MEASURES OF EFFECTIVE LITERACY: A THEORETICAL NOTE

Diganta Mukherjee

Economic Research Unit, Indian Statistical Institute, Kolkata, India
and
Institute for Social and Economic Research, University of Essex, Colchester, UK

and

Manash Ranjan Gupta

Economic Research Unit, Indian Statistical Institute, Kolkata, India

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Institute for Social and Economic Research
University of Essex
Wivenhoe Park
Colchester
Essex
CO4 3SQ UK
Telephone: +44 (0) 1206 872957
Fax: +44 (0) 1206 873151
E-mail: iser@essex.ac.uk
Website: http://www.iser.essex.ac.uk

ABSTRACT

This paper discusses the measurement issue of intra-household externality of literacy. Extending the work of **Basu - Foster (1998)**, we present axiomatic characterisations of several effective literacy measures that endogenizes the extent of externality effect on proximate illiterates. The determinants of externality considered are several attributes of the members of the household such as age, sex, level of education etc.

NON-TECHNICAL SUMMARY

This note wants to pass on the message that, while measuring effective literacy, one should not only look at the literacy status of the members of the household but also look at the gender, age ranking and educational qualification of the literate members. Some extensions of **Basu - Foster (1998)** measure have been developed in this paper; and all of them can be used for the measurement of effective literacy rate in a country like India using either Census or NSS data. **Basu - Foster (1998)** measure with externality parameter set to 0, the traditional literacy rate, always appears as a special case of the class of measures suggested and characterised here.

In **Basu - Foster (1998)**, the externality parameter is exogenous to the formulation and its value is arbitrarily chosen. So the literacy ranking of different regions (districts, states, countries etc.) may be different for different arbitrarily chosen values of it. So choice of the parameter is an important task in measuring the extent of effective literacy. We have suggested some relevant individual characteristics that may impact on the externality effect of a literate person on an illiterate member of the family. In particular, we have characterised certain classes of effective literacy measures that are generalisations of those proposed by **Basu and Foster (1998)** and **Subramanian (2001)** and other general class of measures.

These measures are useful in testing for the effect of the characteristics under discussion on effective literacy and hence for devising appropriate policy in this direction. For example, if education level sensitivity is high, then higher education programme plays a significant role even in basic literacy improvement, in the proximate sense. Similarly, female literacy campaign should be more important for the purpose of generating higher externality if gender sensitivity is significant.

Although our proposed measures are very general in nature and allows many alternative possibilities, the final choice of a measure by a practitioner will depend on her subjective value judgement about the acceptability of alternative axioms set out in this paper and specific parametrisation thereof.

1 Introduction

The literature on literacy says little about intra-household externalities of literacy and education. One might well expect that literacy may have spillover benefits within a household. **Green et. al. (1985)** and **Dreze and Saran (1995)** note how the advantages of literacy can spread to others in the household. But, traditionally the literacy rate is defined as the ratio of number of adult literates to the total number of adults. **Basu and Foster (1998)** (henceforward **BF**) have suggested an alternative measure of effective literacy in which the distinction is made between proximate illiterates (proximate illiterates are the illiterate members of the household with at least one literate member) and isolated illiterates (isolated illiterates come from a household with no literate member). A proximate illiterate is assumed to be α-equivalent to a literate member of the household with $0 < \alpha < 1$. The significance of this alternative measure in designing literacy education programme is also documented by **Basu et al (1999)**. Some theoretical modifications and comments on this measure are also available in **Chakravarty and Majumder (2001), Subramanian (2001), Mishra (2001), Mitra (2001), Valenti (2001)** and **Dutta (2002)**.

However, in all these works, the external effect of literacy on the proximate literates is exogenous and is independent of the set of characteristics of the literate members of the household. In reality, the magnitude of the external effect of literacy on the proximate illiterate depends on the various characteristics of the literate members in the household. Age of the literate member is one such determinant because an elder literate member usually exerts a greater external effect on the illiterate than an younger literate member. A female literate member should have a larger effect than a male literate member because the females play a more active role in the domestic activities of the household. **BF** partially takes care of this point in footnote 5, section 5 and 6 (last paragraph) of their paper. Also, the gender of the recipient illiterate might also matter in the determination of this externality. The literate member who stays in the home should have a greater effect than a migrant literate member. Also the level of education of the literate member is an important determinant of α as an illiterate member has a greater respect for the literate member with higher educational qualification.

In this note we extend the **BF** measure of effective literacy in the light of endogenizing α in terms of the above mentioned characteristics of the literate members of the household. We propose a set of axioms that the ideal measure of literacy should satisfy; and which postulates the effect of the characteristics like age, sex and level of education of literate members on a proximate illiterate. Our suggested extensions of **BF** measure satisfy these axioms; and are reduced to **BF** measure itself in the special case when insensitivity of the external effect to the relevant characteristics is assumed.

The paper is organised as follows. In section 2, the axioms are described and preliminary observations are made. The formulae and characterisation results for a household level effective literacy measure are presented in section 3. We briefly indicate how to aggregate the household level effective literacy measures to arrive at a society wide measure of effective literacy in section 4. In this section, brief comments are made regarding the contrast between our work and the **BF** formulation. We provide an empirical illustration of our methodology using data from Assam, a state of India, in section 5. Conclusions are drawn in section 6.

2 Preliminaries

We will now describe the relevant **set of variables** that we deem are important characteristics of a literate person with respect to the externality effect that she exerts on an illiterate member of the same household. For any person i, $i = 1, ..., N$, in an N-person household, we define the variables in the following way.

(i) Level of education, $e_i \in E = [0, k]$ or $\{0, 1, 2, ..., k\}$ for some $k \geq 1$, integer. k is considered to be the highest attainable level of education. That is, we consider education to be measured on a continuous scale or may allow several discrete levels of education. For technical simplicity, we will do the analysis for the continuum situation, but the results and their demonstration can also be adapted to the discrete case.

(ii) Sex, $s_i \in \{m, f\}$; $m =$ male and $f =$ female.

(iii) Age, $y_i \in R_+$.

Thus, for our purpose, person i is now completely described by the characteristic vector $c_i = (e_i, y_i, s_i)$. Define the set $C^N = (E \times \{m, f\} \times R_+)^N$ for positive integer N (household size), as the set of all possible characteristic vectors for a given household size. Also define $C = \bigcup_{N \geq 1} C^N$ as the union of C^N over all positive integers N. Therefore we can define the measure of household level literacy as a function P defined over all possible sets of characteristic vectors for each individual in the household to a real number between 0 and 1. More precisely,

$P : C \rightarrow [0, 1]$.

