# MODELLING LOW PAY TRANSITION PROBABILITIES, ACCOUNTING FOR PANEL ATTRITION, NON-RESPONSE, AND INITIAL CONDITIONS 

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#### Abstract

We model annual low pay transition probabilities taking account of three potentially endogenous selections: two sample drop-out mechanisms (panel attrition, non-employment) and 'initial conditions' (base-year low pay status). This model, and variants that ignore one or more of these selection mechanisms, are fitted to data for men from the British Household Panel Survey. Tests of the ignorability of the endogenous selection mechanisms suggest that 'economic' selection mechanisms such as initial conditions and retention of employment are more important than the 'survey' selection mechanism (attrition). However, consistent with related US research, relatively simple models provide estimates of covariate effects that differ little from the estimates from the complicated models.


## 1. Introduction

This paper is about how to get good estimates of the determinants of transitions into and out of low pay. Among those who are currently low paid, what are the factors associated with remaining low paid the next year, or becoming high paid? Among those who are currently not low paid, what are the factors associated with becoming low paid? Answers to these questions are an important supplement to the work of bodies such as the Low Pay Commission that focus on the prevalence of low pay in a given year (and its trends) using cross-section survey data. With panel data on transitions, we can see whether it is the same people who are stuck in low pay, or whether there is fluidity in the membership of the low paid group. Persistent low pay exacerbates problems associated with one-off low pay episodes, for example difficulties in making contributions to private pension schemes, getting a mortgage, and saving more generally (Atkinson, 1973). Experience of low pay is associated with higher chances of becoming unemployed in the future (the 'low pay - no pay cycle') and, although the overlap between low pay and poverty is relatively low in any given year, the association between persistent low pay and poverty is much higher (Stewart, 1999).

Multivariate regression models of low pay transition probabilities are an obvious source of evidence for discussing these issues, but their usefulness is contingent on a number of other issues being addressed appropriately - notably non-random panel drop-out (attrition), non-response on the key economic variables, and non-random selection into low pay in the base year (the initial conditions problem). We estimate models that account for these processes, plus variants on these models that illustrate the consequences of neglecting the endogeneities, using data for men from the British Household Panel Survey, waves 1-10.

We offer two contributions. First, there are the substantive results about low pay transition probabilities in Britain, derived from a model that extends the definitive study to date (Stewart and Swaffield 1999). Second, from a methodological perspective, we demonstrate the feasibility (and potential problems) of accounting for multiple endogenous selection mechanisms with panel data, and examine the differences between the general model and simpler models assuming ignorability of one or more selection mechanisms. As far as we are aware, we are the first to consider attrition, non-response and initial conditions simultaneously and jointly with a model of labour market transition probabilities. The results suggest that panel attrition is ignorable. Put another way, when estimating annual low pay transitions, 'economic' selection mechanisms such as whether some has low pay in the base
year (initial conditions), and whether someone is employed, appear to be more important than the 'survey' selection mechanism. Nonetheless, relatively simple models provide estimates of covariate effects that differ little from the estimates from the complicated models.

In Section 2, we briefly review previous research about labour market behaviour using panel data, and its treatment of attrition, non-response, and initial conditions. We distinguish between 'economic non-response' (when data on pay is missing because a respondent does not have a job) and 'survey item non-response' (when someone has a job but data on pay is missing because of a refusal or 'don't know' response). Our statistical models are set out in Section 3, and the British Household Panel Survey data are introduced in Section 4. The estimates of the general model (accounting for three types of endogenous selection) are discussed in Section 5, and models assuming ignorability of one or more selection mechanisms are discussed in Section 6 . Section 7 contains a summary and conclusions.

## 2. Models of labour market behaviour accounting for attrition and non-response

There are four types of endogenous selection mechanisms that arise when modelling labour market behaviour using panel data:

1. Panel drop-out (attrition): the individuals who are retained in the panel from one interview to the next may not be a random sample of the population.
2. 'Economic' item non-response: for example, information about pay (or number of hours worked) is only available for panel members who are employed, and they may not be a random sample.
3. 'Survey' item non-response: for example, among panel members who are employed, those who refuse to provide or don't know their earnings (or work hours) may not be a random sample.
4. Initial conditions: for example, the set of individuals who are low paid in the base year (those at risk of exiting low pay), or the set who are not low paid in the base year (those at risk of entering low pay), may not be a random sample of the population.

The second and third mechanisms arise in cross-section surveys as well. Although the example given under the second heading is framed with reference to models of labour market behaviour, it has analogues in other applications. (For example, one might wish to model expenditures on some disease-specific medicine, expenditure is only observed for those with
the disease, and unobserved factors that influence disease probability also affect expenditure propensities.) The fourth issue arises when one estimates dynamic models, such as models of transition probabilities (like ours).

Each of the mechanisms cited selects of a subset of respondents from the base population of interest, a process that is potentially endogenous because the unobserved individual factors affecting each selection mechanism may be correlated with the unobserved individual factors affecting the economic process of interest (and also correlated across selection mechanisms). A selection mechanism is ignorable if the relevant unobservable correlation(s) are zero. In this case, a model for the economic process estimated using the selected sample will lead to consistent parameter estimates. To get consistent estimates in the case when selection is non-ignorable, economists have typically specified an equation to characterize the selection mechanism jointly with the economic process of interest, allowing for cross-equation correlations of unobservables, and based tests for ignorability on the estimates of these correlations. The most famous example of this general approach is the model of 'economic' item non-response by Nobel-prizewinner James Heckman (1974, 1976, 1979), originally applied to estimation of the determinants of women's work hours and accounting for the fact that not all women were employed.

Models that account for more than one endogenous selection issue are relatively rare in applications to labour market behaviour, as far as we are aware. Table 1 provides an overview of previous research accounting for endogenous selection issues, citing a small number of illustrative examples, each of which used panel data. Initial conditions issues were not relevant to the first three models because the outcomes referred to a point in time rather than changes over time. None of the models considered survey item non-response as an endogenous selection issue: implicitly it has been treated as ignorable, or combined with panel attrition in a manner that we discuss further below.

Hausman and Wise (1979) estimated models of earnings at each of two points in time for a panel of participants in the Gary Income Maintenance Experiment, accounting for panel attrition. Since their focus was on the potential effects on estimates of individuals leaving the experiment, they framed their analysis in terms of panel attrition - the possibility that earnings data were missing due to unemployment spells was not mentioned. The headline finding was that sample attrition was ignorable for analysis of the impact of the experiment on earnings.

Keane et al. (1988) analyzed the correlation between real wages and the business cycle - whether wages varied pro- or counter-cyclically - using 12 waves of data from the National

Longitudinal Study of Young Men. They were concerned that, if the probabilities of being fired during an economic downturn were larger for individuals with low (unobserved) ability than for otherwise comparable workers, then the average productivity among a sample employees would be larger in recessions than at other points of the business cycle, thereby imparting a counter-cyclical bias in average wages. Thus the paper was concerned with the potential impact of economic item non-response (and the effects of panel attrition were not examined). The authors found that unobserved factors that influenced employment propensities were correlated with those influencing earnings: models that ignored this dimension of endogenous selection over-estimated the pro-cyclicality of real wages.

Zabel's (1998) paper is an example of research considering both panel attrition and economic item non-response when modelling labour market outcomes (he considered both wages and work hours, separately). The attrition and employment equations were modelled jointly, from which 'selection correction' terms were derived and employed as additional regressors in the equation for the outcome of interest (à la Heckman, 1979). The model was fitted to data for men from two panels, the Panel Study of Income Dynamics and the Survey of Income and Program Participation. Zabel found evidence of non-random selection into the labour force in the equation for work hours, and the equation for wages, but attrition bias was found only for the wage equation (and not hours). Nonetheless, 'account for attrition bias has little impact on the parameter estimates' (Zabel, 1998, p. 502), echoing Hausman and Wise (1979).

Becketti et al. (1988, p. 490) also found 'no compelling evidence that attrition (or entry) has any effect on estimates of the parameters of the earnings equations ... studied'. Ziliak and Kniesner (1998) reported a similar result when estimating 'lifecycle-consistent' labour supply models. Lillard and Panis (1998, p. 437) stated that '[a]lthough we find evidence of significant selectivity in attrition behavior, the biases that are introduced by ignoring selective attrition are very mild'. (All three studies were based on the PSID.) In addition, using Dutch panel data, Van den Berg and Lindeboom estimated multi-state labour market transition models using hazard regressions accounting for endogenous panel attrition. They found that attrition was non-ignorable, but the estimates of covariate effects in the labor market transition rates do not change a lot when allowing for these relations between labor market durations and attrition' (p. 477).

The next two studies differ from those discussed so far because the dependent variable referred to a change in a labour market outcome, and hence initial conditions were also
relevant. Stewart and Swaffield's (1999) paper is the definitive analysis of low pay dynamics in Britain. Using data for men and women from waves 1-6 of the British Household Panel Survey, they showed that 'exogenous selection into the initial low pay state ... is strongly rejected and that ignoring the endogenous selection ... distorts the estimated coefficients' (1999, p. 40). The marginal effects on the probability of being low paid conditional on initial low pay status were much smaller when initial conditions were accounted for. The importance of accounting for initial conditions was also emphasised by Arulampalam et al. (2000) in their panel model of unemployment dynamics. See Heckman (1981) for a general discussion of the issue.

Cappellari and Jenkins (2004) used a model of transition probabilities similar to Stewart and Swaffield's, except that they added an additional equation to account for attrition as well as initial conditions (and the outcome was conditional low income status rather than conditional low pay status). Attrition was defined to occur either when data were missing because of panel drop-out, or because of incomplete response on income by the respondent or another adult in his or her household. (Poverty status was assessed using a measure of household income, and this could only be calculated for households in which all adults were complete respondents.) There was evidence of both attrition and initial conditions bias. From comparisons of the three-equation model with models assuming that either or both of the endogenous selection mechanisms were ignorable, Cappellari and Jenkins concluded that the estimates suggested that 'neglecting to control for endogeneity of initial poverty status is more problematic than neglecting to control for endogeneity of retention' (2004, p. 15).

The current paper builds on Stewart and Swaffield (1999) and Cappellari and Jenkins (2004). It models low pay transitions, but adds a fourth equation to account for 'economic' survey non-response, and also explores the consequences of alternative assumptions about 'survey' item non-response. Models corresponding to those used the earlier papers, i.e. assuming ignorability in one or more of the endogenous selection mechanisms, are also estimated to investigate the effects of ignoring non-ignorability. The general model is described in the following section.

## 3. A model of low pay transitions accounting for three endogenous selection processes

Consider a sample of men in some base year (year ' $t-1$ '), all of whom are in employment and all of whom have non-missing missing data on pay. Employment is defined as working for an employer. Thus the base year sample excludes men who are self-employed, or unemployed, or economically inactive, or who are employed but for whom pay data is missing. These selection criteria correspond to those used by all previous research on labour market transitions that we are aware of. Thus potential endogenous selection into the labour force, self-employment, or employment, in the base year is ignored. (One exception is Cappellari and Jenkins (2003) who allowed for endogenous selection into employment in the base year for a sample of men who were employed or unemployed.)

