SINGLE MOTHERS

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1 Introduction

Changes in childbearing patterns during the last twenty-five years or so have been remarkable. In 1975 approximately 9% of all births in the US were to unmarried women, whereas in 1999 33% were to unmarried women. There has also been a significant increase in the percentage of couples who choose to cohabit rather than marry (Bumpass and Lu (2000) document the increase). As to be expected, many births outside marriage today are to women who are currently cohabiting. Somewhat surprisingly, however, is that about 60% of births to women who are not married are to women who are also not cohabiting. Given that the improvement in birth control technology and the Roe v Wade decision imply that women have a much greater control over their reproductive system relative to 30 years ago, this increase is even more remarkable.

Who are the women who have children outside a partnership? They are on average a deprived group both before and after the birth. Such women come from poorer family backgrounds, in terms of parents' income and educational attainments, and they score lower on ability tests than women who bear their children within marriage (Rosenzweig 1999). They themselves obtain lower educational attainments (e.g. see Bumpass and Lu (2000)) and earn less when employed. Further, single women who have had a child are less likely to marry than other single women (see Lichter and Graefe (2001) and Upchurch, Lillard and Panis (2001)). The object of the paper is to propose a simple equilibrium theory that can be used to explain at least some of these facts.

It is difficult to argue that such a large number of births to women outside a partnership are all mistakes and therefore it is important to question why women choose to have a child outside marriage. One answer to this question has been proposed by Willis (1999) who considered a frictionless marriage market in which people either marry or remain single forever. Given there is an excess of women relative to men, Willis shows that an equilibrium with non-marital births can exist. In such a situation less desirable women who fail to find a husband may choose to have children rather than remain single and childless.

A somewhat different approach is taken here. The model used here is based on two intuitive ideas. First, following recent developments in the analysis of marriage markets (see, for example, Burdett and Coles (1997, 1999) and Shimer and Smith (2000)), we embed matching frictions into the marriage market considered. At least intuitively, it is difficult to argue that matching frictions do not play a significant role in finding a husband or wife. For most of us finding a spouse takes time and randomness plays a significant role. Second, we exploit the idea that women in the marriage market face different choices than men. When a man and women meet, the man can choose to marry the woman, or not, if she will have him. Of course, a woman faces the same choice when she meets a man. She can also choose, however, to have a child by the man (given he is willing to assist) and then raise it without the father.

Why should a woman choose such an option? Depending on the social welfare system she faces, and whether the father is willing to contribute resources, a woman's utility flow raising a child by herself may be greater than what she obtains when single and childless. Of course, there are also costs in terms of marriage market prospects associated with raising a child alone. A single woman with child may be considered a less desirable wife by men, or the woman may find it more difficult to contact potential husbands while looking after a child. A woman who contacts a man she does not wish to marry, or who will not marry her, will choose to have a child by the man if the short-run gain exceeds the long-term costs in terms of her marriage prospects. Of course, some women may choose to have a child outside marriage if the choice arises, whereas other women may choose not to if faced with the choice. It is to be expected that those women who expect to obtain a significant increase in utility when they marry suffer a greater long term cost by having a child when single than women whose marriage prospects are such that they expect to gain little from marriage. Hence, at least intuitively, it is expected that those women with poorer marriage prospects are more likely to have children outside marriage. It turns out in the equilibrium marriage model used here this intuition is confirmed.

2 The Framework

As the focus is on the decision of women to have children before marriage, all other aspects of the marriage model outlined here are kept as simple as possible. Suppose an equal number of single men and women participate at each moment in time. Focussing on essentials, we assume there are only two types; Gs and Bs. Any woman (man) who marries a G man (G woman) obtains utility flow x_G per unit of time, whereas a woman (man) who marries a B man (B woman) obtains x_B utility flow ($x_G > x_B > 0$).[1] Further, assume an individual is instantly recognizable as a G or a B on contact.

Time is assumed to be continuous. Suppose g new single men and new single women flow into this marriage market per unit of time. Of these, the same proportion, π, of both sexes are Gs; the others Bs. For simplicity, suppose that at least initially the number of B (G) women equals the number of B (G) men. Every now and then a single man contacts a single woman. Let α_m denote the arrival rate of single women faced by a single man. [2] The arrival rate of single men faced by a single woman depends on whether she has a child or not. In particular, let α_w denote the arrival rate of single men faced by a childless single woman, whereas single women with a child contact single men at rate $\beta\alpha_w$, where $0 < \beta < 1$.[3] Given a woman contacts a man, let λ_m denote the probability she contacts a G man. Similarly, λ_w denotes the probability that a man is contacted by a single G woman, given he has a contact.

