benefit is actually received. No use is made of the survey response on the amount of benefit received by claimants, despite the fact that this would be valuable identifying information. Duclos' model also does not consider the source of measurement error in simulated entitlement and the possibility that there is common error in the survey measures of pre-welfare income and assets and the simulated entitlement calculated from them. If the source of measurement error in B_a is error in the survey measures of income and assets, then the assumption (1) cannot be satisfied because of the complex nonlinear nature of the benefit rules.

In the remainder of the paper, we consider the example of pensioners' participation in the British Income Support programme, making explicit allowance for the transmission of errors in directly observed income and assets to simulated entitlement. We use a repeated measures approach, making use of recorded benefit receipts in addition to recorded income and three indicators of asset levels. Despite the complexity of MLME, estimation proves feasible and informative.

3 The welfare system for pensioners in Britain

In our data period, 1997-2000, the principal means-tested benefit available to low-income pensioners was Income Support (IS). Like the earlier SB system analysed by Duclos, the aim of IS is to bring the income of recipients up to a guaranteed minimum level, depending on age, disability and whether single or living as a couple (see Hernandez *et. al.* 2006 for details). A pensioner household with assessed income below the IS level receives the difference between their income and the guaranteed minimum, provided their asset holdings are below £8,000. The definition of income used for this calculation ignores actual investment income but includes a notional income related to asset holdings between £3,000 and £8,000.

Algebraically, the system works as follows. Let the applicable guaranteed minimum standard be M, which depends only on age and the size of the pensioner unit. $K_1 =$ £3,000 and $K_2 =$ £8,000 are the asset thresholds and $\tau = 1/250$ is the conversion rate for converting assets into notional investment income. C is the household's total financial assets; P is the component of original income which can be assumed to be measured without error; Y is all other assessable income. In the application reported below P is defined to include receipts from the basic state pension which generates standard amounts for most recipients and which are normally accurately observed. Y includes receipts of occupational and private pensions which are extremely variable and relate to past earnings and pension contributions. C and Y are non-negative, since deficits and debts are ignored for the purposes of determining eligibility. Let $b(C, Y)$ represent the rules of the IS system which determines official eligibility, B_g, where the dependence on P and M is left implicit. Then:

$$
b(C,Y) = \begin{cases} M - P - Y & \text{if} \quad Y < M - P, \quad C < K_1 \\ M - P - Y - \tau(C - K_1) & \text{if} \quad K_1 < C < \overline{K}(Y) \\ 0 & \text{otherwise} \end{cases} \tag{6}
$$

where $\overline{K}(s) = \min\left\{K_2, K_1 + \frac{M-P-s}{\tau}\right\}$. This can be partially inverted to give:

$$
\widetilde{Y}(b,C) = \begin{cases} M - P - b & \text{if} \quad C < K_1, \quad 0 < b < M - P \\ M - P - b - \tau(C - K_1) & \text{if} \quad K_1 < C < \overline{K}(b), \quad b > 0 \end{cases} \tag{7}
$$

$$
\widetilde{C}(b,Y) = K_1 + \tau^{-1}(M - P - Y - b) \quad \text{if} \quad 0 < Y < M - P - b + \tau K_1, \quad b > 0 \tag{8}
$$

Further details of the IS system are given in Appendix 1.

4 The Family Resources Survey

4.1 Data selection and preparation

We use the Family Resources Survey (FRS), a continuous cross-sectional survey of British households carried out on behalf of the Department for Work and Pensions (DWP) during April 1997 to March 2000. In principle, the FRS gives all information necessary to assess each respondent's entitlement and establish whether they are receiving IS. Although we allow for measurement error, it is important to eliminate data errors as far as possible. We applied the following process of error detection and correction before using the data (and before making the sample deletions listed below). The first step was to reverse data edits and imputations made by DWP, affecting benefit receipts, private pension income and capital holdings. This was done because we detected some inconsistencies in edits to benefit data and because some of the imputation procedures (such as substitution of sample means for missing values) are inappropriate for our purposes. The next stage involved detecting inconsistencies in benefit data and reconciling them where possible. Potential errors in recorded receipts of social security benefits are generally easier to identify than errors in other sources of income or in capital because specified benefit rates and eligibility rules allow consistency checks to be made. Missing values for benefit receipt were imputed where a correct value could be identified unambiguously. For example, some pensioners in the FRS are able to supply a breakdown of their state pension payments which helps to disentangle different benefits received as one combined payment. In other cases it is clear that a payment of IS is included in their pension payment and there is double counting if a separate amount of IS is also recorded. Where it was not possible to correct an inconsistency or to impute a missing value on any reliable basis, the value was left missing. This was true for all missing values for private pension and capital holdings where there is no reliable way to impute an individual-specific value. Full details of this data cleaning process can be found in Hancock and Barker (2005). In addition, cases in which investment income had been imputed were also identified and substituted by missing values.

There were 15,890 potential pensioner units in the FRS sample, defined as single people at least 5 years over state retirement age (60 for women and 65 for men) or couples where either partner is 5 years above retirement age. We focus on older pensioners for several reasons: they are a group with a high poverty rate; they have very little labour market involvement to complicate the welfare participation issue; and, having been retired for a relatively long time, their adjustment to post-retirement circumstances is likely to be complete. This is in contrast with the dynamic modeling issues faced by Blank and Rutter (1996) and Anderson and Meyer (1997). The subsample used for this analysis contains 6,010 benefit units after deleting households which: contained multiple benefit units (2,211); were still re-paying a mortgage (448); received allowances from an absent spouse (11); had employment or self-employment income (419); were certain to be ineligible under our assumptions because state pension income P was above the guaranteed minimum M (5,313); did not respond to survey questions on a core variable such as recorded IS receipt, pension or non-assessable income (1,189); or had recorded benefit higher than the difference between the guaranteed minimum and their non means-tested benefit income (289). These deletions are less serious than they might at first appear. Most (5,313) are simple exclusions of pensioners known to be non-entitled, for whom participation is not an issue. Simulations suggest that virtually all of the 419 pensioners with earnings would be ineligible for IS either with or without their earnings and thus are of no interest to us; only 88 of the 448 mortgagors appear possibly entitled to IS, and we exclude them because of the large measurement problems associated with the calculation of mortgage interest. The last group of 289 deletions was made to exclude cases where there were strong grounds to believe that recorded IS receipt was in error. The most serious of the deletions is likely to be the set of 1,189 cases lost through item non-response (although note that many of these may be IS-ineligible and thus of no interest anyway). We assume that this form of non-response is ignorable. Given the careful data cleaning and sample selection, we believe it is reasonable to assume that the major remaining source of measurement error lies in the survey measurement of assets and private income and it is this source of measurement error that we focus on. We do not exclude any cases because of missing or possibly erroneous responses on private income and assets. Summary statistics for the sample used for estimation of the model are given in Appendix 1.