Whatever follows is actually much more general, as any set of individual characteristics that includes the level of education may be considered as the relevant characteristic set. Our use of age and sex is just one of many possibilities that may be considered relevant. Other possibilities may include the pattern of time use by the literate members of the family. Analysis with just one additional characteristic is also feasible but we have considered two to allow for interactions between characteristics which are often considered relevant.

To facilitate subsequent discussion, denote any set of characteristic vectors $\{c_1, c_2, ..., c_N\}$ of a household of N members by Ω_N. The set of vectors without the i^{th} element $\{c_1, ..., c_{i-1}, c_{i+1}, ..., c_N\}$ by Ω_N^{-i}. We now introduce the set of basic assumptions on our literacy measure P.

A1 (Additivity): The aggregate literacy status of the household is the average of each person's literacy status in the household.

$$P(\Omega_N) = \frac{1}{N}\sum_{i=1}^{N} p(c_i; c_j, j = 1, ..., N, j \neq i) = \frac{1}{N}\sum_{i=1}^{N} p(c_i; \Omega_N^{-i})$$

where $p(c_i; \Omega_N^{-i}) : C \rightarrow [0, 1]$ is the identical literacy indicator function for member i of the household, with characteristic vector c_i and Ω_N^{-i} is the set of characteristics of all other members in the household.

Thus, assumption (A1) provides us with a convenient breakdown of the general effective literacy measure of the household, $P(.)$, in terms of the effective literacy status of each individual in the household, $p(.)$, and postulates that $P(.)$ is a simple average of the $p(.)$ values of all the members of the household. This ensures symmetry among the members of the household with respect to any other individual attributes which are not included in Ω_N. The individual $p(.)$ values in turn depend on the relevant characteristics of the other members of the family. This is the simple formulation of externality that we use in this paper.

A2 (Maximality): The maximum value (= 1) of literacy status for the individual i is attained when $e_i > 0$. That is,

$p((e_i, y_i, s_i); .) = 1$ if and only if $e_i > 0$.

The minimum value (= 0) is attained when there are no literate members in the household. That is,

$p((e_i, s_i, y_i); \{(e_j, s_j, y_j), j = 1, ..., N, j \neq i\}) = 0$ if and only if $e_j = 0$ for all $j = 1, ..., N$.

For all other cases, $0 < p(.) < 1$.

Thus, we are not allowing an illiterate to have equal literacy status as a literate person under any circumstances. Also, the externality value is assumed to be strictly positive.

A3 (Monotone externality of literates): The effective literacy status of any person is determined by his own characteristics and that of the literate members (if any) only and is non-decreasing if more literates are introduced into the household.

That is, we can write $P(.)$ as

$$P(\Omega_N) = \frac{1}{N}\sum_{i=1}^{N} p(c_i; L_N), \tag{1}$$

where $L_N = \{(e_j, s_j, y_j) \in \Omega_N | e_j > 0\}$ = the set of characteristics of the literate members in the household. Denote the size of this set by $l = |L_N|$.

(A3) postulates that each proximate illiterate's effective literacy status, $p(c_i; L_N)$, (weakly) improves if the number of literate members in the household increases.

(A1) - (A3) is our basic set of axioms. Note that (1) is the most general form of effective literacy measure that we consider.

We will now introduce the additional set of axioms, which are dependent on alternative judgements about the externality effect we are trying to model.

A4 (Education level sensitivity) : For any $(e_j, s_j, y_j) \in L_N$

$$\frac{\partial p}{\partial e_j}((0, s_i, y_i); L_N) > 0.$$

That is, the externality effect of any literate member j on the illiterate member i in the household, given other things, is increasing in the level of education of person j. Note that if $L_N = \phi$, the empty set, this axiom is vacuously satisfied.

This assumption postulates that the externality benefits will be larger, higher the education level of a literate member in the household. A more educated person may be able to exert a larger beneficial influence on an illiterate member of the same household. We define neutrality to education level of the effective literacy measure by the following axiom.

A4' (Education level insensitivity) : The externality effect of any literate member j on the illiterate member i in the household is independent of the level of education of j.

A5 (Gender sensitivity) : There can be several variants of such axiom. We present two of those.

(i) For any $c_j = (e, m, y) \in L_N$ and $c_k = (e, f, y) \in L_N$ and $c_i = (0, s_i, y_i)$,

$$p(c_i; L_N^{-j}) \geq p(c_i; L_N^{-k}).$$

That is, female literates has higher externality on the literacy status of illiterates in the household than the male literates. As females spend more time in household activities, this structure is justified.

(ii) : For any $c_j = (e, m, y) \in L_N$ and $c_k = (e, f, y) \in L_N$ and any $c_i = (0, m, y_i) \in \Omega_N$,

$$p(c_i; L_N^{-j}) \leq p(c_i; L_N^{-k})$$

The inequality is reversed if $s_i = f$.

Given other factors, female literates has a higher externality on female illiterates than on male illiterates. Similarly for a male literate.

Basu, Narayan and Ravallion (2002) finds empirical support for this axiom. This variant of the axiom implies that the characteristics of the recipient are also important for measuring the extent of externality that a literate may exert on other members of his/her family.

Again we can define **A5' (Gender insensitivity)** in the same manner as in **(A4')**. This axiom postulates that the externality is independent of gender considerations.

A6 (Age ordering sensitivity) : For an illiterate person i with $c_i = (0, s_i, y_i)$ and any $c_j = (e_j, m, y_i + k) \in L_N$ with $k > 0$,

$$p(c_i; L_N) \geq p(c_i; L_N^{-j} \bigcup \{(e_j, s_j, y_i - h)\}),$$

where $h > 0$.

That is, externality effect is more effective on younger persons than on elders. (Justifying adult education programmes.) This axiom is particularly relevant for measuring effective literacy for households in a traditional society where the intra-household power hierarchy is very much chronologically determined.

Again, we can define **A6' (Age ordering insensitivity)** in an analogous manner. The externality effect on an illiterate person of a literate is independent of their birth order.

A7 (Insensitivity to additional literates with lower externality) : For $(e_1, s_1, y_1), (e_2, s_2, y_2) \in L_N$

$$p(c_i; L_N) = p(c_i; L_N^{-2})$$

for illiterate person i if $p(c_i; \{(e_1, s_1, y_1)\}) \geq p(c_i; \{(e_2, s_2, y_2)\})$.