Suppose a single woman contacts a single man. If both agree, they marry and then leave the marriage market for good - married people do not contact others. On the other hand, if at least one of them chooses not to marry, they don't. In this case the single woman can choose to have a child by this man and then separate (if she has not had a child already).[4] Of course, a woman can choose neither to marry nor

[1]The utilities x_G and x_B include the utility of childbearing within marriage.

[2]Formally, α_m is the parameter of a Poisson process as is α_w described below.

[3]Consistent with the assumption, Lichter and Graefe (2001), Upchurch, Lillard and Panis (2001) and Brien, Lillard and Waite (1999) all provide evidence that, at each age, having a child has a causal impact on the probability that a never married woman marries, which reduces this probability.

[4]By assumption, men are willing to co-operate in this task. We can think of the utility of being

have a child by the man contacted. In this case they separate never to meet again.

For simplicity, we assume a man's utility from marriage to a particular woman does not depend on whether she had a child before they married. Clearly, this is not a particularly satisfactory restriction as men may well prefer childless women over those with a child (or vice-versa). Unfortunately, this would complicate the analysis considerably as it doubles the types of possible women. Therefore, keeping things as simple as possible, we assume the only cost of having a child to a single woman is that it reduces the rate at which she contacts men.

Finally, assume single men and childless single women obtain zero utility flow. A single woman with child, however, obtains utility flow $b > 0$ per unit of time, and we assume that $x_B > b$.[5] All discount the future at rate r.

3 Decisions

Given the marriage market specified above, two implications follow directly. First, all singles are willing to marry a G of the opposite sex - they can do no better. This, of course, implies that when two Gs of the opposite sex make contact, they marry. Second, if G men (women) marry B women (men), then so will B men (women). This implication follows as Bs are at least as constrained as Gs and therefore Gs are at least as selective as Bs.

Three problems remain: (a) Under what conditions will Gs marry Bs of the opposite sex? (b) When will Bs marry each other? and (c) What conditions are required for either type of woman to have a child outside marriage? Answers to these questions are presented below where we consider the decisions made by the various participants.

Each participant utilizes a strategy which specifies what they will do in all possible situations. In particular, a strategy for a man specifies who he is willing to marry if they make contact. Things are slightly more complicated for a single woman. A

a father, even if he has little contact with his child, at least offsetting his cost of fatherhood.

[5]The analysis would be rather trivial if $x_B < b$. Furthermore, this assumption is consistent with the argument that marriage is a more efficient context for raising children.

strategy for a woman specifies not only who she is willing to marry if they make contact but also who she is willing to have a child by and bring it up herself (at least initially). Assume any participant chooses that strategy that maximizes expected discounted future utility given his or her beliefs about the actions of others and the arrival rates of each type of the opposite sex.

In determining the best strategy for individuals we make the following simplifying restriction. If an individual is indifferent between marriage or not, we assume that by convention he or she marries. Hence, if two singles of the opposite sex make contact, then it results in a marriage with probability one or zero. A single will either marry another single with probability one, or not marry that individual with probability one. Similarly, if a woman is indifferent to having a child outside marriage or not, we assume by convention she has a child.

Given the simple model described above, the objective is to describe steady-state equilibria. Hence, below we look at the strategies of the participating individuals where the parameters do not change through time.

3.1 G Men

The arrival rate of single G women faced by a G man is $\alpha_m\lambda_w$.per unit of time. If a G woman is contacted, they marry and the man obtains payoff x_G/r. If a B woman is contacted (and this happens at rate $\alpha_m(1-\lambda_w)$) the G man must decide whether to marry or not. Let U_G denote the expected return to a single G man.[6] Using standard techniques, it follows

$$rU_G = \alpha_m\lambda_w[x_G/r - U_G] + \alpha_m(1-\lambda_w)[\max\{x_B/r, U_G\} - U_G] \tag{1}$$

Simple manipulation of (1) establishes that $x_B/r \geq U_G$ is satisfied if and only if $x_B \geq A_m(x_G)$, where

$$A_m(x_G) = \frac{\alpha_m\lambda_w x_G}{r + \alpha_m\lambda_w} \tag{2}$$

[6]We shall use the term 'expected return' instead of the correct but clumsy 'expected discounted lifetime utility'.