4.2 Assets and income in the FRS

Each adult in the pensioner unit is asked whether he or she receives any state benefit. If in receipt of IS, they are asked follow up questions on the last amount received and how the benefit is paid. Throughout this process, pensioners are encouraged by the interviewer to find and consult documentary evidence such as an order book, a letter from the Benefits Agency or a bank statement. This ensures that the information on benefit receipt is as reliable as possible; in the subsequent analysis, we assume that benefit receipt is accurately recorded in the final dataset.

The FRS questions dealing with asset holdings proceed as follows. Firstly, each respondent is asked whether they have had in the last 12 months a current account or a savings account. If the answer is affirmative they are asked whether they have each of a comprehensive range of these accounts and different types of financial assets. Respondents with any accounts are asked to assign their estimate of the current value of their assets to one of several intervals. Secondly, detailed information about the current values of each of the assets held is collected, provided the initial estimate of total assets was between £1,500 and £20,000. These are aggregated over respondents within the unit and over asset types to give a single capital figure. Item non-response means that these observations for total capital are often not available. Measurement error is likely to be significant even when total capital is observed. In addition to assets, respondents are also asked about investment income. For each one of their asset holdings, they give an estimate of the interest or dividend received in the last 12 months. These are aggregated over all holdings and members of the benefit unit to give total investment income.

Each respondent is asked whether they receive a state pension, with encouragement to consult any relevant documentation to provide the most recent amount received. The FRS also includes questions about occupational and private pensions and any other sources of income. However, not all types of income are included in the calculation of IS entitlement; in particular, non-means-tested disability benefits are excluded from assessable income. In the analysis we assume that the state pension and income excluded from assessable income are accurately observed. All other income types (mainly private and occupational pensions) are subject to error.

5 The model

We use a simple binary response model of welfare participation. One of the core explanatory factors is the amount of benefit entitlement, which cannot be observed directly, but can be simulated by applying the known rules of the benefit system to given levels of income and assets. In the absence of measurement error, probit analysis is applied to the subset of cases with positive simulated entitlement. Since the benefit rules are non-smooth, nonlinear functions of income and assets, the impact of measurement error in income and assets is complex. Moreover, since welfare participation is only possible for those with strictly positive entitlement, measurement errors in income and asset holdings can cause zero entitlement to be wrongly simulated as positive and *vice versa*, generating error in sample selection. Our model differs from that of Duclos (1995), who assumes additive errors in simulated entitlement rather than error in the income and asset variables which are inputs to the entitlement calculation. Like Duclos, we use a MLME approach, since alternative IV estimators have little advantage in terms of simplicity in this case.[2] To implement MLME, we specify a model for the unobserved income and asset variables that underpin IS entitlement. These are defined as the values that are, or would be, reported to IS programme administrators when a claim is made.

[2] If there are (say) additive measurement errors in income and assets, they become complicated non-additive errors in simulated entitlement. Let measured assets and income be C^* and Y^* and let $\Pi(b, C, Y)$ be the take-up probability. The main problem with the IV approach is that $E[\Pi(b(C^*, Y^*), C^*, Y^*) - \Pi(b(C, Y), C, Y)]Z \neq 0$ even for an exogenous instrument Z, owing to the nonlinearity of the benefit function $b(.,.)$. This in turn implies that the IV moment condition $E[R - \Pi(b(C^*, Y^*), C^*, Y^*)]Z = 0$ is violated and simple IV is inconsistent.

5.1 Participation behaviour

The participation decision is represented by a binary probit mechanism:

$$\Pr(R = 1|Z, C, Y) = \Phi\left(Z\alpha + \lambda \ln b(C, Y) + \eta[W + Y + rC]\right) \tag{9}$$

where: R is an indicator of IS receipt; $W = P + N$; P is state pension income; N is non-assessable income; r is the expected rate of return on assets; Z is a vector of explanatory variables; and B is the amount of IS actually received. Observed benefit receipt is $B = R\,b(C, Y)$. The logarithmic specification of the entitlement effect in (9) fits better than a linear form and has generally been preferred in applied work (see Blundell *et. al.*, 1988; Hernandez *et. al.*, 2006).

We assume that the observed amount of IS benefit received is measured without error. This is not an unduly strong assumption, since the sample was subject to very detailed data cleaning. The rates of IS applicable to pensioners involve fixed amounts, which make it possible to identify many errors and amend them unambiguously (see Hancock and Barker, 2005, for details). Moreover, pensioners are encouraged by the interviewer to consult appropriate documentary evidence such as an order book or bank statement. Only a small number (under 2%), did not consult documentation when answering the question about benefit receipt.