For each man in the base year sample, we assume that there is a latent low pay propensity, $L_{t-1}^{*}$, and observed low pay status, $L_{t-1}$, depends on whether this propensity is greater or less than some unobserved threshold (set equal to zero without loss of information). That is, initial conditions are described by:

$$
\begin{gather*}
L_{t-1}^{*}=\beta^{\prime} \boldsymbol{x}_{t-1}+u_{t-1}, u_{t-1} \sim N(0,1)  \tag{1}\\
L_{t-1}=I\left(L_{t-1}^{*}>0\right) \tag{2}
\end{gather*}
$$

where $\boldsymbol{x}_{t-1}$ is a vector of personal characteristics, $\beta$ is a vector of parameters, and $u_{t-1}$ summarises unobserved differences (assumed uncorrelated with observed characteristics). $I\left(L_{t-1}^{*}>0\right)$ is a binary indicator function equal to one if the latent low pay propensity is positive and equal to zero otherwise. Stewart and Swaffield (1999) showed that this specification is equivalent to assuming that there exists some monotonic transformation of observed earnings such that the normality assumption holds.

Now consider outcomes in the following year (the current year, ' $t$ ') for this sample, taking account of potential non-ignorable attrition and economic item non-response. Suppose that there is a latent panel retention propensity, $R_{t}^{*}$, which is a linear function of observed and unobserved characteristics (analogous to that described above), and observed retention status, $R_{t}$, depends on whether this propensity is positive or not:

$$
\begin{gather*}
R_{t}^{*}=\psi^{\prime} \boldsymbol{w}_{t-1}+\varepsilon_{t}, \varepsilon_{t} \sim N(0,1)  \tag{3}\\
R_{t}=I\left(R_{t}^{*}>0\right) \tag{4}
\end{gather*}
$$

where $I($.$) is the binary indicator function, as above. In equation (3), and analogous equations$ below, year $t$ outcomes are parameterized in terms of base year values of explanatory variables so as to avoid simultaneity between changes in outcomes and changes in attributes.

Among the men retained in the panel, a second condition that must be satisfied in order for earnings mobility to be observed, namely being in employment in year $t$. We suppose that there is an employment propensity, $E_{t}^{*}$, that is a linear function of observed and unobserved characteristics, and observed employment status, $E_{t}$, depends on whether this propensity is positive or not:

$$
\begin{gather*}
E_{t}^{*}=\lambda^{\prime} \boldsymbol{h}_{t-1}+\omega, \omega \sim N(0,1)  \tag{5}\\
E_{t}=I\left(E_{t}^{*}>0\right) \text { if } R_{t}=1 ; \text { unobserved otherwise. } \tag{6}
\end{gather*}
$$

For men that drop out of the survey ( $R_{t}=0$ ), equation (5) is incidentally truncated.
Finally, there is the mechanism describing low pay status in the current year $\left(L_{t}^{*}\right)$. In order to characterize low pay transitions, we use a linear index specification again but condition the current year outcome on base year low pay status, thereby defining an endogenous switching regression:

$$
\begin{gather*}
L_{t}^{*}=\left[L_{t-1} \gamma_{1}^{\prime}+\left(1-L_{t-1}\right) \gamma_{2}^{\prime}\right] z_{t-1}+v_{t}, v_{t} \sim N(0,1)  \tag{7}\\
L_{t}=I\left(L_{t}^{*}>0\right) \text { if } R_{t}=1 \text { and } E_{t}=1 ; \text { unobserved otherwise. } \tag{8}
\end{gather*}
$$

Equation (7) is incidentally truncated if either $E_{t}=0$ or $R_{t}=0$. That is, equations (3) and (5) describe two (sequential) selection mechanisms governing whether respondents are in the balanced two-year panel of earners who contribute to the estimation of the low pay transition process.

The combinations of current-year outcomes $\left(R_{t}, E_{t}, L_{t}\right)$ that are possible are shown in Table 2. We distinguish three groups of men $(A, B$, and $C$ ) with different types of likelihood contribution applicable to each group. The log-likelihood contribution of each man, $\mathcal{L}$, has the form:

$$
\begin{equation*}
\log \mathcal{L}=\left(1-R_{t}\right) \log \mathcal{L}_{A}+R_{t}\left(1-E_{t}\right) \log \mathcal{L}_{B}+R_{t} E_{t} \log \mathcal{L}_{C} \tag{9}
\end{equation*}
$$

where $\mathcal{L}_{A}, \mathcal{L}_{B}$, and $\mathcal{L}_{C}$ are the contributions relevant to men in groups $A, B$, and $C$. Assuming that the unobservables $\left(u_{t-1}, v_{t}, \varepsilon_{t}, \omega\right)$ have a four-variate standard normal distribution with correlation matrix $\Sigma$, then the sample log-likelihood contribution of each man can be written:

$$
\begin{gather*}
\log \mathcal{L}=\left(1-R_{t}\right) \log \Phi_{2}\left(\boldsymbol{\Xi}_{-} L_{-} E t ; \Omega_{-L t-E t}\right)+R_{t}\left(1-E_{t}\right) \log \Phi_{3}\left(\Xi_{-L t} ; \Omega_{-L t}\right) \\
+R_{t} E_{t} \log \Phi_{4}\left(L_{t-1} \mathbf{\Xi}_{1}+\left(1-L_{t-1}\right) \Xi_{2} ; \Omega\right) \tag{10}
\end{gather*}
$$

where $\boldsymbol{\Phi}_{j}$ denotes the $j$-variate normal c.d.f., $\boldsymbol{\Xi}_{k}$ for $k=1,2$, is a vector of index functions, and matrices $\Omega_{,} \Omega_{-L_{-} E t}$, and $\Omega_{-L t}$, are derived from $\Sigma$. (The Appendix provides full details.) The _Lt subscript denotes vectors and matrices excluding elements referring to the low pay transition equation, and the _Lt_Et subscript denotes vectors and matrices excluding elements referring to the low pay transition equation and to the employment equation.
<Table 2 near here>
The discussion so far has ignored the possibility of item non-response on current year pay, but this is an empirical reality of course. (There are no men in our data set with $R_{t}=1$ and missing $E_{t}$, and so we ignore this case here.) We took two approaches to this. The first, our leading case, was simply to exclude respondents in group $C$ with non-response on pay from the analysis altogether (Model 1). This strategy, treating survey item non-response as ignorable, is consistent with practice in previous research of this kind, and treats this type of endogenous selection in the same way in both in the current and base years. Our second strategy was to define sample retention to mean no sample drop-out and no non-response on pay in the current year (Model 2), and to check how results differed from the first case. Alternatively, one might define those with survey item non-response on pay having $R_{t}=1$ but $E_{t}=0$, and re-interpret the equation for $E_{t}^{*}$ in terms of combined non-response (economic and survey). Another approach, beyond the scope of this paper, would have been to add equations for survey item non-response to the model.

### 3.1. Identification and ignorability

A sufficient condition for identification of Model 1 (or Model 2), given unconstrained cross-equation correlations, is a set of exclusion restrictions referring to the regressors assumed to be relevant for the various endogenous selection equations but not relevant for the equations that are conditioned upon those processes. (The variables used are described in the next section.) Since the model is also identified by the non-linearities in functional form, the over-identification restrictions concerning the exclusion of the instruments (and hence their validity) can also be tested.

Tests of the ignorability of each endogenous selection mechanism were based on whether the cross-equation correlations associated with each mechanism were jointly equal to zero. Initial conditions are ignorable if $\operatorname{corr}\left(u_{t-1}, \varepsilon_{t}\right)=\operatorname{corr}\left(u_{t-1}, \omega\right)=\operatorname{corr}\left(u_{t-1}, v_{t}\right)=0$,
retention is ignorable if $\operatorname{corr}\left(\varepsilon_{t}, u_{t-1}\right)=\operatorname{corr}\left(\varepsilon_{t}, \omega\right)=\operatorname{corr}\left(\varepsilon_{t}, v_{t}\right)=0$, and economic item nonresponse is ignorable if $\operatorname{corr}\left(\omega_{,}, u_{t-1}\right)=\operatorname{corr}\left(\omega_{,}, \varepsilon_{t}\right)=\operatorname{corr}\left(\omega_{t}, v_{t}\right)=0$.

To assess the consequences of neglecting to account for panel attrition and economic item non-response, we estimated three additional models. Model 3 was analogous to that considered by Cappellari and Jenkins (2004). That is, no distinction was made between panel attrition and economic item non-response, in effect combining groups $A$ and $B$ in Table 2 into one group. (There was no $E_{t}$ equation, and $R_{t}=0$ if there was panel attrition or the respondent was out of the labour force, unemployed or self-employed.) In Model 4, only the base year low pay equation (initial conditions) was estimated jointly with the low pay transition equation. This model corresponds closely to Stewart and Swaffield's (1999). Their model was estimated using information about current year pay status only for respondents who were low paid in the base year (a bivariate probit model with partial observability). By contrast, our model used current pay status information for both high- and low-paid respondents in the base year (a bivariate probit model with endogenous switching). Finally, in Model 5, the low pay transition equation was estimated ignoring all three endogenous selection issues. (The likelihood functions for Models 3-5 are provided in the Appendix.)

### 3.2. Estimation issues

Our sample data consisted of repeated observations on the same individual across successive pairs of years because we pooled transitions from our panel (see later). Hence the standard i.i.d. assumption underlying the maximum likelihood principle is violated. If the individual effects across the pooled transitions could be integrated out, then, this approach would provide consistent and efficient estimates of parameters and their asymptotic standard errors. This was computationally infeasible, and so instead we used an approach providing consistent parameter estimates and adjusted their standard errors using a robust variance estimator.

The sum across all transition-years for all men in the data set of the expression given in equation (10) defines a sample 'partial log likelihood' (or 'pseudo-likelihood'). This is an $M$ estimator problem for which the estimators maximizing the partial likelihood are consistent and asymptotically normal (assuming fixed $T$ and $N \rightarrow \infty$ ), but for which the standard errors need to be adjusted for the correlations between observations (Wooldridge, 2002, chapter 13).

The method is analogous to that used in the survey statistics literature for adjusting the estimates of the parameter covariance matrix to account for clustering induced by survey design. We treated all transitions observed for each man as belonging to the same cluster, thereby allowing for arbitrary correlations between observations on the same individual from different panel transitions.

We evaluated the multivariate standard normal distribution functions using simulation methods based on the GHK simulator (Gourieroux and Monfort, 1996) with 140 random draws. For maximization, we used the modified Gauss-Newton routine implemented in Stata's $m l$ command, together with its cluster option to derive the robust variance estimator (StataCorp, 2003).

## 4. Data: samples and variables from the British Household Panel Survey

We used data from interview waves 1-10, survey years 1991-2000, of the British Household Panel Survey (Taylor, 2001). Data from consecutive waves were used to estimate annual transition rates, using a sample that pooled all transitions. We analysed men aged 18-64, in full time employment and not in full time education in year $t-1$. We restricted attention to men in order to avoid addressing issues of endogenous female labour supply. We defined a man to be unemployed if he was not working and had been looking for a job during the four weeks prior to the interview. The low pay threshold in each year was defined as two-thirds of contemporary sample median hourly earnings, i.e. just under $£ 4.50$ in August 2000 prices, with little trend over the decade. In any given year, the proportion of employed men that was low paid was between 11 and 15 percent.