The decision rule for a G man is simple. Always marry a G woman on contact and if the x_B/x_G ratio is small enough, B women are rejected.

3.2 G Women

A single G woman may, or may not, have a child. Let V_{Gc} denote a G woman's expected return when single with a child, whereas V_{Gs} denotes her expected return when currently childless. It follows

$$rV_{Gc} = b + \beta\alpha_w\lambda_m[x_G/r - V_{Gc}] + \beta\alpha_w(1-\lambda_m)[max(x_B/r, V_{Gc}) - V_{Gc}] \tag{3}$$

and

$$rV_{Gs} = \alpha_w\lambda_m[x_G/r - V_{Gs}] + \alpha_w(1-\lambda_m)[max(x_B/r, V_{Gc}, V_{Gs}) - V_{Gs}] \tag{4}$$

A childless G woman will marry a B man on contact if and only if $x_B/r \geq \max\{V_{Gc}, V_{Gs}\}$. After some manipulation of (3) and (4) it can be shown that this inequality is satisfied if and only if

$$x_B \geq \max\{A_w(x_G), B(b, x_G)\} \tag{5}$$

where

$$A_w(x_G) = \frac{\alpha_w\lambda_m x_G}{r + \alpha_w\lambda_m} \tag{6}$$

and

$$B(b, x_G) = \frac{rb + \beta\alpha_w\lambda_m x_G}{(r + \beta\alpha_w\lambda_m)}$$

It is useful to note two facts that follow from the above. First, $B(b, x_G = b) = b$. Second, $A_w(x_G) \gtrless B(b, x_G)$ as $x_G \gtrless C(b)$, where

$$C(b) = \frac{[r + \alpha_w\lambda_m]b}{\alpha_w\lambda_m(1-\beta)} \tag{7}$$

Suppose G women will not marry B men, i.e., (5) does not hold. Clearly, a single G woman will not marry but have a child by a B man on contact if and only if

$V_{Gc} > \max\{x_B/r, V_{Gs}\}$. From (3) and (4), it follows that this condition is satisfied if and only if

$$x_B < B(b, x_G) \tag{8}$$

and

$$x_G < C(b) \tag{9}$$

A G woman will neither marry nor have a child by a B man if and only if $x_B < A_w(x_G)$ and $x_G \geq C(b)$. Obviously, a single G woman with a child will not marry a B man.

Given the other parameters are held constant, Figure 1 illustrates a single G woman's choices, given possible combinations of the parameters x_B and x_G. As $x_G \geq x_B$ and $x_B \geq b$, by assumption, only those combinations are below the 45^0 line and above b are feasible. In area W (where (5) is satisfied) a G woman marries the first man she meets. In the area marked Y a G woman will not marry, nor have a child by a B man if they make contact. A G woman in this case waits until she contacts a G-man and then marries. In the two areas marked Z (where x_G is such that $C(b) > x_G \geq b$ and x_B satisfies $x_B < B(b, x_G)$), a G woman will have a child by a B man (but not marry him) if she contacts one before a G man. When a G man is contacted later, she marries him.

3.3 B Men

The expected return to a single B man, U_{Bs}, can be written as

$$rU_{Bs} = \alpha_m \lambda_w \theta_{Gb}[x_G/r - U_{Bs}] + \alpha_m(1-\lambda_w)\theta_{Bb}[max(x_B/r, U_{Bs}) - U_{Bs}]$$

where θ_{Gb} denotes the probability a G woman will marry a B man if they make contact, and θ_{Bb} is the probability a B woman will marry a B man if they make contact.[7]

It follows that

$$U_{Bs} = \frac{\alpha_m \lambda_w \theta_{Gb}(x_G/r) + \alpha_m(1-\lambda_w)\theta_{Bb}\max(x_B/r, U_{Bs})}{[r + \alpha_m \lambda_w \theta_{Gb} + \alpha_m(1-\lambda_w)\theta_{Bb}]}$$

[7] Note, these probabilities are either zero or one.

A B man is always willing to marry a G woman. Suppose G women are willing to marry B men, i.e. $\theta_{Gb} = 1$, and therefore $\theta_{Bb} = 1$). In this case a B man acts exactly the same as a G man. In particular, a B man is willing to marry a B woman if they make contact if and only if $x_B \geq A_m(x_G)$.