Our participation analysis is conditional on income and assets, which might be thought endogenous in the sense that people with a distaste for welfare participation may accumulate more pension and other assets in order to reduce IS entitlement, causing a positive bias in the entitlement coefficient. There are two reasons why this simultaneity bias is likely to be small. First, the important decisions governing pension income and asset accumulation (education, occupational choice, marriage, fertility, *etc.*) are made many years earlier than the decision on participation in the IS system which, in any case, did not exist in its present form until later. Moreover, this cohort of people, mainly from the lower occupational classes had limited choice; their access to the personal finance market was limited and the state pension system offered a flat pension that has turned out to be less generous than most would have expected. These factors militate against a strong simultaneity effect. A second restraining factor is the truncation generated by the IS eligibility criterion. An individual with strong distaste for IS participation who responded by accumulating assets would, once beyond the upper asset threshold have zero IS entitlement and thus make no contribution to bias. Moreover, most evidence suggests that benefit claim costs are constant rather than increasing with the benefit amount (Moffitt, 1983; Hernandez *et. al.* 2006); thus a welfare-averse pensioner would have no incentive to reduce their entitlement rather than eliminate it completely. These effects tend to reduce the magnitude of the residual entitlement-participation correlation among IS-eligible pensioners. With available UK data, there is little prospect of investigating this issue further. An analysis of a model endogenising pension and assets (and potentially, housing and education also) would require longitudinal income-asset data that does not currently exist for the UK. In the absence of such data, our attempts to extend the model to allow correlation between (C, Y) and the participation residual were unsuccessful and we conclude that the extended model is essentially unidentified.

5.2 The underlying income and capital processes

In specifying a model for the 'true' levels of financial assets, C, and private income, Y, we need to allow for the possibility of zero levels for both variables. Let X be the vector of explanatory variables. Define four regimes and their associated conditional probabilities: $P^{00}(X) = \Pr(C = 0, Y = 0|X)$, $P^{+0}(X) = \Pr(C > 0, Y = 0|X)$, $P^{0+}(X) = \Pr(C = 0, Y > 0|X)$ and $P^{++}(X) = \Pr(C > 0, Y > 0|X)$. We specify these as a multinomial logit structure:

$$P^{00}(X) = \frac{e^{X\alpha_{00}}}{e^{X\alpha_{00}} + e^{X\alpha_{+0}} + e^{X\alpha_{0+}} + e^{X\alpha_{++}}} \tag{10}$$

with analogous expressions for $P^{0+}...P^{++}$. We normalise $\alpha^{00} = 0$.

Let $g_{c|0}(C|X)$, $g_{y|0}(Y|X)$ and $g_{cy}(C,Y|X)$ be the three regime-specific densities of $(C|C > 0, Y = 0, X)$, $(Y|Y > 0, C = 0, X)$ and $(C, Y|Y > 0, C > 0, X)$ respectively. We specify these as the following lognormal forms:

$$g_{c|0}(C|X) = \frac{1}{C\sigma_1}\phi\left(\frac{\ln C - X\beta_1}{\sigma_1}\right) \tag{11}$$

$$g_{y|0}(Y|X) = \frac{1}{Y\sigma_2}\phi\left(\frac{\ln Y - X\beta_2}{\sigma_2}\right) \tag{12}$$

$$g_{cy}(C,Y|X) = \frac{1}{C\sigma_3 Y\sigma_4}\phi\left(\frac{\ln C - X\beta_3}{\sigma_3}, \frac{\ln Y - X\beta_4}{\sigma_4}; \rho\right) \tag{13}$$

where $\phi(.)$ is the N(0,1) density and $\phi(.,.;\rho)$ is the bivariate standard normal density with correlation ρ.

5.3 The observation process

We assume that IS receipts, B, are accurately observed for recipients. Regular basic state pension income P and non-assessable income N (mainly non-means-tested disability benefits) are also assumed accurate. These assumptions reflect the generally-accepted belief that the FRS is much more accurate in its recording of state benefits than of private income and assets. The transfer payments P and N are mainly paid in standard amounts and our data checking processes are expected to be highly effective in eliminating recording errors.

We assume that the only sources of measurement error are in the private income and asset variables Y and C. Following Duclos, we could make a distinction between the value (Y_a, C_a) reported to FRS interviewers and the value (Y_g, C_g) reported to IS administrators. Since actual IS assessments are based on the latter, we treat these as "quasi-true" values and refer to the difference $(Y_a - Y_g, C_a - C_g)$ as "measurement error". Henceforth, we drop the g-subscript from (Y_g, C_g) and write the composite measurement error $\varepsilon_a - \varepsilon_g$ as ε. We assume the measurement error and quasi-true vales are independent,[3] and specify all measurement errors as multiplicative lognormal variates.

[3] It is possible that both sets of reported values may be in error. Consider income, for example, and write the true value $\overline{Y}$, with $Y_g = \overline{Y} + \varepsilon_g$, $Y_a = \overline{Y} + \varepsilon_a$ and $Y_a - Y_g = \varepsilon_a - \varepsilon_g$. This error is uncorrelated with Y_g if $cov(\varepsilon_a, \overline{Y}) + cov(\varepsilon_a, \varepsilon_g) - cov(\varepsilon_g, \overline{Y}) - var(\varepsilon_g) \approx 0$, requiring either accurate

There is a single survey response, Y^*, on non-pension income and up to three indicators of capital holdings. Write the respondent's initial estimate of total assets as C_1^*; this is observed only as an interval, $A = (A_1, A_2)$, containing C_1^*. Depending on this first-round response, there may be a second-round point estimate (built up from separate constituent balances), reported as C_2^*. Thirdly, each respondent reports annual investment income, V^*. In logarithmic form, we assume:

$$\ln \left(\begin{array}{cccc} C_1^* & C_2^* & V^* & Y^* \end{array} \right) \Big| C, Y \sim N\left(\mu(C,Y), \Omega\right) \tag{14}$$

where: $\mu(C,Y)$ is the vector $\ln(C, C, rC, Y)$; r is the mean rate of return on assets; and $\Omega = \{\omega_{ij}\}$ is a positive definite covariance matrix.[4] In addition to C_1^*, C_2^*, V^* and Y^*, the IS receipt, B, observed for participants is a direct realisation of the function $b(C,Y)$. Thus C_1^*, C_2^*, V^*, Y^* and B constitute a set of five measures of the two unobserved variables C, Y.