This axiom says that multiplicity of identical literates is ineffective with respect to externality effect on illiterates. Also, additional literates do not exert any externality on the illiterate members if their extent of externality is of lower order. This is a sort of crowding out argument that is also embodied in the **BF** externality axiom. **Subramanian (2001)** argues against it and proposes a measure of effective literacy that takes into account the proportion of literates in the household (see below). We will explore the consequences of the presence and absence of this axiom explicitly in our subsequent results.

Note that, if (A4'), (A5') and (A6') are always satisfied, then all literates are treated as identical - similar to that in **BF** and **Subramanian (2001)** (See propositions 1 and 3 below).

Before we proceed to discuss our results in the next section, let us recall the salient measures of household level proximate literacy proposed in the literature. The first two are due to **BF** and are given by (using our notation)

$$P_\alpha = \frac{l + (N - l)\alpha}{N}$$

and their gender sensitive prescription

$$\begin{aligned} P_{\alpha_m, \alpha_f} &= \frac{l + \alpha_m (N - l)}{N} \quad \text{if there are no female literates in the household,} \\ &= \frac{l + \alpha_f (N - l)}{N} \quad \text{otherwise.} \end{aligned}$$

Here $0 < \alpha < 1$, and $0 < \alpha_m < \alpha_f < 1$.

The other important measure of effective literacy that we want to relate our findings to is by **Subramanian (2001)**, defined as

$$P_S = \frac{l + (N - l)\frac{l}{N}}{N}.$$

For a discussion of the properties of this measure, op. cit.

3 The Results

In the following propositions, we identify precise sets of conditions that characterise alternative forms of the effective literacy measure.

Proposition 1 : The form of the effective literacy measure becomes identical to the **BF** P_α formulation if and only if it satisfies (A1) - (A3), (A4'), (A5'), (A6') and (A7).

Proof : For any proof, we assume that $l > 0$ as otherwise the proof becomes trivial as $p(.) = 0$. In that case the household has no literates and the question of proximate literacy does not arise.

Under (A1-3), the form of the individual effective literacy measure becomes as given by (1). Now, if we impose (A5') and (A6') then the measure becomes independent of the sex and age information of the individuals concerned. So, we can redefine the effective literacy measure for illiterate person i, $p((e_i, s_i, y_i), L_N)$, retaining only the education level variable as

$$p^e(0; e_1, e_2, ..., e_l) \tag{2}$$

where l is the number of literates in the household. Now, if we invoke (A4') then the function will become independent of the level of education of each literate and only the information that they are literate will be important. So, effectively, the function p^e can now be redefined as

$$p'(0; 1, 1, ..., 1).$$

Now we finally use (A7) and, as a consequence, the function becomes independent of the number of literates, so we finally get the following form for the effective literacy measure,

$$p"(0, 1) = \alpha, \tag{3}$$

say, where $0 < \alpha < 1$ by (A2). Hence, using (1) to sum over all individuals in the household, the effective literacy status of the household finally becomes,

$$P(\Omega_N) = \frac{l + (N - l)\alpha}{N},$$

which is just the measure P_α. It is easy to verify that P_α satisfies (A1) - (A3), (A4'), (A5'), (A6') and (A7). ■

This is an intuitive and the simplest formulation of the externality effect of literates on a proximate illiterate. We will now explore alternative possibilities that allow for sensitivities with respect to the relevant characteristics in the effective literacy function. In the following proposition, we capture the effect of both sex and age ordering simultaneously and demonstrate the resulting form of the effective literacy measure. To do this in a very general manner, we consider the axioms (A5(ii)) and (A5(ii)). That is, we allow the literacy externality effects to depend on the sex of both the literate and the illiterate member. The age dependence is captured by (A6). To make the subsequent discussion precise, let us define the following.

Definition 1: For any illiterate person i with characteristic $c_i \in \Omega_N$, the set L_N is said to be of type $(f, 1)$ with respect to person i if there exists a $c_j \in L_N$ with $s_j = f$ and $y_j > y_i$. We say that L_N is of type $(f, -1)$ with respect to person i if there exists a $c_j \in L_N$ with $s_j = f$ and $y_j \leq y_i$. analogously define types $(m, 1)$ and $(m, -1)$.

When we do not allow for multiplicity in the presence of literates to be beneficial, we have the following general result.

Proposition 2 : When the effective literacy measure satisfies (A1) - (A3), (A4') and (A7), we have the following equivalences.

(i) The function $p(.)$ satisfies, in addition to the above, axioms (A5(i)) and (A6) if and only if it is defined by the following.

$$
\begin{aligned}
p(c_i; L_N) &= 1 \quad \text{if } e_i > 0, \\
&= p*(f,1) \quad \text{if } e_i = 0,\ L_N \text{ is of type } (f,1) \\
&= p*(f,-1) \quad \text{if } e_i = 0,\ L_N \text{ is not type } (f,1), \text{ not type } (m,1), \text{ but type } (f,-1) \\
&= p*(m,1) \quad \text{if } e_i = 0,\ L_N \text{ is not type } (f,1), \text{ not type } (f,-1), \text{ but type } (m,1) \\
&= max\{p*(m,1), p*(f,-1)\} \quad \text{if } e_i = 0,\ L_N \text{ is not type } (f,1), \\
&\quad\ \text{but type } (m,1) \text{ and type } (f,-1) \\
&= p*(m,-1) \quad \text{if } e_i = 0,\ L_N \text{ is not type } (f,1), \text{ not type } (f,-1), \text{ not type } (m,1), \\
&\quad\ \text{but type } (m,-1),
\end{aligned}
$$

where $0 < p*(m,-1) < p*(f,-1), p*(m,1) < p*(f,1) < 1$ are real constants.

(ii) The function $p(.)$ satisfies, in addition to the above, axioms (A5(i)) and (A6') if and only if the form of the effective literacy measure is identical to the **BF** gender sensitive P_{α_m,α_f} formulation.