### 4.1. Explanatory variables

The sets of explanatory variables were similar to those commonly used in this context: marital status (legally married or cohabiting rather than single), age and age-squared, educational qualifications, health status (number of health problems reported), and region of residence. All were measured using the values pertaining at the interview in the base year ( $t-$ 1 ), and assumed to be pre-determined. These variables were included in each of the vectors $\boldsymbol{x}_{t-}$
${ }_{1}, \boldsymbol{w}_{t-1}, \boldsymbol{h}_{t-1}$, and $\boldsymbol{z}_{t-1}$. In addition, the number of employees in the firm ('firm size'), and whether the respondent was in a skilled occupation was included in $\boldsymbol{x}_{t-1}$ and $z_{t-1}$ (the base- and currentyear low pay equations). Year-of-interview (year $t-1$ ) indicator variables were included in every equation.

The instruments used to define the exclusion conditions for identification were as follows. A dichotomous indicator of whether there was a change in interviewer between the interview at $t-1$ and at $t$ was included as an instrument in the retention equation, but excluded from the equations for current year employment and low pay. (The idea is that interviewer continuation fosters respondent trust and hence survey participation: see inter alia Schräpler, 2003.) Use of this instrument meant that low pay transitions involving survey year 1991 were not used.) Dichotomous indicator variables summarising whether the first employment spell in his working career was in a full-time job, or information missing about this, were used to explain both base year low pay (instrumenting initial conditions, also used by Stewart and Swaffield, 1999) and current year employment (instrumenting endogenous selection into employment), but they were excluded from the low pay transition equation. Finally, we used the number of unemployed people divided by the number of job vacancies in the respondent's travel-to-work area (the 'UV ratio'), a measure of labour market slackness, as an instrument in the current year employment equation, but excluded the variable from the low pay transition equation.

### 4.2. Average transition probabilities

To set the scene, we begin by describing the average transition probabilities for our samples, and show how these differ depending on whether attrition and non-response are treated as potential destination states. See Table 3. When the 'balanced' panel of respondents who provided earnings in two successive years was used to estimate the transition probabilities (panel a), the chances of being low paid in current year were nearly ten times greater for those who were low-paid rather than high-paid in the base year ( 54.7 percent compared with 5.2 percent). There is substantial state dependence in low pay. We do not consider here how much of this aggregate state dependence is genuine, or simply representing observed or unobserved differences between individuals ('heterogeneity'). On this issue, see Cappellari and Jenkins (2003), Stewart (2004), and Stewart and Swaffield (1999).
<Table 3 near here>
The transition probabilities in panel (b) were estimated from a sample enlarged to include individuals who dropped out of the sample altogether, or who did not drop out but for whom there was 'economic' item non-response - they moved from employment in the base year to unemployment, self-employment, or some other status (e.g. economic inactivity, full-time education) in the current year. The number of observations increased by 16.3 percent. Although the relative chances of being low paid in the current year were much the same as for the panel (a) sample, the relative chances of being high paid fell for those who were low paid in the base year (from 0.47 to 0.43 ). The table also shows that exits from the earnings distribution are not evenly distributed according to initial pay states, but were more likely among men who were low paid rather than high paid in the base year ( 22 per cent compared to 13 percent). Differential non-response by base year pay category was greater for 'economic' item non-response than for panel attrition, however. For the former type of nonresponse, base-year low-paid men were twice as likely not to respond (10.0 percent compared with 4.9 percent) - an illustration of the 'low-pay no-pay cycle' - whereas for attrition the corresponding ratio is about 1.5 ( 11.7 percent compared with 7.7 percent).

In panel (c) of Table 3, the sample was enlarged further, to incorporate individuals who did not drop out of the panel, but for whom there was survey item non-response on pay. The prevalence of missing pay data among employees seems to be slightly larger among those were higher paid in the base year rather than lower paid. The increase in sample numbers compared to panel (b) is small, however, only two percent. This suggests that exclusion or inclusion of this group is unlikely to affect model estimates substantially.

The descriptive statistics in Table 3 point to non-trivial rates of non-response and of several different kinds. Much of the rest of the paper is concerned with whether these nonresponses are ignorable when one estimates models of low pay transitions for men. Models 1 and 2 , accounting for three types of endogenous selection are discussed first, and then we consider Models 3-5, which assume ignorability of one or more selection mechanisms.

## 5. Results: models accounting for three types of endogenous selection

5.1. Tests of instrument validity and for ignorability of the endogenous selection mechanisms

The results from our Wald tests of instrument validity, and thence of the identification strategy, are shown in Table 4. The null hypothesis in each case was that the coefficient(s) on the relevant variable tests were equal to zero. A sufficient condition for identification was rejection of the null in the case where instrument(s) were included in an equation(s) and nonrejection of the null in the low pay equation (from which the instruments were excluded). Rows (1)-(5) of the table refer to tests of the former type and, for both Models 1 and 2 , the $p$ values are less than conventional thresholds in every case. Row 6 shows that our exclusion restrictions were supported by the data: the Wald test $\chi^{2}$ values are small for both Model 1 and Model 2, and the associated $p$-values are large ( 0.33 and 0.47 , respectively).
<Table 4 near here>
Estimates of the cross-equation correlations between unobservables provide insights about the endogenous selection processes. They are shown in Table 5. As it happens, none of the correlations associated with the panel retention process was statistically significant at conventional confidence levels, which suggests that such selection process could be ignored when estimating the relationship of economic interest. This is true for both Models 1 and 2. Observe, however, that the inclusion in the estimation sample for Model 2 of respondents with item non-response on current pay (classified among the $R_{t}=0$ cases) switches the sign of the correlation between unobservables in the retention and employment equations from positive to negative. This is consistent with Table 3 (panel c), which showed that men who were low paid in the base-year were more likely that high paid men to drop out of the panel, but more likely to provide earnings information if they stayed in the panel and were employed.

There was a statistically significant correlation between unobserved factors affecting base-year low pay propensities and those affecting current-year employment propensities: 0.139 according to Model $1,-0.130$ according to Model 2 . Men who were more likely to be low-paid, other things being equal, were less likely to remain employed - an association that is consistent with the 'low-pay no-pay cycle' hypothesis. The result suggests that there may be endogenous selection into employment (and we test this shortly).

The correlation between unobservables in the current-year employment and conditional low pay equations was not statistically significant. Nor was the correlation between the unobservable factors associated with base-year low pay propensities and with current-year conditional low pay propensities. Taking the estimates all together, Models 1 and 2 provide a similar picture about the correlation structure: the different treatment of men with item nonresponse on current-year pay appears to have little impact.
<Table 5 near here>
Tests for the ignorability of each selection mechanism were based on a Wald test that every correlation connecting that selection equation to the rest of the model was equal to zero: see the bottom panel of Table 5. The results tell the same story for both Model 1 and Model 2. Panel retention is exogenous, but the null hypotheses of exogeneity of selection into currentyear employment (economic item non-response) and base-year low pay (initial conditions) are each overwhelmingly rejected. The results indicate that, although the sample selection associated with the survey process is not problematic for the analysis of pay dynamics, the economic mechanism moving individuals into and out from the distribution of pay could be a more serious issue. (Comparisons with models assuming ignorability provide evidence about the magnitude of this problem: see Section 6.)

### 5.2. The impacts of the explanatory variables

The estimates of model parameters (other than cross-equation correlations) are summarised in Table 6 (Model 1) and Table 7 (Model 2). We focus on Model 1 in the discussion, and then comment briefly on differences compared with Model 2. To facilitate interpretation, the estimate associated with each covariate in each equation is presented in the form of a marginal effect (ME) rather than a coefficient. An ME shows the effect on the relevant probability of a change in the chosen covariate, whereas the corresponding coefficient shows the effect on the relevant latent propensity.

For a dichotomous variable, the ME was computed as the change in probability induced by a change in value from zero to one, holding all other covariates fixed at their sample mean values. For continuous covariates, the ME was defined as the change in probability induced by a change from the lower quartile to the upper quartile value ( 28 to 46 for age; 4.3 to 18.1 for the unemployment-vacancies ratio).

Calculation of MEs is complicated for the key probabilities of economic interest by the fact that they are conditional probabilities (cf. the probability of current-year low pay conditional on being low paid in the base year). A change in the value of a covariate may influence both the conditional and conditioning events (the numerator and denominator of the conditional probability). In order to keep the probability of a conditioning event constant in the computation of an ME, we set the covariates in the index function(s) for that probability equal to those of the (hypothetical) man who had the average conditioning probability. See Stewart
and Swaffield (1999) or Cappellari and Jenkins (2004) for a more detailed explanation. In all but the simplest model, the MEs (and their standard errors) are functions of the estimated cross-equation correlations as well as estimated coefficients.

ME estimates for Model 1 are shown in Table 6. To help assess whether they are large or small, the table also shows the predicted probabilities of each outcome calculated at mean covariate values.

The effects of changes in covariates on the probability of base-year low pay are large compared to those for the other probabilities: there are many MEs of five percentage points or more (with an average predicted probability baseline of 0.11 ), whereas the MEs in the other equations are smaller (and the average probabilities of retention, employment, and low pay entry are much higher). The results are congruent with those typically found in economists' wage regressions. For example, low pay probabilities were lower for married men compared to singles, lower the higher the educational qualifications held, and lower for those working in large firms or in a skilled occupation. Low pay propensities were higher for those with health problems or who lived outside London and the South-East.

Year-on-year panel retention rates were higher, ceteris paribus, for married men than single men (by about four percentage points), older men compared to younger men, and for men with educational qualifications compared to men with none (by about three percentage points). There appears to be no marked variation in the retention probability with qualification level, however. Retention probabilities did not have statistically significant associations with differences in the prevalence of health problems or residential location. There were statistically significant survey year effects on retention probabilities (not shown for brevity's sake), being higher by about three percentage points for 1993 and thereafter compared to 1992 (the year refers to $t-1$ ).

## <Table 6 near here>

The third selection mechanism relevant to the study of earnings mobility is whether someone was employed in the current year. Probabilities were estimated to be higher for married men compared to single men, and for older men compared to younger men. The more health problems reported, the lower the employment probability. The effect is relatively large: a difference of some eight percentage points for someone with $4+$ health problems compared to a man with none.

In the equations for low pay transition probabilities, the MEs are typically of the expected sign, being negative for the educational qualification variables and positive for the
health problems ones, for example. Few of the MEs were precisely estimated, however, particularly in the low pay entry equation. (In most cases the corresponding regression coefficient was precisely estimated (see the Appendix), with the differences in significance largely reflecting the non-linear transformations involved in calculation of MEs.) The probability of low pay persistence was lower for men with a higher degree (by about 14 percentage points), men living in London (13 percentage points), men working in an establishment with more than 100 workers ( 16 percentage points), and men who worked in skilled occupation (10 percentage points). These are relatively large effects.