The probability structure for observed capital, investment income and non-pension income, conditional on C, Y is:

$$\begin{aligned} &f_1(C_1^* \in A, C_2^*, V^*, Y^* | C, Y) = J^{-1} \phi\left(\ln s_2; \mu_2, \Omega_2\right) \\ &\times \left[\Phi\left(\frac{\ln A_2 - \ln C - m(C,Y)}{\sqrt{V_1}} \right) - \Phi\left(\frac{\ln A_1 - \ln C - m}{\sqrt{V_1}} \right) \right] \end{aligned} \tag{15}$$

where $m(C,Y) = (\ln s_2 - \mu_2)\, \Omega_2^{-1} \omega_{12}$, $\mu_2 = \ln(C, rC, Y)$ and $\Omega = \left(\begin{array}{cc} \omega_{11} & \omega_{12}' \\ \omega_{12} & \Omega_2 \end{array} \right)$.

The conditional variance of the first stage capital estimate is $V_1 = \omega_{11} - \omega_{12}' \Omega_2^{-1} \omega_{12}$, s_2 is the vector containing the observable elements of (C_2^*, V^*, Y^*), the Jacobian term J is the product of the elements of s_2, and $\phi\left(.; \mu_2, \Omega_2\right)$ is the joint pdf of the $N(\mu_2, \Omega_2)$ distribution. Asset questions are often not answered and we assume this non-response is ignorable. If there is no first-stage response to the capital questions, the term in square brackets is deleted from (15) and C_2^* then does not appear in s_2 and J. If any of the capital and income estimates is zero, the relevant components of the distribution (15) become degenerate and (15) is modified accordingly.

5.4 Identification and MLME estimation

There have been no controlled experiments or natural experiments that can identify the impact of entitlement and income on the participation probability. Instead, we use *a priori* structure to give parameter identification. Identification is a serious issue for welfare participation modelling but there has been little explicit consideration of it in the applied literature. First, note that, under the error specification

reporting to administrators ($var(\varepsilon_g) = 0$) or that the variance term is offset by some combination of positive covariance between the two types of reporting error ($cov(\varepsilon_a, \varepsilon_g) > 0$) and strategic under-reporting of high income ($cov(\varepsilon_g, \overline{Y}) < 0$). If there is error in reporting of income and assets to IS administrators, these offsetting covariances are also likely to exist.

[4] This formulation of the measurement process implies: (i) if measured capital is zero, then measured investment income is also zero; and (ii) if measured capital is positive, then measured investment income is positive. The former holds exactly in the sample, whilst the latter is violated for very few cases. Inspection of individual records suggests that the latter are really missing, rather than zero, investment income figures. We have dealt with this by recoding them as missing.

(14), we can determine from the capital and income responses which of the regimes (10)-(13) is operative. The four probabilities $P^{00}(X)...P^{++}(X)$ are thus nonparametrically identifiable. The terms $X\beta_j$ in (11)-(13) are identified from the empirical distributions of measured capital and income variables, since they are conditional means of log income and assets and the measurement errors are additive in log terms. The linearity of $X\beta_j$ is inessential. Write the participation model (9) as $\Pi\left(Z, b, W+Y+rC\right)$ and consider the take-up probability for groups of individuals classified by asset-income regime:

$$\Pr(R=1|Z,P,W,C=0,Y=0)=\Pi\left(Z,b(0,0,P,Z),W\right) \tag{16}$$

$$\begin{aligned}\Pr(R &= 1|Z,P,W,X\beta_1,C>0,Y=0)= \\ &\int \Pi\left(Z,b(C,0,P,Z),W+rC\right)g_{c|0}\left(C|X\beta_1\right)dC\end{aligned} \tag{17}$$

$$\begin{aligned}\Pr(R &= 1|Z,P,W,X\beta_2,C=0,Y>0)= \\ &\int \Pi\left(Z,b(0,Y,P,Z),W+Y\right)g_{y|0}\left(Y|X\beta_2\right)dY\end{aligned} \tag{18}$$

$$\begin{aligned}\Pr(R &= 1|Z,P,W,X\beta_3,X\beta_4,C>0,Y>0)= \\ &\int\int \Pi(Z,b(C,Y,P,Z),W+Y+rC)g_{cy}\left(C,Y|X\beta_3,X\beta_4\right)dCdY\end{aligned} \tag{19}$$

where $b(C,Y,P,Z)$ is the benefit entitlement rule (6). Note that the vector Z enters $b(.)$ through its influence on the guaranteed amount M.

Equation (16) shows that the whole function $\Pi(.)$ is identified nonparametrically from the subpopulation with $C=Y=0$, using a nonparametric regression of R on Z, $b(0,0,P,Z)$ and W, provided there is enough independent variation in Z, P and N within this subpopulation for the arguments of Π to range independently over the whole space $\{Z,b:b\in(0,M(Z)]\}\times R_1^+$. With $\Pi(.)$ known and adequate independent variation in Z, P and N, the mixing distributions $g_{c|0}$, $g_{y|0}$ and $g_{c,y}$ can then be recovered nonparametrically from (17)-(19), subject to a mean or median normalisation. Finally, the measurement error distribution $f(.)$ can be identified from the conditional data distribution by virtue of the variation in X:

$$\begin{aligned}f(C_1^* &\in A,C_2^*,V^*,Y^*|X)= \\ &\int\int f_1(C_1^*\in A,C_2^*,V^*,Y^*|C,Y)g_{cy}\left(C,Y|X\beta_3,X\beta_4\right)dCdY\end{aligned} \tag{20}$$

This structure is over-identified since the reduced-form distributions which give identifying information are defined on higher-dimensional spaces than the structural distributions of $(R|Z,b,W)$, $(C,Y|X)$ and $(C_1^*,C_2^*,V^*,Y^*|C,Y)$. Thus, even with inadequate independent variation to identify $\Pi(.)$ fully from the $C=Y=0$ subpopulation, identification may still be achievable from other parts of the structure. In practice, secure identification requires sufficient exogenous variation to determine the structural parameters with acceptable precision. We will be content, provided the Hessian matrix of the maximised log-likelihood function is clearly invertible and confidence intervals reasonably narrow. The log-likelihood for n observations is:

$$\ln L = \sum_{i=1}^{n} \ln g(B_i, C_{1i}^*, C_{2i}^*, V_i^*, Y_i^* | X_i) \tag{21}$$

The distribution $g(B, C_1^*, C_2^*, V^*, Y^*|X)$ is complicated by the nonlinearity of the benefit rules and requires 2-dimensional integration, evaluated by quadrature. A derivation of $g(B, C_1^*, C_2^*, V^*, Y^*|X)$ is given in Appendix 2.