Suppose G women will not marry B men but B women will, i.e., $\theta_{Gb} = 0$ and $\theta_{Bb} = 1$. In this case it is simple to show a B man will always marry a B woman on contact - by assumption it is more desirable than remaining single forever.

3.4 B Women

The expected return to a single B woman with child, V_{Bc}, can be written as

$$rV_{Bc} = b + \beta\alpha_w\lambda_m\phi_{Gb}[\frac{x_G}{r} - V_{Bc}] + \beta\alpha_w(1-\lambda_m)\phi_{Bb}[\max\{\frac{x_B}{r}, V_{Bc}\} - V_{Bc}]$$

where ϕ_{Gb} denotes the probability a G man is willing to marry a B woman, and ϕ_{Bb} is the probability a B man will marry a B woman. The expected return to a childless single B woman, V_{Bs}, can be expressed as

$$\begin{aligned} rV_{Bs} &= \alpha_w\lambda_m[\phi_{Gb}(\frac{x_G}{r} - V_{Bs}) + (1-\phi_{Gb})(\max\{V_{Bc}, V_{Bs}\} - V_{Bs})] \\ &+ \alpha_w(1-\lambda_m)[\phi_{Bb}(\max\{\frac{x_B}{r}, V_{Bs}, V_{Bc}\} - V_{Bs}) \\ &+ (1-\phi_{Bb})(\max\{V_{Bc}, V_{Bs}\} - V_{Bs})] \end{aligned}$$

The first thing to notice is that if G men will marry B women (i.e., $\phi_{Gb} = 1$ and therefore $\phi_{Bb} = 1$[8]), then B women act exactly the same as G women. In particular, if $\phi_{Gb} = 1$ and $\phi_{Bb} = 1$, then B women reject B men if and only if $x_B < \max\{A_w(x_G), B(b, x_G)\}$. Further, even if B women reject B men, they are still willing to have a child by a B man if one is contacted before a G man if and only if $b \leq x_G < C(b)$ and $x_B < B(b, x_G)$.

Suppose now $\phi_{Gb} = 0$ and $\phi_{Bb} = 1$, i.e., G men will not marry them but B men will. In this case a single B women will marry the first B man contacted, and are willing to have a child by a G man if they make contact if and only if

[8]Remember, if $\phi_{Gb} = 1$, then $\phi_{Bb} = 1$

$$b \leq x_B < D(b) \tag{10}$$

where

$$D(b) = \frac{[r + \alpha_w(1 - \lambda_m)]b}{\alpha_w(1 - \lambda_m)(1 - \beta)} \tag{11}$$

In words, it is worthwhile to have a child while waiting for a B man to marry, even though she will probably have to wait longer, because the return when she finds him is not that much greater than being a single mother. Note B women may have a child outside marriage by either a G man, or a B man but never for the same parameter configuration.

4 Steady-States

There is one final element that needs to be specified before considering market equilibrium - the matching function. This specifies the number of encounters between single men and women given what we shall term the effective numbers of men and women participating in the marriage market. The difference between the actual and effective number of participating women follows as those women with children face a lower contact rate than those without children. There is no difficulty for men as all single men face the same encounter rate.

Keeping things as simple as possible, we use a constant returns to scale Cobb-Douglas matching function with equal exponents such that

$$e = \Omega {N_m}^{0.5} {N_w}^{0.5} \tag{12}$$

where $\Omega > 0$, and N_m and N_w are the effective numbers of men and women respectively. All men contact women at the same rate, α_m, by assumption. Hence, without loss of generality, we assume that the effective number of men equals the actual number of men, i.e., $N_m = N$. If some women have children before they marry, then all women do not contact men at the same rate. Let U^B and U^G denote the steady-state number of single B and G women who do not have children, and let S^B and S^G be the steady-state number of single B and G women with a child. In this case, the effective number of women can be written as

$$N_w = U^B + U^G + \beta[S^G + S^B]$$

Hence, given $\alpha_m N$ men contact single women per unit of time, (12) implies

$$\alpha_m = \frac{e}{N} = \Omega\left(\frac{N_w}{N}\right)^{0.5} \tag{13}$$

Further,

$$\alpha_w = \Omega\left(\frac{N_w}{N}\right)^{-0.5} \tag{14}$$

As $N_w \leq N$, $\alpha_m \leq \alpha_w$.