How do the results change if men with item non-response on current-year pay are treated as $R_{t}=0$ cases, rather than dropped from the analysis? The answer, according to Model 2, is: not much. Compare Table 7 with Table 6. Unsurprisingly, the average predicted probability of retention is now lower ( 0.908 rather than 0.925 ), and also the average predicted probabilities of remaining, and falling into low pay, are smaller. However, estimated MEs are remarkably similar to their counterparts in Table 6 for the base-year low pay, retention, and employment equations. The most noticeable differences are for the low pay transition equations: Model 2 MEs are generally less precisely estimated than Model 1 MEs, though of the same sign. Overall, the results suggest that accounting for survey item non-response in this manner makes little difference. Of course, this conclusion may not be applicable to situations where the prevalence of this type of non-response is much greater.
<Table 7 near here>

## 6. Results: models assuming ignorability of one or more endogenous selection mechanisms

How would estimates differ if some or all of the endogenous selection mechanisms were not taken into account when modelling low pay transitions? Models 3-5 are informative about this. Table 8 reports estimates of the cross-equation correlations and the low pay transition equations from each of these models. (Retained panel members in employment with item nonresponse on pay are now dropped, as for Model 1. Estimates for the other equations are provided in the Appendix.)

Model 3 is the one in which no distinction was made between panel attrition and nonresponse on pay due to non-employment. The estimated correlation between unobservables in
the base-year low pay and the retention equation, -0.089 , lies in between the correlation between base-year low pay and current-year employment propensities ( -0.139 ) and the correlation between panel retention and base-year low pay propensities ( -0.031 ) that was estimated by Model 1 (Table 5). The correlation between base-year low pay and current-year conditional low pay propensities $(0.165,|t|=0.82)$ is similar to its Model 1 counterpart ( $0.173,|t|=1.1$ ); so too is the correlation between retention and current-year conditional low pay (cf. $0.336,|t|=2.42$ with $0.226,|t|=1.35$ ). In addition, it is remarkable how similar corresponding MEs (and their precision) are in Models 1 and 3. Overall, these results suggest that distinguishing between panel attrition and economic item non-response has little impact on the estimates of the low pay transition probabilities, as long as the combination of the two selection processes is modelled along with the process of interest (and so too are initial conditions).
<Table 8 near here>
Model 4 is the one in which initial conditions were the only endogenous selection mechanism accounted for. The estimate of the correlation between unobservables affecting base-year and current-year low pay propensities is smaller in magnitude than in Models 1 and 3 , and less precisely estimated $(0.156,|t|=0.52)$. Observe, however, that the estimated MEs, and their precision, are little different from their counterparts from Models 1 and 3.

Model 5 incorporates the assumption that all selection mechanisms (including initial conditions) are ignorable. Now there may be some more perceptible differences. In both the low pay persistence and low pay entry equations, it appears that the MEs from Model 5 are generally larger in magnitude than their counterparts from the other models (with the exception of the ME for age), and the statistical precision is higher, so that many estimates in the low pay entry equation become statistically significant at conventional levels. However, the differences in the size of corresponding MEs remains relatively small. For example, men with an 'other higher degree' are estimated to have a low pay persistence probability some 11 percentage points lower than men with no educational qualifications according to Model 5 and 14 percentage points lower according to Model 1. Living in London reduced the low pay persistence probability by 10 percentage points according to Model 5 and 13 percentage points according to Model 1.

## 7. Summary and conclusions

Our starting point was the argument that, in order to get good estimates of the determinants of transitions into and out of low pay, one needs to account for the potential non-ignorability of selection processes. We have built on previous research by extending the number of selection mechanisms considered to three: two related to sample drop-out (panel attrition, nonemployment), plus 'initial conditions' (base year low pay status). This model, and variants that ignore one or more of these selection mechanisms, were fitted to data for men from the British Household Panel Survey. Tests of the ignorability of the selection processes suggested that, when estimating models for annual low pay transition probabilities, 'economic' selection mechanisms such as initial conditions and retention of employment are more important than the 'survey' selection mechanism (attrition).

From their model of low pay transitions also estimated using BHPS data (but for men and women, and data from waves 1-5), and controlling only for initial conditions, Stewart and Swaffield concluded that '[t]ypically, the estimated marginal effects on the conditional probability of remaining low paid (and the asymptotic $t$-ratios on the maximum likelihood coefficient estimates) are much reduced when allowance is made for endogenous selection.' (1999, p. 40). Our results are consistent with Stewart and Swaffield's, but more clearly regarding the reduction in precision than the magnitude of marginal effects. Few MEs in the low pay transition equations were statistically significant according to Model 1, whereas many were according to Model 5. Put another way, a model of low pay transitions that assumes ignorability of all selection processes may lead to misleading inferences about the effects of covariates on low pay transition probabilities.

More positively, however, the relatively small differences in corresponding MEs across models are also striking. It appears that relatively simple models provide estimates of covariate marginal effects that differ little from estimates from the complicated models. (In addition, it appears that ignoring survey item non-response on pay makes little difference, perhaps because its prevalence is relatively low.) In this sense our results are consistent with the findings for the related US studies summarised in Section 2.

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Table 1
Models of labour market behaviour with endogenous selection: examples

| Paper | Outcome of interest | Endogenous selection issues addressed? |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Attrition | Economic item nonresponse | Survey item nonresponse | Initial conditions |
| Hausman and Wise (1979) | Earnings | $\sqrt{ }$ |  |  |  |
| Keane et al. (1988) | Wages |  | $\sqrt{ }$ |  |  |
| Zabel (1998) | Wages, work hours | $\checkmark$ | $\checkmark$ |  |  |
| Stewart and | Low pay transitions |  |  |  | $\checkmark$ |
| Swaffield (1999) |  |  |  |  |  |
| Cappellari and | Low income | $\sqrt{*}$ |  |  | $\sqrt{ }$ |
| Jenkins (2004) | transitions |  |  |  |  |
| This paper | Low pay transitions | $\checkmark$ | $\checkmark$ |  | $\checkmark$ |

*: Attrition defined as sample drop-out or survey item non-response on income (see text).

Table 2
Year $t$ outcome combinations, and the treatment of item non-response on pay

| Grou <br> p | Retention | Employment | Low pay | Interpretation |
| :--- | :--- | :--- | :--- | :--- |
| $A$ | $R_{t}=0$ | Unobserved | Unobserved | Panel attrition |
| $B$ | $R_{t}=1$ | $E_{t}=0$ | Unobserved | Retained; OLF\|U|SE* |
| $C$ | $R_{t}=1$ | $E_{t}=1$ | $L_{t}=0$ | Retained; high-paid employee |
|  | $R_{t}=1$ | $E_{t}=1$ | $L_{t}=1$ | Retained; low-paid employee |
| Notes. *: Out of the labour force or unemployed or self-employed. Year $t-1$ |  |  |  |  |
| sample: men with $E_{t-1}=1$ and no non-response on $L_{t-1}$. Year $t$ sample, Model 1: |  |  |  |  |
| men with $E_{t}=1$ but non-response on $L_{t}$ excluded from estimation. Year $t$ |  |  |  |  |
| sample, Model 2: men with $E_{t}=1$ but non-response on $L_{t}$ included as cases with |  |  |  |  |
| $R_{t}=0$. |  |  |  |  |

Table 3
Average transition probabilities for British men (BHPS data)

| Row percentages |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\text { Year } t-1$state | High Low <br> pay pay |  |  | Year $t$ s | tate |  |  |  |
|  |  |  | 'Economic' item non-responseon pay |  |  | Attrition | 'Survey' item nonresponse | $\begin{aligned} & \text { All } \\ & \text { (col. } \\ & \% \text { ) } \end{aligned}$ |
|  |  |  | Selfemployed | Unemployed | Other |  |  |  |
|  | (C) ${ }^{*}$ | (C) | (B) | (B) | (B) | (A) |  |  |
| (a) Balanced panel of wage earners ( $N=13,967$ ) |  |  |  |  |  |  |  |  |
| High pay | 94.9 | 5.2 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 86.2 |
| Low pay | 45.3 | 54.7 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 13.9 |
| All | 88.0 | 12.0 |  |  |  |  |  | 100.0 |
| (b) Unbalanced panel, distinguishing by source of missing pay data ( $N=16,249$ ) |  |  |  |  |  |  |  |  |
| High pay | 82.8 | 4.5 | 1.6 | 1.3 | 2.0 | 7.7 |  |  |
| Low pay | 35.4 | 42.9 | 2.2 | 3.7 | 4.1 | 11.7 |  | 84.8 |
|  |  |  |  |  |  |  |  | 15.2 |
| All | 75.6 | 10.3 | 1.7 | 1.7 | 2.3 | 8.3 |  | 100.0 |
| (c) Unbalanced panel, distinguishing by source of missing pay data $(N=16,523)$ |  |  |  |  |  |  |  |  |
| High pay | 81.4 | 4.4 | 1.6 | 1.3 | 2.0 | 7.6 | 1.7 |  |
| Low pay | 34.9 | 42.3 | 2.1 | 3.7 | 4.0 | 11.5 | 1.6 |  |
|  |  |  |  |  |  |  |  | 15.2 |
| All | 73.4 | 10.2 | 1.7 | 1.6 | 2.3 | 8.2 | 1.7 | 100.0 |

Notes. *: Groups $A, B, C$, as described in Table 2. Pooled transitions from British Household Panel Survey, waves $1-10$. Men aged 16-64, excluding full-time students and self-employed (or missing pay data at $t-1$ ). The low pay threshold is two-thirds of median contemporary hourly earnings.

Table 4
Tests for identification and instrument validity (Low pay transitions for British men, BHPS data, Models 1 and 2)

| Tes | Instrument(s) | Equation | d.f. | Model 1 |  | Model 2 |
| :--- | :--- | :--- | :--- | ---: | :---: | ---: |
| t |  |  |  |  |  |  |

Notes. Wald tests that coefficient estimate(s) for instrument(s) are zero for in equation. *: Instruments are as follows. FW: respondent's first work post-school was in full-time employment, or not known (two variables). IC: Interviewer changed. UV: unemployment-vacancies ratio in respondent's travel-to-work area. Differences between Models 1 and 2 explained in Table 2 and text.