6 Estimates

Table 1 gives four alternative estimates of the participation component of a simple illustrative model. The first three are standard probit estimates, computed from alternative samples of individuals simulated to be entitled to IS, ignoring measurement error in income and assets. These three samples differ in the measure of entitlement used. Sample 1 (the smallest of the three due to missing values in income and assets) uses simulated entitlements calculated using the survey measures of income and assets. Samples 2 and 3 are intended to reduce the impact of measurement errors and missing values by using survey information on actual benefit receipt rather than simulated entitlement: sample 2 whenever available and sample 3 only when the respondent has consulted documentation to support their response. The final column of Table 1 gives MLME estimates, which are given fully in Tables 2-4.

Table 1: Estimates of participation parameters
(standard errors in parentheses)

Covariate	Probit estimates (1)	(2)	(3)	MLME estimates
Intercept	0.3764	0.5296	0.4446	0.4245
	(0.1883)	(0.1691)	(0.1722)	(0.1850)
Single female	0.1836	0.1775	0.1781	0.1725
	(0.0745)	(0.0723)	(0.0727)	(0.0807)
Head educated past 14	-0.3216	-0.3250	-0.3253	-0.3717
	(0.0755)	(0.0739)	(0.0742)	(0.0833)
Home owner	-0.6267	-0.5942	-0.6050	-0.6888
	(0.0676)	(0.0659)	(0.0663)	(0.0763)
Disabled	-0.0905	-0.0243	-0.0746	0.0281
	(0.1332)	(0.1202)	(0.1221)	(0.1270)
In entitlement (λ)	0.3006	0.2341	0.2603	0.1539
	(0.0345)	(0.0313)	(0.0320)	(0.0342)
Private income (η)	-0.0083	-0.0080	-0.0076	-0.0072
	(0.0017)	(0.0015)	(0.0016)	(0.0016)
n	1977	2033	2024	6010

Note: sample 1 uses simulated entitlement; sample 2 substitutes simulated entitlement by recorded benefit if available; sample 3 substitutes only if IS documentation is consulted when answering.

The choice of explanatory covariates is based on our earlier study (Hernandez *et. al.*, 2006). They represent household structure, education, housing tenure, disability, income and log entitlement. Other factors such as age, location and greater detail on household structure were found to be insignificant. Households consisting of a single female have a significantly raised probability of participation in IS, while education and home ownership are associated with a significantly reduced claim probability. Log entitlement and original income have a highly significant positive and negative impact respectively.

The estimated measurement error variances are high. However, it is difficult to distinguish reliably the variances of the underlying random components of income and assets ($\widehat{\sigma}_1^2 ... \widehat{\sigma}_4^2$) from the measurement error variances ($\widehat{\omega}_{11} ... \widehat{\omega}_{44}$).[5] The parame-

[5] This is perhaps to be expected. For example, in a model with additive measurement errors and entitlement linear in income and assets, we can only identify the variance of the sum of the measurement error and the corresponding income or asset residual, not their separate variances. Despite this, provided there are restrictions which prevent collinearity between entitlement and the other explanatory variables, the entitlement coefficient would be identified.

ter covariance matrix estimated from the Hessian of the log likelihood implies large negative correlations (up to -0.50) between the two sets of variance estimates, but the estimated participation coefficient $\widehat{\lambda}$ is robust since it has low correlation with $\widehat{\omega}_{11}...\widehat{\omega}_{44}$ (at most 0.06 in magnitude). The measurement error structure implied by the MLME estimates (Table 4) is plausible. The ordering of variables by measurement inaccuracy is: investment income, then the first and second-round asset responses, then private income ($\omega_{33} > \omega_{11} > \omega_{22} > \omega_{44}$). The estimated correlation between measurement errors on C_1^* and C_2^* is very high (roughly 0.9), those between (C_1^*, C_2^*) and V^* are lower (around 0.7) and the correlation between the errors on the asset (C_1^*, C_2^*, V^*) and income variable Y^* are much smaller (0.3-0.4).

Comparison of the measurement error model with the probit results shows a large difference in the entitlement coefficient, which is reduced by a third to a half. Other coefficients, including that of income, are remarkably robust. The use of different samples in estimating the simple probit model has little impact on the estimates.

Table 2 MLME estimates: censoring probabilities for C, Y

	MNL coefficients (std. err.)					
Covariate	$P^{0+}(X)$		$P^{+0}(X)$		$P^{++}(X)$	
Intercept	0.155	(0.389)	1.348	(0.262)	2.988	(0.252)
Single male household	-0.894	(0.322)	-0.466	(0.243)	-0.778	(0.230)
Single female household	-1.522	(0.266)	-0.436	(0.205)	-1.547	(0.197)
Retirement years	-0.007	(0.017)	0.019	(0.008)	0.006	(0.008)
Head educated below 15	0.116	(0.252)	-0.303	(0.146)	-0.754	(0.142)
Head educated past 18	-0.608	(0.849)	-0.132	(0.439)	0.280	(0.428)
Owner occupier	0.816	(0.212)	1.326	(0.138)	1.807	(0.136)
London and South East	-0.310	(0.254)	0.182	(0.137)	0.241	(0.135)
Wales and Scotland	0.185	(0.220)	-0.531	(0.142)	-0.357	(0.136)