(iii) The function $p(.)$ satisfies, in addition to the above, axioms (A5(ii)) and (A6) if and only if it is defined by the following.

$$p(c_i; L_N) = 1 \quad \text{if } e_i > 0$$

Otherwise, if $e_i = 0$, we consider two alternative possibilities. First, suppose that $s_i = f$. Then, $p(.)$ is defined as follows.

$$\begin{aligned}
p(c_i; L_N) &= p_f * (f,1) \quad \text{if } L_N \text{ is of type } (f,1) \\
&= p_f * (f,-1) \quad \text{if } L_N \text{ is not type } (f,1), \text{ not type } (m,1), \text{ but type } (f,-1) \\
&= p_f * (m,1) \quad \text{if } L_N \text{ is not type } (f,1), \text{ not type } (f,-1), \text{ but type } (m,1) \\
&= max\{p_f * (m,1), p_f * (f,-1)\} \quad \text{if } L_N \text{ is not type } (f,1), \\
&\quad\ \text{but type } (m,1) \text{ and type } (f,-1) \\
&= p_f * (m,-1) \quad \text{if } L_N \text{ is not type } (f,1), \text{ not type } (f,-1), \text{ not type } (m,1), \\
&\quad\ \text{but type } (m,-1),
\end{aligned}$$

where $0 < p_f * (m,-1) < p_f * (f,-1), p_f * (m,1) < p_f * (f,1) < 1$ are real constants. For $s_i = m$, we have the following.

$$\begin{aligned}
p(c_i; L_N) &= p_m * (m,1) \quad \text{if } L_N \text{ is of type } (m,1) \\
&= p_m * (m,-1) \quad \text{if } L_N \text{ is not type } (m,1), \text{ not type } (f,1), \text{ but type } (m,-1) \\
&= p_m * (f,1) \quad \text{if } L_N \text{ is not type } (m,1), \text{ not type } (m,-1), \text{ but type } (f,1) \\
&= max\{p_m * (f,1), p_m * (m,-1)\} \quad \text{if } L_N \text{ is not type } (m,1), \\
&\quad\ \text{but type } (f,1) \text{ and type } (m,-1) \\
&= p_m * (f,-1) \quad \text{if } L_N \text{ is not type } (m,1), \text{ not type } (m,-1), \text{ not type } (f,1), \\
&\quad\ \text{but type } (f,-1),
\end{aligned}$$

where $0 < p_m * (f,-1) < p_m * (m,-1), p_m * (f,1) < p_m * (m,1) < 1$ are real constants.

(iv) The function $p(.)$ satisfies, in addition to the above, axioms (A5(ii)) and (A6') if and only if it is defined by the following.

$$\begin{aligned}
p(c_i; L_N) &= 1 \quad \text{if } e_i > 0 \\
&= p_f * (f) \quad \text{if } e_i = 0,\ s_i = f,\ L_N \text{ is of type } (f,1) \text{ or type } (f,-1) \\
&= p_f * (m) \quad \text{if } e_i = 0,\ s_i = f,\ L_N \text{ is not type } (f,1), \text{ not type } (f,-1) \\
&= p_m * (m) \quad \text{if } e_i = 0,\ s_i = m,\ L_N \text{ is of type } (m,1) \text{ or type } (m,-1) \\
&= p_m * (f) \quad \text{if } e_i = 0,\ s_i = m,\ L_N \text{ is not type } (m,1), \text{ not type } (m,-1),
\end{aligned}$$

where $0 < p_f * (m) < p_f * (f) < 1$ and $0 < p_m * (f) < p_m * (m) < 1$ are real constants.

Proof : (i) Due to (A1)-(A3) and (A4'), the level of education becomes unimportant for the members of L_N. Then one can simplify and redefine the individual effective literacy measure as

$$p^{sa}((s_i, y_i); (s_1, y_1), ..., (s_l, y_l)). \tag{4}$$

We now invoke (A6) on (4). Now, given (A6), we need only consider whether, for each literate j, $y_j > y_i$ or not. We denote this event by a variable I_J in the set of arguments of p' that takes a value of "1" when the condition hold and in case of the converse event it equals "-1". As before, we redefine p^{sa} and write it as

$$p'(s_i; (s_1, I_1), ..., (s_l, I_l)) \tag{5}$$

Now given (A5(i)) and (A6), we need to consider four alternative possibilities of externality effects, namely those due to the presence of an older female literate (type $(f,1)$) or older male literate (type $(m,1)$) or younger female literate (type $(f,-1)$) or, finally, a younger male literate (type $(m,-1)$) in L_N. Note that, due to the conjunction of (A5(i)) and (A6), an older female can exert the highest influence on an illiterate household-member. The younger male literate has the least influence and older males and younger females are in the intermediate zone. A comparison between the externality effects of a younger literate female and that of an older literate male is ambiguous given our assumptions.

In our notation, if $\{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\} \ni (f,1)$ and given that due to (A5(i)) and (A6), this configuration has the highest externality, then due to (A7), $p'(s_i, (s_1, I_1), ..., (s_l, I_l)) =$

$p'(s_i; (f,1)) = p*(f,1)$ say, as this is a constant independent of s_i.

Otherwise, if $\{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\} \not\ni (f,1)$, then we check for the presence of $(m,1)$ or $(f,-1)$, the two intermediate effects in terms of (A5(i)) and (A6). This would give rise to the following three possibilities and corresponding externality effect parameters due to (A7) again.

(a) $p'(s_i; (s_1, I_1), ..., (s_l, I_l)) = p'(s_i; (f,-1)) = p*(f,-1)$ if $(f,-1) \in \{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\}$ and $(m,1) \notin \{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\}$. (b) $p'(s_i; (s_1, I_1), ..., (s_l, I_l)) = p'(s_i; (m,1)) = p*(m,1)$ if $(m,1) \in \{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\}$ and $(f,-1) \notin \{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\}$. And, (c) $p'(s_i; (s_1, I_1), ..., (s_l, I_l)) = p'(s_i; (f,-1), (m,1)) = max\{p*(f,-1), p*(m,1)\}$ if $(f,-1) \in \{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\}$ and $(m,1) \in \{(s_i, y_i); (s_1, y_1), ..., (s_l, y_l)\}$.

Finally we have the residual case $p'(s_i; (m,-1)) = p*(m,-1)$.

It is easy to verify the reverse implication.

(ii) The proof of this part can be deduced from that of part (i). In the presence of (A6'), the two externality effects $p*(f,1)$ and $p*(f,-1)$ will now be equal. So we redefine them as $p*(f,1) = p*(f,-1) = \alpha_f$, say. Similarly, we have the situation $p*(m,1) = p*(m,-1) = \alpha_m$. Due to (A5(i)), $\alpha_f > \alpha_m$. Note that, the presence or otherwise of a female literate in the household now affects all the illiterates in the same manner. So, using (1) to sum over all i, we finally get the effective literacy measure in this case as

$$\begin{aligned} P(\Omega_N) &= \frac{l + \alpha_m(N-l)}{N} \quad \text{if there are no female literates in the household,} \\ &= \frac{l + \alpha_f(N-l)}{N} \quad \text{otherwise.} \end{aligned}$$

This is evidently same as the Basu-Foster gender sensitive prescription P_{α_m, α_f}. It is easy to check that P_{α_m, α_f} satisfies (A1) - (A3), (A4'), (A5(i)), (A6) and (A7). Thus we have the desired result.