We are interested here in steady-state equilibria. Two conditions need to be satisfied in such a situation. First, in an equilibrium all utilize an optimal strategy and have correct beliefs about the actions of others. Second, the behavior of those in market is such that they generate the steady-state arrival rates assumed to hold by the participants.

The objective is to identify and analyze pure strategy equilibria What are the possible types of equilibria? Five possible types of equilibria are listed below.[9]

Type 1: Only G/G and B/B marriages form and some B women have a child before marriage.

Type 2: Only G/G and B/B marriages form and some G women have a child before marriage.

Type 3: Only G/G and B/B marriages form and some G and B women have babies before marriage.

Type 4: Only G/G and B/B marriages form and nobody has a child before marriage.

Type 5: All marry the first person of the opposite sex they meet.

Analyzing equilibrium in the situation specified above is a two-step procedure. First, given a particular type of equilibrium is assumed to hold, we first derive the

[9] As we show later, there is another possible equilibrium. This, however, cannot exist in steady-state.

steady-state associated with this type of equilibrium. This imposes restrictions on the arrival rates faced by participants. Second, we identify the parameter restrictions required for the particular type of equilibrium to be generated by optimizing behaviour. Given these restrictions, we then investigate when participants choose to act in the assumed way.

The first goal is to calculate the steady-state values of the endogenous variables that hold when a type k equilibrium exists (if one does). The parameters of the model are $(\pi, g, \beta, \Omega, x_B, x_G, r, b)$. Given these parameters and the assumed behavior of a type k equilibrium it is possible to calculate the steady-state values of each of the endogenous variables (μ_k, $\lambda_w(k)$, $\lambda_m(k)$, $\alpha_m(k)$, $\alpha_w(k)$, $N(k)$, $U^B(k)$, $S^B(k)$, $U^G(k)$, $S^G(k)$), where μ_k is the steady-state proportion of G-people in the marriage market, which must be the same for both sexes. This is a mechanical and tedious task that has been relegated to an Appendix. The results are presented in Table 1.

5 Equilibrium

The utility maximizing decisions for the different types of individuals have been described above. Note, the function $A_m(x_G)$ plays a critical role in determining the behavior of men, whereas G women use the functions $A_w(x_G)$, $B(b, x_G)$, and $C(b)$ when deciding whether to marry a man, or have a child outside marriage. B women utilize the function $D(b)$, if G men will not marry them. The magnitude of the functions used in making decisions depends on the particular steady-state involved. Hence, we define $A_{mk}(x_G)$, $A_{wk}(x_G)$, $B_k(b, x_G)$, $C_k(b)$, and $D_k(b)$ as the magnitude of these functions when evaluated in a type k steady-state.

5.1 Type 1 Equilibrium (Only B woman have children outside marriage)

In this case only G/G and B/B marriages form and B women who contact a G man before a B man have a child before marriage. The relevant steady-states are given in

Table 1. First, we require that G men reject B women. This occurs if and only if

$$x_B < A_{m1}(x_G) \tag{15}$$

Second, we want G women to reject B men and not have children outside marriage. This holds if and only if

$$x_B < A_{w1}(x_G) \text{ and } x_G \geq C_1(b) \tag{16}$$

Finally, we require B women to have a child by G men if one is contacted before a B man. This holds if and only if

$$x_B < D_1(b) \tag{17}$$

Can these inequalities can co-exist? In particular, for any given allowable parameter values we need to check that there exists a set of doubletons (x_B, x_G) which satisfy these inequalities and thus generate a type 1 equilibrium. To achieve this goal, we first establish a result that holds in any equilibrium.

Define $E(k)$ by $A_{wk}(x_G = E(k)) = D_k(b)$. From (6) and (11) it follows

$$E(k) = \frac{[r + \alpha_w(k)\mu_k][r + \alpha_w(k)(1-\mu_k)]b}{\alpha_w(k)^2\mu_k(1-\mu_k)(1-\beta)} \tag{18}$$

Using (18) and (7) when $k = 1$ yields

$$E(1) - C_1(b) = \frac{r(r + \alpha_w(1)\mu_1)b}{\alpha_w(1)^2(1-\mu_1)\mu_1(1-\beta)} > 0 \tag{19}$$

This is illustrated in Figure 2. This implies that there always exist doubletons (x_B, x_G) which satisfy (15), (16), and (17) and thus generate a type 1 equilibrium. The shaded area in Figure 2 illustrates the pairs of (x_G, x_B) which generate a type 1 equilibrium. Note, in this type of equilibria $A_{w1}(x_G) = A_{m1}(x_G)$.