Table 3 MLME estimates: the C, Y distribution

	Coefficient (std. err.)			
Covariate	Capital, C		Income, Y	
Distribution $g_{cy}(C,Y\|X)$				
Intercept	0.879	(0.126)	3.022	(0.079)
Single male household	-0.058	(0.090)	-0.036	(0.057)
Single female household	-0.573	(0.077)	-0.048	(0.046)
Retirement years	0.015	(0.005)	-0.003	(0.003)
Head educated below 15	-0.584	(0.068)	-0.138	(0.046)
Head educated past 18	0.496	(0.158)	0.097	(0.159)
Owner occupier	1.319	(0.073)	0.151	(0.046)
London and South East	0.131	(0.068)	0.029	(0.036)
Wales and Scotland	-0.078	(0.096)	0.080	(0.049)
Std deviation: σ_3, σ_4	0.619	(0.086)	0.552	(0.007)
Interest rate r		0.023	(0.001)	
Correlation ρ		0.029	(0.116)	
Distribution:	$g_{c\|0}\,(C\|X)$		$g_{y\|0}\,(Y\|X)$	
Intercept	0.707	(0.190)	3.050	(0.657)
Single male household	-0.027	(0.181)	-0.346	(0.258)
Single female household	-0.161	(0.145)	-0.076	(0.313)
Retirement years	0.012	(0.006)	0.011	(0.024)
Head educated below 15	-0.308	(0.106)	-0.162	(0.512)
Head educated past 18	0.623	(0.261)	0.147	(17.00)
Owner occupier	0.989	(0.096)	-0.124	(0.300)
London and South East	-0.164	(0.090)	-0.228	(0.343)
Wales and Scotland	-0.116	(0.123)	-0.182	(0.231)
Std deviation: σ_1, σ_2	1.058	(0.064)	0.794	(0.069)

Table 4 MLME estimates: measurement error covariances

Std deviation (std. err.)			Correlation (std. err.)		
$\sqrt{\omega_{11}}$	1.822	(0.046)	ρ_{12}	0.895	(0.007)
$\sqrt{\omega_{22}}$	1.663	(0.056)	ρ_{13}	0.710	(0.012)
$\sqrt{\omega_{33}}$	2.115	(0.038)	ρ_{14}	0.415	(0.030)
$\sqrt{\omega_{44}}$	1.324	(0.020)	ρ_{23}	0.698	(0.013)
			ρ_{24}	0.404	(0.035)
			ρ_{34}	0.288	(0.029)

7 Implicit claim costs and poverty measures

Our aim here is to produce an estimate of the welfare loss incurred by IS claimants as a result of the claim costs associated with means-testing. We use an argument based on the compensating variation principle and we take account of the self-selected nature of the claimant population (see Hernandez *et. al.*, 2006 for further elaboration of this). The participation probability (9) can be rewritten:

$$\begin{aligned} \Pr(R=1|Z,C,Y) &= \Pr\left(Z\alpha + \lambda \ln b(C,Y) + \eta[W+Y+rC] + \varepsilon > 0\right) \\ &= \Pr\left(b(C,Y) > \Gamma\right) \end{aligned} \tag{22}$$

where ε is a $N(0,1)$ random error and $\Gamma = \exp\{-\left[Z\alpha + \eta[W+Y+rC] + \varepsilon\right]/\lambda\}$. Since participation occurs whenever the amount of entitlement $b(C,Y)$ exceeds Γ, the latter is interpretable as the compensating variation - the cash equivalent of the social stigma, application costs or information search costs that are responsible for non-participation. These claim costs represent a welfare loss borne by welfare participants and it is possible to adjust income for claimants by subtracting Γ from observed income. Write $\Pi(b)$ as the relationship between the participation probability and entitlement, for given values of Z and $W+Y+rC$. Then $\Pi(b) = \Pr(\Gamma \leq b)$ so the pdf of Γ is $\Pi'(\Gamma)$. Thus, for IS participants $E(\Gamma|\Gamma \leq b) = \int_0^b \Gamma\Pi'(\Gamma)d\Gamma/\Pi(b)$ and, integrating by parts:

$$E(\Gamma|\Gamma < b) = b - \int_0^b \Pi(\Gamma)d\Gamma \Big/ \Pi(b) \tag{23}$$

Table 5 shows the implications of measurement error for estimated claim costs. For each of the four sets of parameter estimates, we evaluate (23) for three alternative values for (C,Y,X). In each case, we assume a household consisting of a couple aged over 75, with no education beyond age 14, not disabled and receiving a full basic state pension of $P =$ £112.55. The 'low income, no assets' assumption sets $C = 0$ and $N = Y = 0$. The 'mid-income and assets' assumption sets $C =$ £4,000, $N = 0$ and $Y =$ £10. The 'upper income & assets' assumption sets $C =$ £5,500, $N = 0$ and $Y =$ £25. For these three cases, weekly IS entitlements are respectively £47.85, £33.85 and £12.85. Table 3 suggests that the effect of neglecting measurement error in estimation is to overstate the magnitude of implied claim costs for participants by 20-35%. Over-estimation is greatest for households with relatively high income and low IS entitlement. After taking account of measurement errors in estimation, claim costs for these illustrative households are 11-14% of benefit entitlement for home renters and 16-19% for home owners. Although MLME estimation significantly reduces estimated claim costs, they remain large enough to be an important factor in welfare measurement for this group of welfare participants.