(iii) We again proceed as for the proof of part (i). Given (A1)-(A3), (A4') and (A6), $p(.)$ can be redefined as in (5). Due to (A5(ii)), now the recipient's gender also matter in the determination of the extent of externality. So, we analyse the two possibilities separately.

When $s_i = m$, we look at

$$p'(m; (s_1, I_1), ..., (s_l, I_l)). \tag{6}$$

Now we proceed with (6) in exactly the same manner as with (5) in part (i). Again, due to the conjunction of (A5(ii)) and (A6), we now see that type $(m,1)$ now has the highest externality effect and type $(f,-1)$ the lowest with $(m,-1)$ and $(f,1)$ being the two intermediates. Thus, we finally arrive at the parameters $p'(m;(m,1)) = p_m*(m,1)$, $p'(m;(m,-1)) = p_m*(m,-1)$, $p'(m;(f,1)) = p_m*(f,1)$, $p'(m;(f,1),(m,-1)) = max\{p_m*(f,1), p_m*(m,-1)\}$ and $p'(m;(f,-1)) = p_m*(f,-1)$.

The case $s_i = f$ is dealt with similarly and we omit the details. Again, reverse implication is easy to check.

(iv) Again, this part is reduced from part (iii), in the presence of (A6'), in exactly the same way as the reduction of part (ii) from part (i). The parameters $p_x*(y,1)$ and $p_x*(y,-1)$ are equalised and denoted by $p_x*(y)$; $x,y = m,f$.

The normalisation of all the relevant constants between 0 and 1 is due to (A2). ■

In each of the cases analysed above, the measures will be completely characterised by a well defined set of parameters. The actual form will depend on the presence of certain combinations in the set of literates and the sex of the illiterate member. The parameters can be ordered unambiguously if we impose more structure on comparability. For example, being able to rank the externality effect of $(f,-1)$ and $(m,1)$ from a female illiterate's point of view. That is, whether we can rank $p*_f(f,-1)$ and $p*_f(m,1)$.

Below, we illustrate the above proposition by an example.

Example 1: (a) This corresponds to the most general situation, as depicted in part (iii) of Proposition 2. Consider the situation where $p_f^*(f,1) = p_m^*(m,1) = \alpha_f$, $p_f^*(f,-1) = p_m^*(m,-1) = \beta\alpha_f$, $p_f^*(m,1) = p_m^*(f,1) = \alpha_m$ and $p_f^*(m,-1) = p_m^*(f,-1) = \beta\alpha_m$.

(b) Now, to illustrate part (i) of Proposition 2, consider $p*(f,1) = \alpha_f$, $p*(m,1) = \alpha_m$, $p*(f,-1) = \beta\alpha_f$ and $p*(m,-1) = \beta\alpha_m$.

(c) Finally, illustrating part (iv) of Proposition 2, suppose $p_f*(f) = p_m*(m) = \alpha_f$, $p_f*(m) = p_m*(f) = \alpha_m$.

Here, $0 < \beta < 1$ and $0 < \alpha_m < \alpha_f < 1$.

To have a precise numerical illustration, let us take $\alpha_f = 0.6$, $\alpha_m = 0.4$ and $\beta = 0.8$, then we have

(a) $p_f^*(f,1) = p_m^*(m,1) = 0.6$, $p_f^*(f,-1) = p_m^*(m,-1) = 0.48$, $p_f^*(m,1) = p_m^*(f,1) = 0.4$ and $p_f^*(m,-1) = p_m^*(f,-1) = 0.32$.

(b) $p*(f,1) = 0.6$, $p*(m,1) = 0.4$, $p*(f,-1) = 0.48$ and $p*(m,-1) = 0.32$.

(c) $p_f*(f) = p_m*(m) = 0.6f$, $p_f*(m) = p_m*(f) = 0.4$.

We will now look at alternative forms of the effective literacy measure that arises when we consider effect of the education level of the literates to be important in determining the effective literacy level of the proximate illiterates. To keep the discussion tractable, we will assume that gender and age considerations are not relevant in the context of the following result. In what follows, we will sometimes allow for presence of multiple literates in the household to be beneficial for the illiterate members. This is more in the spirit of **Subramanian (2001)** who supports the beneficial impact of having a higher number of literates in the household.

Proposition 3 : When the effective literacy measure satisfies (A1) - (A3), (A5') and (A6'), we have the following equivalences.

(i) The function $p(.)$ satisfies, in addition to the above, axiom (A4) if and only if the form of the effective literacy measure becomes

$$P_e(\Omega_N) = \frac{l + (N-l)g_1(e_1, e_2, ..., e_l)}{N}, \tag{7}$$

for a suitably defined function $g_1 : \bigcup_{l \geq 1}[0,k]^l \rightarrow [0,1]$ increasing in each of its arguments.

(ii) The function $p(.)$ satisfies, in addition to the above, axioms (A4) and (A7) if and only if the form of the effective literacy measure becomes

$$P_{me}(\Omega_N) = \frac{l + (N-l)g_2(e_{Max})}{N}, \tag{8}$$

where $e_{Max} = Max_{1 \leq j \leq l}\ e_j$ and for a suitably defined increasing function $g_2 : [0,k] \rightarrow [0,1]$.

(iii) The function $p(.)$ satisfies, in addition to the above, axiom (A4') if and only if the form of the effective literacy measure becomes a generalisation of the **Subramanian (2001)** measure P_S.[2]

Proof : Given (A1) - (A3), (A5') and (A6'), the individual externality effect for each illiterate takes the form (2), as already shown in Proposition 1. Now, this effect is identical for each illiterate, so aggregating over all members of the household, the form of (1) becomes

$$P(\Omega_N) = \frac{1}{N}(l + (N-l)p^e(0; e_1, e_2, ..., e_l)). \tag{9}$$

(i) Equation (9) can be rewritten as (7) for a function g_1 as defined above. The range of the

[2]**Valenti (2001)** characterises similar functional forms. **Dutta (2002)** reproduces these results.

function is a consequence of (A2). The increasingness of g_1 in each of its arguments follow from (A4). Reverse implication is easy to check. Hence we have part (i).