Table 5
Estimated correlations of unobservables, and tests of the ignorability of endogenous selection mechanisms
(Low pay transitions for British men, BHPS data, Models 1 and 2)

| Correlations of unobservables |  | Model 1 |  | Model 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Estimate | \|t | Estimate | \|t |
| Retention, low pay at $t-1$ | $\left(\rho_{1}\right)$ | -0.031 | [1.32] | -0.016 | [0.69] |
| Retention, employee at $t$ | $\left(\rho_{2}\right)$ | 0.275 | [1.33] | -0.297 | [1.65] |
| Retention, low pay at $t$ | $\left(\rho_{3}\right)$ | 0.226 | [1.35] | 0.268 | [1.51] |
| Low pay at $t-1$, employee at $t$ | $\left(\rho_{4}\right)$ | -0.139 | [5.28] | -0.130 | [4.82] |
| Low pay at $t-1$, low pay at $t$ | ( $\rho_{5}$ ) | 0.173 | [1.13] | -0.129 | [0.69] |
| Employee at $t$, low pay at $t$ | ( $\rho_{6}$ ) | -0.431 | [1.82] | -0.311 | [0.68] |
| Wald tests of ignorability |  | $\chi^{2}$ | $p$-value | $\chi^{2}$ | $p$-value |
| Initial conditions $\quad \mathrm{H}_{0}: \rho_{1}=\rho_{4}=\rho_{5}=0$ |  | 30.04 | 0.000 | 24.21 | 0.000 |
| Retention $\quad \mathrm{H}_{0}: \rho_{1}=\rho_{2}=\rho_{3}=0$ |  | 5.71 | 0.127 | 5.42 | 0.144 |
| Economic item non-response |  |  |  |  |  |
| $\mathrm{H}_{0}: \rho_{2}=\rho_{4}=\rho_{6}=0$ |  | 36.10 | 0.000 | 30.80 | 0.000 |

Notes. Estimated marginal effects are shown in Tables 6 and 7 below. $|t|$ is the absolute asymptotic $t$-ratio.

Table 6. Low pay transitions for British men (BHPS data): estimated marginal effects from Model 1

| Covariate | $\operatorname{Pr}($ low pay $t-1)$ |  | $\operatorname{Pr}$ (retention) |  | $\operatorname{Pr}($ employee $t$ ) |  | $\operatorname{Pr}$ (low pay $t \mid$ low pay $t-1$ ) |  | $\operatorname{Pr}($ low pay $t \mid$ high pay $t-1$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ME | \|t| | ME | \|t| | ME | \|t| | ME | \|t| | ME | \|t| |
| Predicted probability | 0.105 |  | 0.925 |  | 0.951 |  | 0.617 |  | 0.041 |  |
| Married | -0.034 | [2.34] | 0.039 | [3.16] | 0.014 | [1.82] | -0.074 | [1.85] | -0.011 | [1.01] |
| Age | 0.000 | [0.78] | 0.018 | [19.05] | 0.001 | [1.89] | 0.000 | [0.77] | -0.001 | [0.93] |
| Educ. qual.: other | -0.043 | [2.81] | 0.026 | [2.87] | 0.008 | [1.11] | -0.033 | [0.57] | -0.020 | [1.13] |
| Educ. qual.: O-level(s) | -0.060 | [3.24] | 0.018 | [2.35] | 0.006 | [0.89] | -0.086 | [1.53] | -0.027 | [1.14] |
| Educ. qual.: A-level(s) | -0.064 | [3.23] | 0.038 | [3.52] | -0.004 | [0.45] | -0.077 | [1.27] | -0.031 | [1.10] |
| Educ. qual.: other higher degree | -0.083 | [3.33] | 0.027 | [3.02] | 0.007 | [1.22] | -0.140 | [2.23] | -0.042 | [1.15] |
| Educ. qual.: first degree or higher | -0.104 | [3.08] | 0.013 | [1.65] | 0.013 | [1.67] | -0.113 | [1.35] | -0.054 | [1.06] |
| No. health problems: 1 | 0.010 | [1.28] | -0.003 | [0.67] | -0.016 | [2.04] | 0.026 | [0.92] | 0.012 | [1.09] |
| No. health problems: 2 | 0.016 | [1.22] | 0.008 | [1.11] | -0.028 | [2.13] | 0.070 | [1.44] | 0.013 | [0.98] |
| No. health problems: 3 | 0.034 | [1.47] | -0.004 | [0.26] | -0.039 | [1.87] | 0.144 | [1.83] | 0.057 | [1.23] |
| No. health problems: 4+ | 0.063 | [1.44] | -0.023 | [0.85] | -0.077 | [1.96] | 0.067 | [0.55] | 0.085 | [1.25] |
| Lived in South East | -0.037 | [2.80] | 0.003 | [0.60] | 0.006 | [1.26] | -0.065 | [1.70] | -0.017 | [1.11] |
| Lived in London | -0.053 | [2.87] | -0.012 | [1.32] | 0.004 | [0.55] | -0.128 | [2.03] | -0.019 | [1.06] |
| Firm size > 100 | -0.105 | [3.97] |  |  |  |  | -0.163 | [2.83] | -0.030 | [1.32] |
| Skilled occupation | -0.100 | [3.90] |  |  |  |  | -0.097 | [2.00] | -0.049 | [1.36] |
| First spell: full time employment | -0.010 | [0.92] |  |  | 0.015 | [1.91] |  |  |  |  |
| First spell: missing information | 0.017 | [1.32] |  |  | -0.004 | [0.53] |  |  |  |  |
| Unemployment-vacancies ratio |  |  |  |  | 0.000 | [0.28] |  |  |  |  |
| Interviewer changed |  |  | -0.033 | [3.35] |  |  |  |  |  |  |
| Model chi-squared (d.f. = 117) | 2257.58 | $p=0.00$ |  |  |  |  |  |  |  |  |
| Log Likelihood | -16,491 |  |  |  |  |  |  |  |  |  |
| Number of observations | 16,131 |  |  |  |  |  |  |  |  |  |

Notes. Model 1 described in text. Predicted probabilities calculated at mean covariate values. Regressions also included year dummies. $|t|$ is the absolute asymptotic $t$-ratio. The ME for age-squared was negligible for all models, and is not reported.

Table 7. Low pay transitions for British men (BHPS data): estimated marginal effects from Model 2

| Covariate | $\operatorname{Pr}($ low pay $t-1)$ |  | $\operatorname{Pr}($ retention) |  | $\operatorname{Pr}($ employee $t$ ) |  | $\operatorname{Pr}$ (low pay $t \mid$ low pay $t-1$ ) |  | $\begin{gathered} \operatorname{Pr}(\text { low pay } t \mid \text { high } \\ \text { pay } t-1) \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ME | $\|t\|$ | ME | $\|t\|$ | ME | \|t| | ME | $\|t\|$ | ME | \|t |
| Predicted probability | 0.105 |  | 0.908 |  | 0.949 |  | 0.519 |  | 0.038 |  |
| Married | -0.034 | [2.35] | 0.038 | [3.33] | 0.010 | [1.51] | -0.052 | [1.32] | -0.007 | [0.83] |
| Age | 0.000 | [0.79] | 0.018 | [11.22] | 0.002 | [2.17] | -0.003 | [2.12] | -0.009 | [1.64] |
| Educ. qual.: other | -0.043 | [2.79] | 0.031 | [3.16] | 0.005 | [0.69] | -0.005 | [0.09] | -0.015 | [0.86] |
| Educ. qual.: O-level(s) | -0.061 | [3.24] | 0.017 | [2.11] | 0.004 | [0.62] | -0.052 | [1.05] | -0.022 | [0.86] |
| Educ. qual.: A-level(s) | -0.064 | [3.22] | 0.038 | [3.66] | -0.009 | [0.98] | -0.041 | [0.77] | -0.026 | [0.82] |
| Educ. qual.: other higher degree | -0.083 | [3.33] | 0.025 | [2.85] | 0.005 | [0.79] | -0.091 | [1.70] | -0.035 | [0.86] |
| Educ. qual.: first degree or higher | -0.104 | [3.08] | 0.009 | [1.03] | 0.012 | [1.62] | -0.037 | [0.46] | -0.046 | [0.80] |
| No. health problems: 1 | 0.010 | [1.29] | 0.000 | [0.09] | -0.016 | [2.12] | 0.022 | [0.75] | 0.010 | [0.92] |
| No. health problems: 2 | 0.017 | [1.28] | 0.006 | [0.75] | -0.029 | [2.33] | 0.062 | [1.24] | 0.010 | [0.89] |
| No. health problems: 3 | 0.031 | [1.42] | -0.007 | [0.48] | -0.039 | [1.95] | 0.129 | [1.51] | 0.045 | [1.02] |
| No. health problems: 4+ | 0.067 | [1.51] | -0.024 | [0.85] | -0.073 | [1.99] | 0.028 | [0.22] | 0.066 | [1.09] |
| Lived in South East | -0.037 | [2.79] | 0.004 | [0.59] | 0.006 | [1.20] | -0.046 | [1.33] | -0.013 | [0.87] |
| Lived in London | -0.054 | [2.88] | -0.012 | [1.31] | 0.005 | [0.67] | -0.095 | [1.76] | -0.015 | [0.82] |
| Firm size > 100 | -0.105 | [3.96] |  |  |  |  | -0.122 | [2.88] | -0.021 | [0.96] |
| Skilled occupation | -0.101 | [3.90] |  |  |  |  | -0.059 | [1.55] | -0.037 | [0.98] |
| First spell: full time employment | -0.011 | [1.01] |  |  | 0.016 | [1.98] |  |  |  |  |
| First spell: missing information | 0.017 | [1.26] |  |  | -0.004 | [0.55] |  |  |  |  |
| Unemployment-vacancies ratio |  |  |  |  | 0.000 | [0.29] |  |  |  |  |
| Interviewer changed |  |  | -0.030 | [3.42] |  |  |  |  |  |  |
| Model chi-squared (d.f. = 117) | 2142.30 | $p=0.00$ |  |  |  |  |  |  |  |  |
| Log Likelihood | -17,279 |  |  |  |  |  |  |  |  |  |
| Number of observations | 16,404 |  |  |  |  |  |  |  |  |  |

Notes. Model 2 described in text. Predicted probabilities calculated at covariate mean values. Regressions also included year dummies. $|t|$ is the absolute asymptotic $t$-ratio. The ME for age-squared was negligible for all models, and is not reported.