5.2 Type 2 Equilibrium (Only G woman have children outside marriage)

Here, only G/G and B/B marriages form and those G women who contact a B man before a G man have a child before marriage. Using the steady-state values of the relevant endogenous variables presented in Table 1, G men reject B women if and only if

$$x_B < A_{m2}(x_G) \tag{20}$$

We also require single G women not to marry B men but have a child by one on contact. This occurs if and only if

$$x_B < B_2(b, x_G) \text{ and } x_G < C_2(b) \tag{21}$$

Finally, we require B women do not have children by G men. This occurs if and only if

$$x_B \geq D_2(b) \tag{22}$$

Hence, if a type 2 equilibrium is to exist we require $x_B < B_2(b, x_G)$ for $x_G < C_2(b)$ and $x_B \geq D_2(b)$. Can these restrictions be satisfied simultaneously? First, note that from (A6) in the Appendix we have $\alpha_w(1-\mu) = \alpha_m(1-\lambda_w)$ and therefore

$$\alpha_w\mu - \alpha_m\lambda_w = \alpha_w - \alpha_m$$

Further, as some G women by assumption have children outside marriage, (13) and (14) imply $\alpha_w > \alpha_m$. Hence, $\alpha_w\mu - \alpha_m\lambda_w > 0$, and therefore $A_{w2}(x_G) > A_{m2}(x_G)$.

Given $E(2)$ defined in (18) and (7), we have[10]

$$E(2) - C_2(b) = \frac{r[r + \alpha_w(2)\mu_2]b}{\alpha_w(2)^2\mu_2(1-\mu_2)(1-\beta)} > 0$$

Therefore if x_B is large enough for B women not to have children by G men, i.e., $x_B \geq D_2(b)$, then this x_B is too large for G women to reject B men but have a child by them, i.e., $x_B > B_2(b, x_G)$ for $x_G < C_2(b)$. In other words, if G women find that x_G is small enough to make it worthwhile to have a child and slow down their search, then so must B women, because $x_B < x_G$. Hence, a type 2 equilibrium cannot exist for any allowable parameter values. Only G women having children outside marriage is not an equilibrium configuration.

[10]The key value of x_G in this context is that value at which $D_2(b) = B_2(b, x_G)$, but Figure 2 indicates that this value always exceeds $E(2)$. Thus, showing that $E(2)$ exceeds $C_2(b)$ is sufficient.

5.3 Type 3 Equilibrium (Both types of woman have children outside marriage)

In this case only G/G and B/B marriages form and some B and G women have a child before marriage. In this case we require

$$x_B < A_{m3}(x_G) \tag{23}$$

so G men reject B women,

$$x_B < B_3(b, x_G) \text{ and } x_G < C_3(b) \tag{24}$$

so G women reject B men and but have children by them, and

$$x_B < D_3(b) \tag{25}$$

so B women have children by G men.

Below we check if such inequalities can co-exist. First, we show $A_{w3}(x_G) > A_{m3}(x_G)$ for $x_G > 0$. To see this note from (2) and (6) that $A_{w3}(x_G) > A_{m3}(x_G)$ if $\Psi = \alpha_w(3)\mu_3 - \alpha_m(3)\lambda_w(3) > 0$. However, using (13), (14), it follows

$$\begin{aligned}\Psi &= \Omega(\frac{N_w}{N})^{-0.5}\mu_3 - \Omega(\frac{N_w}{N})^{0.5}\lambda_w(3)\\ &= \Omega(\frac{N_w}{N})^{0.5}\left[\frac{N}{N_w}\mu_3 - \lambda_w(3)\right]\end{aligned}$$

Substituting (A11) and (A12) in the Appendix into the above establishes

$$\Psi = \Omega(\frac{N_w}{N})^{0.5}\left[\frac{(1-\beta)(1-\mu)\mu[(1-\mu)\beta+\mu]}{[2\beta\mu(1-\mu)+\mu^2+(1-\mu)^2]\beta}\right] > 0$$

Hence, $A_{w3}(x_G) > A_{m3}(x_G)$.