Table 5: Estimated claim costs for claimants (£ per week)

	Probit			
	(1)	(2)	(3)	MLME
	Home renters			
Low income, no assets	6.81	5.65	6.09	5.07
Mid-income & assets	5.67	4.63	5.00	3.99
High income & assets	2.81	2.23	2.43	1.78
	Home owners			
Low income, no assets	10.33	8.52	9.21	7.68
Mid income & assets	8.19	6.70	7.25	5.85
High income & assets	3.75	3.02	3.28	2.50

8 Conclusions

In this paper we have developed a maximum likelihood estimator for a model of welfare participation and applied it to a model of pensioner participation in the British Income Support using Family Resources Survey data for the tax years 1997-2000. The model incorporates the relationship between the amount of entitlement and the household's level of income and financial assets. The ML estimator allows for measurement error in the survey respondent's estimates of both income and capital assets, and corrects a technical error in the influential study by Duclos (1995). Measurement errors are particularly important in participation models since they contaminate the simulated level of welfare entitlement and distort the selection of welfare-entitled households for conventional participation modelling. The estimates presented here should only be seen as illustrative but they do make the point that results are sensitive to the treatment of measurement error. Making allowance for measurement error has a large effect on our estimate of the impact of the extent of entitlement on the propensity to claim. We have used these estimates to calculate implicit claim costs through an application of the compensating variation principle, and found them to be significantly smaller than estimates obtained using conventional probit estimates of participation behaviour. Despite this, claim costs still represent an important factor in welfare measurement for welfare participants.

References

[1] Anderson, P. M. and Meyer, B. D. (1997). Unemployment insurance take-up rates and the after-tax value of benefits, *Quarterly Journal of Economics* , 913-937.

[2] Ashenfelter, O. (1983). Determining participation in income-tested social programs, *Journal of the American Statistical Association* **78**, 517-526.

[3] Blank, R. M. and Ruggles, P. (1996). When do women use Aid to Families with Dependent Children and Food Stamps?, *Journal of Human Resources* 31, 57-89.

[4] Blundell, R. W., Fry, V. and Walker, I. (1988). Modelling the take-up of means-tested benefits: the case of housing benefits in the United Kingdom, *Economic Journal (Conference Papers)* **98**, 58-74.

[5] Bollinger, C. R. and David, M. H. (1997). Modeling discrete choice with response error: Food Stamp participation. *Journal of the American Statistical Association* **92**, 827-835.

[6] Carroll, R. J., Ruppert, D. and Stefanski, L. A. (1995). *Measurement Error in Nonlinear Models*. London: Chapman and Hall.

[7] Duclos, J.-Y. (1995). Modelling the take-up of state support, *Journal of Public Economics* **58**, 391-415.

[8] Duclos, J.-Y. (1997). Estimating and testing a model of welfare participation: the case of supplementary benefits in Britain, *Economica* **64**, 81-100.

[9] Fry, V. and Stark, G. (1993). *The Take-up of Means-Tested Benefits, 1984-90.* London: Institute for Fiscal Studies.

[10] Hancock, R. M. and Barker, G. (2005). The quality of social security benefit data in the British Family Resources Survey: Implications for investigating income support take-up by pensioners. *The Journal of the Royal Statistical Society series A* **168**, 63-82.

[11] Hernandez, M. and Pudney, S. E. (2003). Modelling welfare participation with measurement error in entitlements: the take-up of Income Support by British pensioners, University of Leicester working paper, http://www.le.ac.uk/economics/sep2/mlme.pdf.

[12] Hernandez, M., Pudney, S. E. and Hancock, R. M. (2006). The welfare cost of means-testing: pensioner participation in Income Support. *Journal of Applied Econometrics*, forthcoming.

[13] Keane, M. and Moffitt, R. (1998). A structural model of multiple welfare program participation and labor supply, *International Economic Review* **39**, 553-589.

[14] Moffitt, R. (1983). An economic model of welfare stigma, *American Economic Review* **73**, 1023-1035.

[15] Pudney, S. E. (2003). Measurement error in a structural model of welfare participation: A comment on a study by Duclos, University of Leicester working paper.

[16] Rodgers, W. L., Brown, C. and Duncan, G. J. (1993). Errors in survey reports of earnings, hours worked, and hourly wages, *Journal of the American Statistical Association* **88**, 1208-1218.

Appendix 1 Additional tables

Table A1 Weekly rates of Income Support applicable to pensioners from 1997 to 2000

	£ per week		
	1997/8	1998/9	1999/0
Single pensioner under 75	68.80	70.45	75.00
Single pensioner 75-79	71.00	72.65	77.30
Single pensioner 80+	75.70	77.55	82.25
Single pensioner with SDP[1]	112.85	116.05	122.00
Couple, both under 75	106.80	109.35	116.60
Couple, one or both 75-79	109.90	112.55	119.85
Couple, one or both 80+	115.15	117.90	125.30
Couple, one or both 75-79, one with CP[1]	123.25	126.20	133.80
Couple, one or both 80+, with CP[1]	128.50	131.55	139.25
Couple, both with SDP[1]	189.45	194.90	204.80
Sample mean of guarantee level M in FRS	86.21	88.78	94.65

[1]CP (Carer Premium) and SDP (Severe Disability Premium) are non-means-tested benefits available to the disabled. They are disregarded in the assessment of IS eligibility.

Table A2 Sample means of main variables

Variable	Estimate	Std.err.
Single male household	0.1602	0.0047
Single female household	0.6180	0.0063
Retirement years	16.9812	0.0828
Head educated below 15	0.6536	0.0061
Head educated 16-18	0.2942	0.0059
Head educated past 18	0.0522	0.0029
Home owner	0.5413	0.0064
London and South East	0.2759	0.0058
Wales and Scotland	0.1506	0.0046
Disabled (receiving disability benefits	0.1770	0.0049
IS amount received (recipients only)	4.4280	0.1691
Proportion receiving IS	0.2068	0.0052
Pension income P	73.7877	0.2912
Other assessable income Y	46.6480	1.1636
Non-assessable income N	7.4865	0.2122

Appendix 2 Sample probabilities

There are four regions in (C, Y) space requiring distinct treatment: $R_{00} = \{C = Y = 0\}$; $R_{0+} = \{C = 0, Y > 0\}$; $R_{+++0} = \{C > 0, Y = 0\}$; $R_{++} = \{C > 0, Y > 0\}$. The probabilities of these unobservable events are P^{00}, P^{0+}, P^{+0} and P^{++}. In terms of benefit receipt, there are three regimes: $B = 0, 0 < B < M - P$ and $B = M - P$. Thus B has a 3-part distribution with a central continuous density and two extreme probability masses. Define the following binary variables:

$$\begin{aligned}
\xi_0 &= \mathbb{I}(B = 0) \\
\xi_1 &= \mathbb{I}(0 < B < M - P) \\
\xi_2 &= \mathbb{I}(B = M - P)
\end{aligned}$$

where $\mathbb{I}(.)$ is the indicator function.