Table 8. Low pay transitions for British men (BHPS data): estimated marginal effects from Models 3-5

| Covariate | Model 3 <br> (Retention \& non-employment combined) |  |  |  | Model 4 <br> (Retention \& non-employment assumed to be ignorable) |  |  |  | Model 5 <br> (All endogenous selections assumed to be ignorable) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\operatorname{Pr}($ low pay $t \mid$ low pay $t-1$ ) |  | $\operatorname{Pr}($ low pay $t \mid$ high pay $t-1$ ) |  | $\operatorname{Pr}$ (low pay $t \mid$ low pay $t-1$ ) |  | $\operatorname{Pr}($ low pay $t \mid$ high pay $t-1$ ) |  | $\operatorname{Pr}($ low pay $t \mid$ low pay $t-1$ ) |  | $\operatorname{Pr}($ low pay $t \mid$ high pay $t-1$ ) |  |
|  | ME | \|t| | ME | \|t| | ME | \|t| | ME | \|t| | ME | \|t| | ME | \|t| |
| Predicted probability | 0.598 |  | 0.038 |  | 0.579 |  | 0.037 |  | 0.549 |  | 0.037 |  |
| Married | -0.070 | [1.68] | -0.008 | [0.88] | -0.080 | [1.81] | -0.009 | [0.73] | -0.072 | [2.44] | -0.008 | [1.32] |
| Age | 0.000 | [1.18] | -0.005 | [1.88] | 0.000 | [0.92] | -0.003 | [1.33] | -0.001 | [1.89] | -0.007 | [2.61] |
| Educ. qual.: other qualification | -0.028 | [0.48] | -0.018 | [0.98] | -0.033 | [0.54] | -0.018 | [0.75] | -0.021 | [0.48] | -0.017 | [1.97] |
| Educ. qual.: O-level(s) | -0.080 | [1.38] | -0.025 | [0.99] | -0.084 | [1.29] | -0.025 | [0.74] | -0.066 | [1.75] | -0.023 | [2.08] |
| Educ. qual.: A-level(s) | -0.076 | [1.20] | -0.029 | [0.95] | -0.082 | [1.16] | -0.029 | [0.72] | -0.062 | [1.50] | -0.028 | [2.07] |
| Educ. qual.: other higher degree | -0.131 | [2.01] | -0.038 | [1.00] | -0.137 | [1.82] | -0.038 | [0.76] | -0.112 | [2.94] | -0.036 | [2.12] |
| Educ. qual.: first degree or higher | -0.108 | [1.23] | -0.049 | [0.92] | -0.104 | [1.03] | -0.048 | [0.69] | -0.066 | [1.17] | -0.047 | [2.01] |
| No. health problems: 1 | 0.018 | [0.63] | 0.009 | [0.91] | 0.023 | [0.80] | 0.009 | [0.71] | 0.021 | [0.81] | 0.009 | [1.56] |
| No. health problems: 2 | 0.063 | [1.26] | 0.008 | [0.74] | 0.068 | [1.33] | 0.008 | [0.63] | 0.066 | [1.55] | 0.008 | [1.03] |
| No. health problems: 3 | 0.129 | [1.59] | 0.044 | [1.03] | 0.135 | [1.57] | 0.044 | [0.80] | 0.133 | [1.88] | 0.043 | [1.82] |
| No. health problems: 4+ | 0.024 | [0.19] | 0.058 | [1.02] | 0.043 | [0.32] | 0.065 | [0.86] | 0.026 | [0.24] | 0.061 | [1.57] |
| Lived in South East | -0.062 | [1.57] | -0.015 | [0.97] | -0.065 | [1.52] | -0.015 | [0.76] | -0.055 | [1.80] | -0.014 | [1.92] |
| Lived in London | -0.123 | [1.99] | -0.017 | [0.95] | -0.121 | [1.86] | -0.016 | [0.75] | -0.104 | [1.97] | -0.015 | [1.78] |
| Firm size > 100 | -0.166 | [2.54] | -0.027 | [1.18] | -0.164 | [2.06] | -0.026 | [0.92] | -0.143 | [5.84] | -0.023 | [2.20] |
| Skilled occupation | -0.098 | [1.80] | -0.046 | [1.21] | -0.097 | [1.43] | -0.044 | [0.93] | -0.077 | [2.98] | -0.040 | [2.33] |
| Corr(retention, low pay at $t-1$ ) | -0.089 | [4.45] |  |  |  |  |  |  |  |  |  |  |
| Corr(retention, low pay at $t$ ) | 0.336 | [2.42] |  |  |  |  |  |  |  |  |  |  |
| Corr(low pay at $t$, low pay at $t-1$ ) | 0.165 | (0.82] |  |  | 0.156 | [0.53] |  |  |  |  |  |  |
| Model chi-squared | $($ d.f. $=93$ ) | 1981.76 |  |  | (d.f. $=71$ ) | 1241.94 |  |  | (d.f. $=23$ ) | 125.29 | (d.f. $=23$ ) | 492.09 |
| Log Likelihood | -15,219 |  |  |  | -7,874. |  |  |  | -1,268 |  | -2,196. |  |
| Number of observations | 16,24 |  |  |  | 13,967 |  |  |  | 1,933 |  | 12,03 |  |

Notes. Models described in text. Predicted probabilities calculated at covariate mean values. Regressions also included year dummies. Model 3 Wald test of ignorability of retention: $\chi^{2}=25.63$ (d.f. $=2$ ), $p=0.000$. Model 3 Wald test of initial conditions: $\chi^{2}=20.65$ (d.f. $=2$ ), $p=0.000$. $|t|$ is the absolute asymptotic $t$-ratio. The ME for age-squared was negligible for all models, and is not reported.

## APPENDIX

A1. Likelihood functions for Models 1-5
A2. Estimated coefficients for Models 1-5

## A1. Likelihood functions for Models 1-5

## Models 1 and 2

The equations characterising Models 1 and 2 are given in equations (1)-(8) of the main text. We assumed that unobservables had a four-variate standard normal distribution:

$$
\left(u_{t-1}, v_{t}, \varepsilon_{t}, \omega\right) \sim N(0, \Sigma)
$$

where symmetric correlation matrix $\Sigma$ has diagonal elements equal to unity and off-diagonal elements equal to $\sigma_{i j}, i \neq j$, where subscripts $i$ and $j$ refer to the $i^{\text {th }}$ and $j^{\text {th }}$ elements of the ordered list $\left\{R, E, L_{t-1}, L_{t}\right\}$. For example, $\sigma_{34}=\sigma_{43}$ is the correlation between unobservables determining base-year low pay $\left(L_{t-1}\right)$ - the initial condition - and current-year low pay $\left(L_{t}\right)$. (Note that $\sigma_{43}$ is referred to as $\rho_{5}$ in Table 5.)

There are three types of year $t$ outcome, implying three types of likelihood contribution: men with $R_{t}=0($ group $A$ in Table 2$)$, men with $R_{t}=1 \& E_{t}=1(\operatorname{group} B)$, and men with $R_{t}=1 \& E_{t}=0($ group $C)$.

Define a set of indices, $k_{j}$, for $j=1,2,3,4$, where

$$
\begin{aligned}
k_{1} & =2 R_{t}-1 \\
k_{2} & =2 E_{t}-1 \\
k_{3} & =2 L_{t-1}-1 \\
k_{4} & =2 L_{t}-1 .
\end{aligned}
$$

We shall use these indices to sign the arguments of multivariate normal c.d.f.s entering the likelihood function. Let the vector $\boldsymbol{k}=\left(k_{1}, k_{2}, k_{3}, k_{4}\right)$, and $K$ be the $4 \times 4$ diagonal matrix with the elements of $\boldsymbol{k}$ on the leading diagonal and zeros elsewhere. Define the vectors of index functions:

$$
\begin{aligned}
& \boldsymbol{\Xi}_{1}=\left(k_{4} \gamma_{1}^{\prime} z_{t-1}, k_{2} \lambda^{\prime} \boldsymbol{h}_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}, k_{1} \boldsymbol{\psi}^{\prime} \boldsymbol{w}_{t-1}\right)^{\prime} \\
& \boldsymbol{\Xi}_{2}=\left(k_{4} \gamma_{2}^{\prime} z_{t-1}, k_{2} \lambda^{\prime} \boldsymbol{h}_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}, k_{1} \boldsymbol{\psi}^{\prime} \boldsymbol{w}_{t-1}\right)^{\prime}
\end{aligned}
$$

Then, for each man with $R_{t}=1 \& E_{t=} 1$ (group $C$ ), the likelihood contribution takes the form:

$$
\mathcal{L}_{C}=\Phi_{4}\left(L_{t-1} \Xi_{1}+\left(1-L_{t-1}\right) \Xi_{2} ; \Omega\right)
$$

where $\Omega=K \Sigma K$ and $\Phi_{m}($.$) denotes an m$-variate normal c.d.f.

For each man with $R_{t}=1 \& E_{t}=0(\operatorname{group} B)$, the likelihood will be truncated because earnings in year $t$ are not observed. Let

$$
\boldsymbol{\Xi}_{-L t}=\left(k_{2} \lambda^{\prime} \boldsymbol{h}_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}, k_{1} \psi^{\prime} \boldsymbol{w}_{t-1}\right)^{\prime}
$$

and

$$
\Omega_{-L t}=M_{1} \Omega M_{1}^{\prime}
$$

where

$$
M_{1}=\left(\begin{array}{llll}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

Thus, subscript $\_L t$ denote vectors and matrices deprived of elements referring to the year $t$ low pay equation. The likelihood contribution for each group $B$ member can then be written as

$$
\mathcal{L}_{B}=\Phi_{3}\left(\Xi_{-L t} ; \Omega_{-L t}\right) .
$$

For each man with $R_{t}=1 \& E_{t}=0(\operatorname{group} A)$, there is additional truncation. Let

$$
\boldsymbol{\Xi}_{-L_{t} E t}=\left(k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}, k_{1} \psi^{\prime} \boldsymbol{w}_{t-1}\right)^{\prime}
$$

and

$$
\Omega_{-t_{-} E t}=\mathrm{M}_{2} \Omega \mathrm{M}_{2}^{\prime}
$$

where

$$
M_{2}=\left(\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

Thus, subscript _Lt_Et denotes vectors and matrices deprived of elements referring to the year $t$ low pay and employment equations. The likelihood contribution for each group $A$ member can then be written as

$$
\mathcal{L}_{A}=\Phi_{2}\left(\Xi_{-L t_{-} E t} ; \Omega_{-L t_{-} E t}\right) .
$$

We may combine the expressions for the different types of likelihood contribution for each man:

$$
\log \mathcal{L}=\left(1-R_{t}\right) \log \mathcal{L}_{A}+R_{t}\left(1-E_{t}\right) \log \mathcal{L}_{B}+R_{t} E_{t} \log \mathcal{L}_{C} .
$$

This is equation (9) in the main text.

## Model 3

In the main text we discussed additional models nested within Models 1 and 2. Model 3 collapsed panel retention and employment into a single process, yielding a single selection
mechanism that discriminated between sample observations according to whether $L_{t}$ was observable or not. In this model, therefore, the sample retention process (equations 2 and 3) refers to having observable earnings in year $t$, and there is no equation for employment propensity in $t$. The conditioning set for sample retention is the same as in equation (2).