(ii) Given (A7), the externality function as given in (2) only depends on the highest education level among the literate members of the household, or e_{Max} as defined above. Thus, in this case, (9) becomes

$$P(\Omega_N) = \frac{1}{N}(l + (N-l)p^e(0; e_{Max})).$$

Now this can be redefined as (8) for a function g_2 as defined above. The range is due to (A2). The increasingness of g_2 follows from (A4). Reverse implication is easy to check. Hence we have part (ii).

(iii) Again, following the proof of Proposition 1, given (A4'), $p^e(.)$ can be redefined as $p'(0; 1, 1, ..., 1)$. Now, the only variable part in the arguments' set is l, the number of "1"s. So, one can rewrite this as $f(l) : Z_+ \to [0, 1]$. f is increasing due to (A3). Then (9) can be rewritten again as

$$P(\Omega_N) = \frac{1}{N}(l + (N-l)f(l)). \tag{10}$$

If we consider the particular case $f(l) = \frac{l}{N}$, the traditional measure of literacy, (10) simplifies to the **Subramanian (2001)** measure,

$$P_S = \frac{l + (N-l)\frac{l}{N}}{N}.$$

Once again, the reverse implication is easy to verify. ■

Proposition 3 explicitly brings into focus the impact of the presence or otherwise of (A7) in the set of axioms. Comparing part (i) and (ii), it is easy to see that, when (A7) is assumed, the effective literacy level of a proximate illiterate depends only on the highest level of education available in the household. Other literates exert no influence on the illiterates whatsoever. While, they do have nontrivial impact in this respect, in the absence of (A7). One can consider several interesting special cases of (7) and (8). Below, we illustrate with two of these.

Example 2 : (a) Suppose we postulate $\frac{\partial g_1}{\partial e_j} = \beta$, say. This can be thought of as a special case of (A4) when the marginal contribution of education is constant across educational levels and across individuals. Then, by repeated integration[3], one obtains

$$g_1(.) = \beta \sum_{j=1}^{l} e_j.$$

Due to (A2), $0 < \beta \sum_{j=1}^{l} e_j < 1 \ \forall e_1, e_2, ..., e_l$. So, we get $0 < \beta l k < 1$. So, one may take $\beta = \frac{1}{kN}$ to get

$$P_e(\Omega_N) = \frac{l}{N} + \frac{(N-l)}{N} \frac{\sum_{j=1}^{l} e_j}{Nk}. \tag{7.1}$$

(b) Similarly, if we take $g_2' = \frac{1}{N}$ and simplify analogously, we may arrive at a special case of (8) given by

$$P_{me}(\Omega_N) = \frac{l}{N} + \frac{(N-l)}{N} \frac{e_{Max}}{N}. \tag{8.1}$$

Such examples are important in the sense that they are simply parametrised, easily computable and hence very useful for policy purposes. These would be amenable to empirical exercises using real life data, for measuring the impact of literacy programmes and evaluating related policy.

4 Discussion

4.1 Aggregate Social Effective Literacy

So far we have only discussed the effective literacy status of a household in isolation. But, to be an useful tool for empirical purposes, the measures of effective literacy should be extendable to a society aggregate. We will now take up this issue. Let us consider a society (some well defined unit such as village, town, district, county, state, nation etc.) of M households, where M is any positive integer. Let each individual household be described by the set of individual characteristic vectors, using our notation,

$$\Omega_{N_i}^i = \{c_1^i, c_2^i, ..., c_{N_i}^i\}, i = 1, ..., M.$$

Define the society wide set of such characteristics as

$$A^M = \{\Omega_{N_1}^1, \Omega_{N_2}^2, ..., \Omega_{N_M}^M\}.$$

[3]This demonstration is similar to the proof of Lemma 3 in **Mukherjee (2001)**. We do not discuss it in detail here.

Then, one can define the *aggregate social effective literacy measure* by $P^A(A^M) : C \rightarrow [0,1]$. One can now once again appeal to *decomposability* properties of such measures and define this measure P^A as an average of the effective literacy status of each household in the society. (For discussion on such issues, see for example, the pioneering work of **Foster, Greer and Thorbecke (1984)** and **Chakraborty, Pattanaik and Xu (2002)** for a recent treatment. Given these very thorough works, we will not give a repetitious detailed discussion of such issues and will only outline the basic argument we put forward.)

A simple postulated form would be

$$P^A(A^M) = \frac{1}{M}\sum_{j=1}^{M} P^A(A_j^1),$$

where $A_j^1 = \{\Omega_{N_j}^j\}$. One can then simply postulate that the aggregate effective literacy measure P^A coincides with the household level measure P for any society with a single household. That is,

$$P^A(A_j^1) = P(\Omega_{N_j}^j).$$

This formulation presents a consistent method of reducing the computation of the aggregate measure of effective literacy of a society to combining the calculations for each constituent household in a simple fashion. This formulation ensures that the measure is normalised between 0 and 1, it is anonymous, monotonically nondecreasing with respect to the number of literates in the society and the externality is restricted to household level only.

4.2 A Contrast with the BF formulation

One can now contrast this formulation with that of **BF**. If we look back at the measures discussed in section 2 like P_α, P_{α_m,α_f} and P_S, and use the decomposable structure put forth above, it can be checked that social aggregate versions of these measures will indeed be the same as proposed in **BF** and **Subramanian (2001)**. If we look at the axioms put forth in this paper and theirs, it is easy to match our (A2) with **BF** *Normalisation.* In the absence of sensitivity with respect to the characteristics that we discuss (as this is not considered by **BF**), our axiom (A1), along with decomposability, has the same consequences as the axioms *externality* and *decomposability* in **BF** (as a consequence of **BF, Theorem 1**. To focus on the endogenisation of the externality parameter, we have not gone into the details of the externality mechanism in detail, as they have done, but instead we took the consequences of that as a primitive in our discussion.

If we consider household splits in an analogous manner to **BF**, then one can compare the externality axiom with our axioms (A3) and (A7). Again, in the absence of sensitivity with respect to gender, sex and education level considerations, in the case of an externality neutral household split, due to (A7), the effective literacy status of the proximate literates in the new households will be the same as before. Just as envisaged by the **BF** *externality* axiom. For an externality reducing split, due to (A3), the illiterates who now become isolated are worse off. Hence, due to decomposability, the society will now have a lower level of aggregate effective literacy as a whole. Again, the same effect is postulated in **BF** *externality.* Note that, in the absence of (A7), this relation may not hold. For example, the measures postulated in proposition 3 may not be externality neutral *a la* **BF**.

5 Empirical Analysis

The importance of these alternative theoretical measures of literacy is strengthened when the literacy ranking of different geographical areas becomes different with different measures. We now turn to provide an example.