Consider the distribution of B, C_1^*, C_2^*, V^* and Y^* for a generic case:

$$g(B, C_1^*, C_2^*, V^*, Y^*|X) = g^{00} + g^{0+} + g^{+0} + g^{++} \tag{24}$$

where g^{00}, g^{0+}, g^{+0} and g^{++} correspond to the four regimes $R_{00}...R_{++}$.

The term g^{00} corresponds to zero income and capital:

$$\begin{aligned}
g^{00} &= (1 - \xi_1)P^{00}\, f_1(s|C = 0, Y = 0) \\
&\quad \times \Phi\left(-[Z\alpha + \lambda \ln B + \eta[W]]\right)^{\xi_0}\ \Phi\left(Z\alpha + \lambda \ln B + \eta[W]\right)^{\xi_2}
\end{aligned} \tag{25}$$

where $s = \{C_1^*, C_2^*, V^*, Y^*\}$ is the vector of observed measures. For cases with positive income and zero capital:

$$
\begin{aligned}
g^{0+} &= (1-\xi_2)P^{0+} \\
&\times \left[f_1\left(s|C=0, Y=\widetilde{Y}(B,0)\right)\Psi_1\left(B,\widetilde{Y}(B,0)\right)g_{y|0}\left(\widetilde{Y}(B,0)\right)\right]^{\xi_1} \\
&\times \left[\int_0^\infty \Psi_1(0,Y)\, f_1(s|C=0,Y)\, g_{y|0}(Y)dY\right]^{\xi_0}
\end{aligned}
$$

where:

$$
\Psi_1(B,Y) = \begin{cases} \Phi\left(Z\alpha + \lambda \ln B + \eta[W + \widetilde{Y}(B,0)\right) & \text{if } B > 0 \\ \Phi\left(-[Z\alpha + \lambda \ln b(0,Y) + \eta(W+Y)]\right) & \text{if } Y < M - P \\ & \quad \text{and } B = 0 \\ 1 & \text{otherwise} \end{cases} \tag{26}
$$

and the functions $b(0,Y)$ and $\widetilde{Y}(B,0)$ are defined by (6) and (7) respectively. Note that, for computational purposes, the integral in (26) should be broken into two sub-integrals over $(0, M-P)$ and $(M-P, \infty)$.

For cases with zero income and positive capital:

$$
\begin{aligned}
g^{+0} = P^{+0} &\left\{\left[\int_0^\infty \Psi_2(0,C)\, f_1(s|C, Y=0) g_{c|0}(C)dC\right]^{\xi_0}\right. \\
&\times \left[\Psi_2\left(B,\widetilde{C}(B,0)\right)\, f_1\left(s|C=\widetilde{C}(B,0), Y=0\right)\, \tau^{-1} g_{c|0}\left(\widetilde{C}(B,0)\right)\right]^{\xi_1} \\
&\times \left.\left[\int_0^{K_1} \Psi_2(M-P, C)\, f_1(s|C, Y=0)\, g_{c|0}(C)dC\right]^{\xi_2}\right\}
\end{aligned}
$$

The term τ^{-1} is the Jacobian of the transformation $C \to B$ and the function $\Psi_2(C)$ is:

$$\Psi_2(B,C) = \begin{cases} \Phi\left(-[Z\alpha + \lambda \ln b(C,0) + \eta(W + rC)]\right) & \text{if } B = 0 \text{ and } C \leq \overline{K}(0) \\ \Phi\left(Z\alpha + \lambda \ln B + \eta[W + rC]\right) & \text{if } B > 0 \\ 1 & \text{if } B = 0 \text{ and } C > \overline{K}(0) \end{cases} \tag{27}$$

The fourth likelihood component is relevant to individuals with positive levels of capital and income:

$$g^{++} = (1 - \xi_2)\, P^{++} \left\{ \left[\int_0^\infty \int_0^\infty \Psi_3\,(C,Y)\;\, f_1(s|C,Y)\, g_{cy}(C,Y)\, dC\, dY \right]^{\xi_0} \right.$$

$$\left. \times \left[\int_0^{\overline{K}(0)} \Psi_4\,(C)\;\, f_1\left(s|C, Y = \widetilde{Y}(B,C)\right)\, g_{cy}\left(\widetilde{Y}(B,C), C\right)\, dC \right]^{\xi_1} \right\} \tag{28}$$

where:

$$\Psi_3(C,Y) = \begin{cases} \Phi\left(-[Z\alpha + \lambda \ln b(C,Y) + \eta(W + Y + rC)]\right) & \\ \qquad \text{if } \{C < K_1 \text{ and } Y < M - P\} & \\ \qquad \text{or } \{K_1 < C < \overline{K}(0) \text{ and } Y < \widetilde{Y}(0,C)\} & \\ 1 \qquad \text{otherwise} & \end{cases} \tag{29}$$

$$\Psi_4(C) = \Phi\left(Z\alpha + \lambda \ln B + \eta[W + \widetilde{Y}(B,C) + rC]\right) \tag{30}$$

The double integral in (28) should be computed as the sum of sub-integrals over five regions: $\{(0, K_1) \times (0, M - P)\}$; $\{(0, K_1) \times (M - P, \infty)\}$; $\{K_1 < C < \overline{K}(0), 0 < Y < \widetilde{Y}(0, C)\}$; $\{(\overline{K}(0), \infty) \times (0, \infty)\}$ and $\{K_1 < C < \overline{K}(0), \widetilde{Y}(0, C) < Y < \infty\}$. Note that the last of these can be expressed as the product of two univariate integrals. The last term in (28) should be broken up into sub-integrals over $(0, K_1)$ and $(K_1, \overline{K}(0))$.