For each man with earnings observed in year $t$, likelihood contributions take the form

$$
\mathcal{L}_{D}=\Phi_{3}\left(L_{t-1} \mathbf{\Xi}_{1_{-} E t}+\left(1-L_{t-1}\right) \mathbf{\Xi}_{2-E t} ; \Omega_{-E t}\right),
$$

where

$$
\begin{aligned}
& \boldsymbol{\Xi}_{1_{-} E t}=\left(k_{4} \gamma_{1}^{\prime} z_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}, k_{1} \psi^{\prime} \boldsymbol{w}_{t-1}\right)^{\prime} \\
& \boldsymbol{\Xi}_{2_{-} E t}=\left(k_{4} \gamma_{2}^{\prime} z_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}, k_{1} \psi^{\prime} \boldsymbol{w}_{t-1}\right)^{\prime}
\end{aligned}
$$

and

$$
\Omega_{-E t}=M_{3} \Omega M_{3}{ }^{\prime}
$$

where

$$
M_{3}=\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right) .
$$

For men with earnings not observed in year $t$, the likelihood contribution is $\mathcal{L}_{A}$.
The likelihood contribution for each man can therefore be written as

$$
\log \mathcal{L}=\left(1-R_{t}\right) \log \mathcal{L}_{A}+R_{t} \log \mathcal{L}_{D} .
$$

## Model 4

Model 4 assumes, in addition, that the availability of earnings information in year $t$ is exogenous. The likelihood contribution for each man can be written as

$$
\log \mathcal{L}=\log \left[\Phi_{2}\left(L_{t-1} \mathbf{\Xi}_{1_{-} E t_{-} R t}+\left(1-L_{t-1}\right) \mathbf{\Xi}_{2_{-} E t_{-} R t} ; k_{4} k_{3} \sigma_{43}\right)\right]
$$

where

$$
\begin{aligned}
& \boldsymbol{\Xi}_{1-E t-R t}=\left(k_{4} \gamma_{1}^{\prime} z_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}\right)^{\prime} \\
& \boldsymbol{\Xi}_{2_{-} E t_{-} R t}=\left(k_{4} \gamma_{2}^{\prime} z_{t-1}, k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}\right)^{\prime} .
\end{aligned}
$$

The likelihood contribution may be written alternatively as

$$
\log \mathcal{L}=\log \left[L_{t-1} \Phi_{2}\left(\mathbf{\Xi}_{1-E t-R t} ; k_{4} k_{3} \sigma_{43}\right)+\left(1-L_{t-1}\right) \Phi_{2}\left(\mathbf{\Xi}_{2_{-} E t-R t} ; k_{4} k_{3} \sigma_{43}\right)\right] .
$$

By contrast, the likelihood contribution in Stewart and Swaffield's (1999) partial observability model was

$$
\log \mathcal{L}=\log \left[L_{t-1} \Phi_{2}\left(\mathbf{\Xi}_{1-E t-R t} ; k_{4} k_{3} \sigma_{43}\right)+\left(1-L_{t-1}\right) \Phi_{1}\left(k_{3} \beta^{\prime} \boldsymbol{x}_{t-1}\right)\right] .
$$

## Model 5

Finally, Model 5 treated no selection mechanism as endogenous. The equation for year $t$ low pay status, with coefficients differing according to base-year low pay status, could therefore estimated via two univariate probit models for year $t$ low pay, one for the men who were low paid of year $t-1$, and the other for those who were not high paid. The likelihood contribution for each man may be written as

$$
\log \mathcal{L}=\log \left[L_{t-1} \Phi_{1}\left(k_{4} \boldsymbol{\gamma}_{1}^{\prime} z_{t-1}\right)+\left(1-L_{t-1}\right) \Phi_{1}\left(k_{4} \boldsymbol{\gamma}_{2}^{\prime} z_{t-1}\right)\right]
$$

A2 Estimated coefficients for Models 1-5
Appendix Table. Models 1-5: estimated coefficients

|  | Appendix Table. Models 1-5: estimated coefficients |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Model 1 |  | Model 2 |  | Model 3 |  | Model 4 | Model 4 (S\&S) |  |  | Model 5 |  |
|  | Coeff. | \|t| | Coeff. | \|t| | Coeff. | $\|t\|$ | Coeff. | \|t| | Coeff. | \|t| | Coeff. | \|t| |
| Retention ( $\boldsymbol{R}_{t}$ ) |  |  |  |  |  |  |  |  |  |  |  |  |
| Married | 0.265 | [7.17] | 0.226 | [6.50] | 0.236 | [7.12] |  |  |  |  |  |  |
| Age | 0.061 | [6.45] | 0.051 | [5.69] | 0.110 | [14.12] |  |  |  |  |  |  |
| Age-squared | -0.001 | [5.80] | -0.001 | [5.22] | -0.001 | [14.97] |  |  |  |  |  |  |
| Educ. qual.: other qualification | 0.210 | [3.16] | 0.215 | [3.41] | 0.150 | [2.56] |  |  |  |  |  |  |
| Educ. qual.: O-level(s) | 0.132 | [2.39] | 0.109 | [2.05] | 0.095 | [1.92] |  |  |  |  |  |  |
| Educ. qual.: A-level(s) | 0.314 | [5.21] | 0.264 | [4.57] | 0.147 | [2.83] |  |  |  |  |  |  |
| Educ. qual.: other higher degree | 0.204 | [3.93] | 0.157 | [3.20] | 0.144 | [3.18] |  |  |  |  |  |  |
| Educ. qual.: first degree or higher | 0.096 | [1.69] | 0.055 | [1.01] | 0.084 | [1.66] |  |  |  |  |  |  |
| No. health problems: 1 | -0.024 | [0.69] | -0.003 | [0.09] | -0.084 | [2.87] |  |  |  |  |  |  |
| No. health problems: 2 | 0.062 | [1.11] | 0.039 | [0.75] | -0.098 | [2.16] |  |  |  |  |  |  |
| No. health problems: 3 | -0.025 | [0.27] | -0.043 | [0.50] | -0.164 | [2.18] |  |  |  |  |  |  |
| No. health problems: 4+ | -0.146 | [0.96] | -0.136 | [0.94] | -0.363 | [2.94] |  |  |  |  |  |  |
| Lived in South East | 0.023 | [0.60] | 0.021 | [0.59] | 0.042 | [1.26] |  |  |  |  |  |  |
| Lived in London | -0.078 | [1.46] | -0.072 | [1.42] | -0.038 | [0.79] |  |  |  |  |  |  |
| Year $t-1$ : 1993 | 0.198 | [3.55] | 0.152 | [2.89] | 0.206 | [4.29] |  |  |  |  |  |  |
| Year $t-1: 1994$ | 0.188 | [3.48] | 0.144 | [2.85] | 0.217 | [4.61] |  |  |  |  |  |  |
| Year $t-1: 1995$ | 0.428 | [7.47] | 0.343 | [6.44] | 0.334 | [6.95] |  |  |  |  |  |  |
| Year $t-1: 1996$ | 0.323 | [5.72] | 0.303 | [5.71] | 0.332 | [6.87] |  |  |  |  |  |  |
| Year $t-1: 1997$ | 0.188 | [3.47] | 0.162 | [3.17] | 0.267 | [5.68] |  |  |  |  |  |  |
| Year $t-1: 1998$ | 0.239 | [4.27] | 0.225 | [4.26] | 0.260 | [5.43] |  |  |  |  |  |  |
| Year $t-1: 1999$ | 0.236 | [4.21] | 0.213 | [4.05] | 0.316 | [6.49] |  |  |  |  |  |  |
| Interviewer changed | -0.227 | [7.09] | -0.178 | [5.86] | -0.148 | [5.39] |  |  |  |  |  |  |
| Constant | -0.222 | [1.27] | -0.065 | [0.39] | -1.217 | [8.20] |  |  |  |  |  |  |

## Continued over page

|  | Model 1 |  | Model 2 |  | Model 3 |  | Model 4 |  | Model 4 (S\&S) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | \|t| | Coeff. | $\|t\|$ | Coeff. | \|t| | Coeff. | \|t| | Coeff. | \|t| |
| Base year low pay ( $L_{t-1}$ ) |  |  |  |  |  |  |  |  |  |  |
| Married | -0.185 | [3.77] | -0.184 | [3.79] | -0.189 | [3.86] | -0.175 | [3.23] | -0.176 | [3.23] |
| Age | -0.186 | [15.91] | -0.184 | [15.79] | -0.187 | [16.06] | -0.201 | [14.92] | -0.200 | [14.55] |
| Age-squared | 0.002 | [14.22] | 0.002 | [14.10] | 0.002 | [14.34] | 0.002 | [13.47] | 0.002 | [13.19] |
| Educ. qual.: other qualification | -0.276 | [3.47] | -0.273 | [3.42] | -0.279 | [3.51] | -0.284 | [3.23] | -0.282 | [3.22] |
| Educ. qual.: O-level(s) | -0.385 | [5.64] | -0.388 | [5.70] | -0.388 | [5.69] | -0.392 | [5.16] | -0.392 | [5.14] |
| Educ. qual.: A-level(s) | -0.427 | [5.82] | -0.426 | [5.82] | -0.429 | [5.85] | -0.458 | [5.58] | -0.457 | [5.57] |
| Educ. qual.: other higher degree | -0.526 | [7.68] | -0.525 | [7.70] | -0.526 | [7.68] | -0.558 | [7.28] | -0.556 | [7.27] |
| Educ. qual.: first degree or higher | -0.805 | [9.52] | -0.815 | [9.65] | -0.809 | [9.56] | -0.826 | [8.81] | -0.826 | [8.79] |
| No. health problems: 1 | 0.053 | [1.41] | 0.053 | [1.43] | 0.050 | [1.33] | 0.049 | [1.18] | 0.049 | [1.19] |
| No. health problems: 2 | 0.084 | [1.36] | 0.088 | [1.44] | 0.089 | [1.45] | 0.060 | [0.86] | 0.061 | [0.86] |
| No. health problems: 3 | 0.167 | [1.74] | 0.157 | [1.66] | 0.169 | [1.76] | 0.103 | [0.93] | 0.103 | [0.92] |
| No. health problems: 4+ | 0.290 | [1.80] | 0.307 | [1.91] | 0.289 | [1.79] | 0.373 | [1.99] | 0.375 | [2.00] |
| Lived in South East | -0.220 | [4.44] | -0.219 | [4.43] | -0.223 | [4.50] | -0.218 | [3.94] | -0.220 | [3.98] |
| Lived in London | -0.351 | [5.09] | -0.357 | [5.20] | -0.349 | [5.08] | -0.359 | [4.56] | -0.361 | [4.58] |
| Year $t-1$ : 1993 | 0.058 | [1.23] | 0.056 | [1.19] | 0.051 | [1.10] | 0.046 | [0.90] | 0.045 | [0.88] |
| Year t-1: 1994 | 0.115 | [2.30] | 0.111 | [2.25] | 0.109 | [2.19] | 0.072 | [1.28] | 0.071 | [1.25] |
| Year t-1: 1995 | 0.124 | [2.48] | 0.117 | [2.35] | 0.117 | [2.37] | 0.114 | [2.00] | 0.110 | [1.93] |
| Year $t-1$ : 1996 | 0.056 | [1.05] | 0.046 | [0.87] | 0.051 | [0.96] | 0.085 | [1.44] | 0.084 | [1.39] |
| Year t-1: 1997 | 0.054 | [1.00] | 0.046 | [0.87] | 0.049 | [0.92] | 0.071 | [1.18] | 0.069 | [1.13] |
| Year $t-1$ : 1998 | -0.007 | [0.13] | -0.017 | [0.31] | -0.007 | [0.12] | -0.009 | [0.14] | -0.010 | [0.16] |
| Year $t-1$ : 1999 | -0.046 | [0.82] | -0.044 | [0.79] | -0.046 | [0.83] | -0.023 | [0.37] | -0.025 | [0.40] |
| Firm size > 100 | -0.506 | [13.00] | -0.506 | [13.13] | -0.510 | [13.19] | -0.518 | [12.04] | -0.517 | [11.99] |
| Skilled occupation | -0.459 | [10.39] | -0.460 | [10.40] | -0.461 | [10.47] | -0.472 | [9.63] | -0.471 | [9.62] |
| First spell: full time employment | -0.054 | [0.94] | -0.062 | [1.04] | -0.051 | [0.91] | -0.051 | [0.80] | -0.055 | [0.84] |
| First spell: missing information | 0.092 | [1.61] | 0.093 | [1.54] | 0.079 | [1.39] | 0.070 | [1.08] | 0.074 | [1.07] |
| Constant | 3.864 | [17.97] | 3.843 | [17.85] | 3.902 | [18.22] | 4.128 | [16.75] | 4.118 | [16.44] |