The data we use are the primary data collected from the seven villages of Narayanpur block of Lakhimpur district of Assam, which is a state of India. All the households of each of these seven villages have been surveyed. We consider adult literacy, i.e., we consider those individuals with age not less than fifteen years.

Tables 1 to 6 (see below) present the literacy rates of the villages and their ranking according to alternative measures of literacy with alternative sets of values of α, α_F, α_M and β. Different literacy measures are defined as follows:

LBAG1: Illiterate: 0, Literate: 1.

LBAG2: Isolated Illiterate: 0, Proximate Illiterate: α (predefined), Literate: 1.

LBAG3: Isolated Illiterate: 0, Proximate Illiterate: α, Literate: 1 where α = L/T; L: numbers of literates in a family, T: total numbers of members in a family.

LBAG4 Isolated Illiterate: 0, Proximate Illiterate: α, Literate: 1 where α = L/T; L: sum of literacy level, T: total numbers of members in a family $\times$ 4 (number of literacy levels considered).

LBAG5: Isolated Illiterate: 0, Proximate Illiterate: α_F and α_M, Literate: 1. Value is α_F if family has at least one female literate, otherwise, α_M if family has at least one male literate.

LBAG6: Isolated Illiterate: 0, Proximate Illiterate: α_{OF}, α_{YF}, α_{OM} and α_{YM}, Literate : 1, where the value is

α_{OF}: if family has female older literate,

α_{YF}: $(\beta \times \alpha_{OF})$ if family has female younger literate,

α_{OM}: if family has male older literate, and

α_{YM}: $(\beta \times \alpha_{OM})$ if family has male younger literate.

Literacy measures of a village are the average of the literacy measures of the households in that village. An unbracketed number in any table shows the literacy rate of that village according to the selected literacy measure and the bracketed number shows the rank of that village.

The interesting result is that two of these seven villages - Borpather and Jarabari - often interchange their ranking as we move on from one formula to other and from one set of exogenous values of α, α_F, α_M and β to the other set. However, the ranking of the other five villages - Buhabuhi, Kasaripather, Kathajan, Majorsapori and Singia - remains unchanged.

According to the traditional literacy rate shown in column 1 of each of these tables, we find that Borpather is marginally behind Jarabari. However, the percentage of proximate illiterates and the female literacy rate are higher in Borpather than those in Jarabari. This explains the interchanging of ranking of those two villages following the other literacy measures, which take care of proximate illiterates.

6 Conclusion

This note wants to pass on the message that, while measuring effective literacy, one should not only look at the literacy status of the members of the household but also look at the gender, age ranking and educational qualification of the literate members. Some extensions of **BF** measure have been developed in this paper; and all of them can be used for the measurement of effective literacy rate in a country like India using either Census or NSS data. **BF** measure with $\alpha = 0$, the traditional literacy rate, always appears as a special case of the class of measures suggested and characterised here.

In **BF**, α is exogenous to the formulation and its value is arbitrarily chosen. So the literacy ranking of different regions (districts, states, countries etc.) may be different for different arbitrarily chosen values of α. So choice of α is an important task in measuring the extent of effective literacy. We have suggested some relevant individual characteristics that may impact on the externality effect of a literate person on an illiterate member of the family. In particular, we have characterised certain classes of effective literacy measures that are generalisations of those proposed by **Basu and Foster (1998)** and **Subramanian (2001)** and other general class of measures.

These measures are useful in testing for the effect of the characteristics under discussion on effective literacy and hence for devising appropriate policy in this direction. For example, if education level sensitivity is high, then higher education programme plays a significant role even in basic literacy improvement, in the proximate sense. Similarly, female literacy campaign should be more important for the purpose of generating higher externality if gender sensitivity is significant.

Although our proposed measures are very general in nature and allows many alternative possibilities, the final choice of a measure by a practitioner will depend on her subjective value judgement about the acceptability of alternative axioms set out in this paper and specific parametrisation thereof.

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TABLE 1 : LITERACY RANKING OF THE VILLAGES

alpha=0.5	alpha(f)=0.75		alpha(m)=0.25		beta=0.6	
VILLAGES	**LITERACY INDEX**					
	LBAG1	**LBAG2**	**LBAG3**	**LBAG4**	**LBAG5**	**LBAG6**
Buhabuhi	0.854264	0.920155	0.937351	0.912044	0.928682	0.911085
	.(4)	.(4)	.(4)	.(4)	.(4)	.(4)
Borpather	0.907563	0.952941	0.969876	0.953034	0.969748	0.951096
	.(2)	.(2)	.(1)	.(2)	.(1)	.(2)
Kasaripather	0.601124	0.747191	0.75485	0.698209	0.765449	0.715449
	.(7)	.(7)	.(7)	.(7)	.(7)	.(7)
Kathajan	0.842185	0.908194	0.929138	0.903847	0.924507	0.895068
	.(5)	.(5)	.(5)	.(5)	.(5)	.(5)
Jarabari	0.92723	0.957746	0.968839	0.958806	0.964789	0.951643
	.(1)	.(1)	.(2)	.(1)	.(2)	.(1)
Majarsapori	0.7713	0.869955	0.895262	0.853082	0.901345	0.859417
	.(6)	.(6)	.(6)	.(6)	.(6)	.(6)
Singia	0.883588	0.930344	0.950685	0.93226	0.947996	0.925668
	.(3)	.(3)	.(3)	.(3)	.(3)	.(3)

Source : Field work

TABLE 2 : LITERACY RANKING OF THE VILLAGES

alpha=0.5	alpha(f)=0.75		alpha(m)=0.25		beta=0.8	
VILLAGES	**LITERACY INDEX**					
	LBAG1	**LBAG2**	**LBAG3**	**LBAG4**	**LBAG5**	**LBAG6**
Buhabuhi	0.854264	0.920155	0.937351	0.912044	0.928682	0.926279
	.(4)	.(4)	.(4)	.(4)	.(4)	.(3)
Borpather	0.907563	0.952941	0.969876	0.953034	0.969748	0.962057
	.(2)	.(2)	.(1)	.(2)	.(1)	.(1)
Kasaripather	0.601124	0.747191	0.75485	0.698209	0.765449	0.740449
	.(7)	.(7)	.(7)	.(7)	.(7)	.(7)
Kathajan	0.842185	0.908194	0.929138	0.903847	0.924507	0.909788
	.(5)	.(5)	.(5)	.(5)	.(5)	.(5)
Jarabari	0.92723	0.957746	0.968839	0.958806	0.964789	0.958216
	.(1)	.(1)	.(2)	.(1)	.(2)	.(2)
Majarsapori	0.7713	0.869955	0.895262	0.853082	0.901345	0.880381
	.(6)	.(6)	.(6)	.(6)	.(6)	.(6)
Singia	0.883588	0.930344	0.950685	0.93226	0.947996	0.925668
	.(3)	.(3)	.(3)	.(3)	.(3)	.(4)

Source : Field work

TABLE 3 : LITERACY RANKING OF THE VILLAGES

alpha=0.5	alpha(f)=0.6		alpha(m)=0.4		beta=0.8	
VILLAGES	**LITERACY INDEX**					
	LBAG1	**LBAG2**	**LBAG3**	**LBAG4**	**LBAG5**	**LBAG6**
Buhabuhi	0.854264	0.920155	0.937351	0.912044	0.928682	0.916279
	.(4)	.(4)	.(4)	.(4)	.(4)	.(4)
Borpather	0.907563	0.952941	0.969876	0.953034	0.959664	0.953862
	.(2)	.(2)	.(1)	.(2)	.(2)	.(2)
Kasaripather	0.601124	0.747191	0.75485	0.698209	0.754494	0.733034
	.(7)	.(7)	.(7)	.(7)	.(7)	.(7)
Kathajan	0.842185	0.908194	0.929138	0.903847	0.914719	0.902094
	.(5)	.(5)	.(5)	.(5)	.(5)	.(5)
Jarabari	0.92723	0.957746	0.968839	0.958806	0.960563	0.955117
	.(1)	.(1)	.(2)	.(1)	.(1)	.(1)
Majarsapori	0.7713	0.869955	0.895262	0.853082	0.882511	0.865381
	.(6)	.(6)	.(6)	.(6)	.(6)	.(6)
Singia	0.883588	0.930344	0.950685	0.93226	0.937405	0.928473
	.(3)	.(3)	.(3)	.(3)	.(3)	.(3)

Source : Field work

TABLE 4 : LITERACY RANKING OF THE VILLAGES

alpha=0.25	alpha(f)=0.6		alpha(m)=0.4		beta=0.8	
VILLAGES	**LITERACY INDEX**					
	LBAG1	**LBAG2**	**LBAG3**	**LBAG4**	**LBAG5**	**LBAG6**
Buhabuhi	0.854264	0.887209	0.937351	0.912044	0.928682	0.916279
	.(4)	.(4)	.(4)	.(4)	.(4)	.(4)
Borpather	0.907563	0.930252	0.969876	0.953034	0.959664	0.953862
	.(2)	.(2)	.(1)	.(2)	.(2)	.(2)
Kasaripather	0.601124	0.674157	0.75485	0.698209	0.754494	0.733034
	.(7)	.(7)	.(7)	.(7)	.(7)	.(7)
Kathajan	0.842185	0.87519	0.929138	0.903847	0.914719	0.902094
	.(5)	.(5)	.(5)	.(5)	.(5)	.(5)
Jarabari	0.92723	0.942488	0.968839	0.958806	0.960563	0.955117
	.(1)	.(1)	.(2)	.(1)	.(1)	.(1)
Majarsapori	0.7713	0.820628	0.895262	0.853082	0.882511	0.865381
	.(6)	.(6)	.(6)	.(6)	.(6)	.(6)
Singia	0.883588	0.906966	0.950685	0.93226	0.937405	0.928473
	.(3)	.(3)	.(3)	.(3)	.(3)	.(3)

Source : Field work

TABLE 5 : LITERACY RANKING OF THE VILLAGES

alpha=0.75	alpha(f)=0.6		alpha(m)=0.4		beta=0.8	
VILLAGES	LITERACY INDEX					
	LBAG1	LBAG2	LBAG3	LBAG4	LBAG5	LBAG6
Buhabuhi	0.854264	0.953101	0.937351	0.912044	0.928682	0.916279
	.(4)	.(4)	.(4)	.(4)	.(4)	.(4)
Borpather	0.907563	0.97563	0.969876	0.953034	0.959664	0.953862
	.(2)	.(2)	.(1)	.(2)	.(2)	.(2)
Kasaripather	0.601124	0.820225	0.75485	0.698209	0.754494	0.733034
	.(7)	.(7)	.(7)	.(7)	.(7)	.(7)
Kathajan	0.842185	0.941199	0.929138	0.903847	0.914719	0.902094
	.(5)	.(5)	.(5)	.(5)	.(5)	.(5)
Jarabari	0.92723	0.973005	0.968839	0.958806	0.960563	0.955117
	.(1)	.(2)	.(2)	.(1)	.(1)	.(1)
Majarsapori	0.7713	0.919283	0.895262	0.853082	0.882511	0.865381
	.(6)	.(6)	.(6)	.(6)	.(6)	.(6)
Singia	0.883588	0.953721	0.950685	0.93226	0.937405	0.928473
	.(3)	.(3)	.(3)	.(3)	.(3)	.(3)

Source : Field work

TABLE 6: LITERACY RANKING OF THE VILLAGES

alpha=0.5	alpha(f)=0.6		alpha(m)=0.4		beta=0.6	
VILLAGES	**LITERACY INDEX**					
	LBAG1	**LBAG2**	**LBAG3**	**LBAG4**	**LBAG5**	**LBAG6**
Buhabuhi	0.854264	0.920155	0.937351	0.912044	0.928682	0.905302
	.(4)	.(4)	.(4)	.(4)	.(4)	.(4)
Borpather	0.907563	0.952941	0.969876	0.953034	0.959664	0.943462
	.(2)	.(2)	.(1)	.(2)	.(2)	.(2)
Kasaripather	0.601124	0.747191	0.75485	0.698209	0.754494	0.713258
	.(7)	.(7)	.(7)	.(7)	.(7)	.(7)
Kathajan	0.842185	0.908194	0.929138	0.903847	0.914719	0.890926
	.(5)	.(5)	.(5)	.(5)	.(5)	.(5)
Jarabari	0.92723	0.957746	0.968839	0.958806	0.960563	0.949765
	.(1)	.(1)	.(2)	.(1)	.(1)	.(1)
Majarsapori	0.7713	0.869955	0.895262	0.853082	0.882511	0.850404
	.(6)	.(6)	.(6)	.(6)	.(6)	.(6)
Singia	0.883588	0.930344	0.950685	0.93226	0.937405	0.920687
	.(3)	.(3)	.(3)	.(3)	.(3)	.(3)

Source : Field work