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## Continued over page

|  | Model 1 |  | Model 2 |  | Model 3 |  | Model 4 |  | Model 4 (S\&S) |  | Model 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | \|t| | Coeff. | \|t | Coeff. | \|t| | Coeff. | \|t| | Coeff. | \|t| | Coeff. | \|t| |
| Low pay\|past low pay |  |  |  |  |  |  |  |  |  |  |  |  |
| Married | -0.182 | [1.96] | -0.128 | [1.33] | -0.172 | [1.83] | -0.202 | [2.07] | -0.172 | [1.25] | -0.182 | [2.39] |
| Age | -0.111 | [3.82] | -0.067 | [1.54] | -0.086 | [2.44] | -0.103 | [2.25] | -0.071 | [0.60] | -0.080 | [4.59] |
| Age-squared | 0.001 | [3.87] | 0.001 | [1.63] | 0.001 | [2.44] | 0.001 | [2.36] | 0.001 | [0.66] | 0.001 | [4.30] |
| Educ. qual.: other qualification | -0.079 | [0.60] | -0.011 | [0.09] | -0.069 | [0.50] | -0.083 | [0.57] | -0.039 | [0.19] | -0.052 | [0.48] |
| Educ. qual.: O-level(s) | -0.209 | [1.76] | -0.128 | [1.06] | -0.198 | [1.59] | -0.210 | [1.50] | -0.148 | [0.59] | -0.166 | [1.77] |
| Educ. qual.: A-level(s) | -0.187 | [1.42] | -0.101 | [0.77] | -0.186 | [1.35] | -0.206 | [1.31] | -0.134 | [0.48] | -0.155 | [1.51] |
| Educ. qual.: other higher degree | -0.338 | [2.63] | -0.225 | [1.66] | -0.322 | [2.34] | -0.343 | [2.08] | -0.254 | [0.75] | -0.281 | [2.95] |
| Educ. qual.: first degree or higher | -0.270 | [1.48] | -0.091 | [0.46] | -0.264 | [1.33] | -0.260 | [1.10] | -0.126 | [0.26] | -0.165 | [1.18] |
| No. health problems: 1 | 0.065 | [0.94] | 0.053 | [0.76] | 0.045 | [0.64] | 0.059 | [0.83] | 0.052 | [0.69] | 0.054 | [0.80] |
| No. health problems: 2 | 0.178 | [1.51] | 0.153 | [1.28] | 0.160 | [1.33] | 0.176 | [1.44] | 0.167 | [1.34] | 0.170 | [1.53] |
| No. health problems: 3 | 0.384 | [2.03] | 0.325 | [1.63] | 0.340 | [1.77] | 0.359 | [1.83] | 0.345 | [1.67] | 0.350 | [1.79] |
| No. health problems: 4+ | 0.169 | [0.55] | 0.069 | [0.22] | 0.060 | [0.19] | 0.111 | [0.33] | 0.048 | [0.12] | 0.066 | [0.24] |
| Lived in South East | -0.157 | [1.90] | -0.112 | [1.34] | -0.153 | [1.77] | -0.163 | [1.75] | -0.128 | [0.86] | -0.139 | [1.82] |
| Lived in London | -0.305 | [2.21] | -0.235 | [1.71] | -0.300 | [2.13] | -0.302 | [1.98] | -0.245 | [1.00] | -0.262 | [1.96] |
| Year $t-1: 1993$ | 0.123 | [1.04] | 0.114 | [0.93] | 0.141 | [1.21] | 0.108 | [0.90] | 0.099 | [0.77] | 0.102 | [0.82] |
| Year $t-1: 1994$ | 0.090 | [0.76] | 0.076 | [0.63] | 0.125 | [1.07] | 0.088 | [0.74] | 0.074 | [0.57] | 0.079 | [0.64] |
| Year $t-1: 1995$ | -0.022 | [0.19] | -0.053 | [0.46] | -0.006 | [0.05] | -0.062 | [0.54] | -0.085 | [0.62] | -0.079 | [0.65] |
| Year $t-1: 1996$ | -0.083 | [0.74] | -0.084 | [0.71] | -0.035 | [0.31] | -0.092 | [0.79] | -0.112 | [0.84] | -0.106 | [0.89] |
| Year $t-1: 1997$ | -0.199 | [1.73] | -0.208 | [1.74] | -0.151 | [1.29] | -0.209 | [1.76] | -0.229 | [1.75] | -0.224 | [1.88] |
| Year $t-1: 1998$ | -0.033 | [0.28] | -0.031 | [0.24] | 0.021 | [0.17] | -0.034 | [0.28] | -0.040 | [0.32] | -0.038 | [0.31] |
| Year $t-1$ : 1999 | -0.072 | [0.59] | -0.057 | [0.45] | -0.017 | [0.14] | -0.073 | [0.60] | -0.078 | [0.64] | -0.077 | [0.63] |
| Firm size > 100 | -0.405 | [4.89] | -0.301 | [3.24] | -0.417 | [4.35] | -0.420 | [3.42] | -0.340 | [1.13] | -0.364 | [6.14] |
| Skilled occupation | -0.242 | [2.82] | -0.145 | [1.65] | -0.248 | [2.58] | -0.248 | [2.03] | -0.174 | [0.63] | -0.196 | [3.04] |
| Constant | 2.521 | [5.12] | 1.825 | [2.30] | 1.952 | [3.32] | 2.433 | [3.43] | 1.949 | [1.07] | 2.094 | [6.61] |

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|  | Model 1 |  | Model 2 |  | Model 3 |  | Model 4 |  | Model 4 (S\&S) | Model 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | \|t| | Coeff. | \|t | Coeff. | \|t| | Coeff. | \|t | Coeff. | Coeff. | \|t |
| Low pay\|past high pay |  |  |  |  |  |  |  |  |  |  |  |
| Married | -0.110 | [1.89] | -0.080 | [1.38] | -0.090 | [1.51] | -0.108 | [1.81] |  | -0.098 | [2.02] |
| Age | -0.119 | [5.57] | -0.090 | [2.86] | -0.097 | [4.22] | -0.109 | [3.58] |  | -0.095 | [7.25] |
| Age-squared | 0.001 | [5.02] | 0.001 | [2.45] | 0.001 | [3.79] | 0.001 | [3.35] |  | 0.001 | [6.22] |
| Educ. qual.: other qualification | -0.256 | [2.83] | -0.218 | [2.51] | -0.253 | [2.70] | -0.267 | [2.71] |  | -0.251 | [3.22] |
| Educ. qual.: O-level(s) | -0.354 | [4.55] | -0.316 | [4.26] | -0.359 | [4.44] | -0.369 | [4.21] |  | -0.346 | [5.30] |
| Educ. qual.: A-level(s) | -0.438 | [5.07] | -0.398 | [4.97] | -0.464 | [5.25] | -0.480 | [4.95] |  | -0.455 | [6.19] |
| Educ. qual.: other higher degree | -0.541 | [6.94] | -0.491 | [6.67] | -0.541 | [6.52] | -0.558 | [5.94] |  | -0.528 | [8.22] |
| Educ. qual.: first degree or higher | -0.951 | [8.31] | -0.876 | [8.19] | -0.955 | [8.02] | -0.969 | [7.29] |  | -0.929 | [9.97] |
| No. health problems: 1 | 0.122 | [2.53] | 0.110 | [2.18] | 0.099 | [2.07] | 0.108 | [2.23] |  | 0.106 | [2.30] |
| No. health problems: 2 | 0.125 | [1.63] | 0.109 | [1.32] | 0.085 | [1.12] | 0.094 | [1.21] |  | 0.091 | [1.24] |
| No. health problems: 3 | 0.424 | [3.80] | 0.394 | [3.36] | 0.375 | [3.30] | 0.391 | [3.39] |  | 0.387 | [3.52] |
| No. health problems: 4+ | 0.559 | [2.91] | 0.511 | [2.50] | 0.460 | [2.44] | 0.516 | [2.65] |  | 0.496 | [2.74] |
| Lived in South East | -0.194 | [3.36] | -0.170 | [2.99] | -0.189 | [3.18] | -0.193 | [3.08] |  | -0.182 | [3.53] |
| Lived in London | -0.245 | [2.72] | -0.209 | [2.40] | -0.241 | [2.60] | -0.239 | [2.42] |  | -0.220 | [2.78] |
| Year $t-1: 1993$ | -0.023 | [0.28] | -0.019 | [0.23] | 0.019 | [0.25] | -0.006 | [0.08] |  | -0.010 | [0.12] |
| Year $t-1: 1994$ | 0.044 | [0.57] | 0.040 | [0.51] | 0.089 | [1.17] | 0.067 | [0.86] |  | 0.061 | [0.77] |
| Year $t-1: 1995$ | -0.004 | [0.05] | -0.017 | [0.22] | 0.026 | [0.34] | -0.008 | [0.11] |  | -0.017 | [0.20] |
| Year $t-1: 1996$ | -0.086 | [1.06] | -0.075 | [0.85] | -0.030 | [0.39] | -0.063 | [0.78] |  | -0.071 | [0.86] |
| Year $t-1: 1997$ | -0.099 | [1.25] | -0.092 | [1.08] | -0.039 | [0.50] | -0.070 | [0.89] |  | -0.077 | [0.94] |
| Year $t-1: 1998$ | -0.091 | [1.15] | -0.085 | [1.01] | -0.047 | [0.61] | -0.079 | [1.01] |  | -0.083 | [1.01] |
| Year $t-1: 1999$ | -0.048 | [0.59] | -0.034 | [0.38] | 0.017 | [0.22] | -0.018 | [0.23] |  | -0.020 | [0.26] |
| Firm size > 100 | -0.277 | [4.80] | -0.224 | [4.32] | -0.283 | [4.44] | -0.282 | [3.66] |  | -0.251 | [5.68] |
| Skilled occupation | -0.398 | [6.36] | -0.349 | [5.91] | -0.408 | [6.01] | -0.412 | [5.15] |  | -0.383 | [7.63] |
| Constant | 1.941 | [3.68] | 1.099 | [1.44] | 1.437 | [2.37] | 1.750 | [2.11] |  | 1.358 | [5.43] |

Notes. Models and estimation methods explained in main text. $|t|$ is the absolute asymptotic $t$-ratio. See Tables 5-8 in main text for estimated cross-equation correlations, log-likelihoods, and marginal effects. Model 4 (S\&S) uses the model specification of Stewart and Swaffield (1999, equation 8): see main text for discussion of difference between this model and Model 4 . The estimated corr(low pay at $t$, low pay at $t-1$ ) for Model 4 (S\&S) was -0.063 [ $|t|=0.08]$, and the loglikelihood value was $-5,677$. The omitted categories for categorical variables are: not married, no educational qualifications, no health problems, lived outside London and South East, year $t-1$ : 1992, firm size $\leq 100$, non-skilled occupation.