Second, in this case given $E(k)$ defined in (18) $E(3) - C_3(b)$ can be written as

$$E(3) - C_3(b) = \frac{r[r+\alpha_w(3)\mu_3]b}{\alpha_w(3)^2\mu_3(1-\mu_3)(1-\beta)} > 0$$

The above results imply that a type 3 equilibrium exists if $b \leq x_B < A_{m3}(x_G)$ and $x_G < C_3(b)$. Finally, we need to check that $A_{m3}(C_3(b)) - b > 0$ can be satisfied. As

$$A_{m3}(C_3(b)) - b = b\left[\frac{\alpha_m(3)\lambda_w(3)[r+\alpha_w(3)\mu_3]}{[r+\alpha_m(3)\lambda_w(3)]\alpha_w(3)\mu_3(1-\beta)} - 1\right]$$

It is now simple to show that $A_{m3}(C_3(b)) - b > 0$ can hold for r small enough. The set of (x_B, x_G) which generate a type three equilibria is illustrated in Figure 3 by the triangular shaded area.

Finally, we consider two possible equilibria where women do not have children before marriage.

5.4 Type 4 Equilibrium (Only B/B and G/G marriages and no children outside marriage)

In this case only G/G and B/B marriages form and nobody has a child outside marriage. Again, we require that G men to reject B women and this occurs if and only if

$$x_B < A_{m4}(x_G) \tag{26}$$

Further, G women reject B men and do not have a child outside marriage if and only if

$$x_B < A_{w4}(x_G) \text{ and } x_G \geq C_4(b) \tag{27}$$

Finally, we require B women do not have children outside marriage and this holds if and only if

$$x_B \geq D_4(b) \tag{28}$$

As no women have children outside marriage, $N_w = N$. It follows that $\alpha_w = \alpha_m = \Omega$ and $\lambda_w = \mu_4$ (as shown in the Appendix). This, of course, implies $A_{m4}(x_G) = A_{w4}(x_G)$. Further, given (18), it follows that $E(4) - C_4(b) > 0$. Hence, such a type 4 equilibrium exists if and only if (26) and (28) hold.

Figure 4 illustrates the pairs (x_B, x_G) that generate the above type of equilibria, given the other parameters are held constant.

5.5 Type 5 Equilibrium (All marry first person they meet)

In this case all marry the first person of the opposite sex they contact. G men marry B women on contact if and only if

$$x_B \geq A_{m5}(x_G),$$

whereas single G women marry B men on contact if and only if

$$x_B \geq \max\{A_{w5}(x_G), B_5(b, x_G)\} \tag{29}$$

As shown in the Appendix, $A_{m5}(x_G) = A_{w5}(x_G)$. Hence, such an equilibrium exists if (29) is satisfied. Such an equilibrium is illustrated in Figure 5. In particular, Figure 5 illustrates the pairs (x_B, x_G) which generate this equilibrium. Not surprisingly, a type 5 equilibrium results if the utility from marrying a B is not that dissimilar to marrying a G.

Consider for a moment the area indicated by K in Figure 5. If (x_B, x_G) is in this area, G men are willing to marry B women but G women prefer to have a child by a B man (but not marry him) if they make contact. This possible equilibrium situation is now briefly considered. Suppose G men are willing to marry B women. This implies B men are also willing to marry B women. Hence, all men are willing to marry both G and B women. As they face the same constraints G and B women will make the same decisions. In particular, if G women will not marry B men, neither will B women. As B men do not marry, it is straightforward to show there exists no steady-state and therefore there is no steady-state equilibrium of this type.

6 Multiple Equilibria

So far we have presented a reasonably complete characterization of each of the four possible types of equilibria (types 1, 3, 4, 5). The parameter values required for each type of equilibria to exist have been presented. If a particular type of equilibrium exists for a given set of parameters, it was clearly unique. The objective here is to show briefly that there are parameter values which generate at least two types of equilibria, or no pure strategy equilibrium.

Two factors generate the possibility of multiple equilibria. First, given (β, π), the proportion of Gs, μ, of both sexes depends on which type of equilibrium holds. Second, the arrival rate of contacts with the opposite sex also depends on the type of equilibrium ruling in the market. These two factors lead to a sorting externality that generates the possibility of multiple equilibria.

We start by investigating the relationships between the proportion of Gs in the market, μ_k, and the parameters (β, π) for each type of equilibrium. Table 1 defines the four functions and they are illustrated in Figure 6. Simple calculation then establishes the following relationships:

(a) $\mu_1 = \pi$ if and only if $\pi = 0, \beta^{1/2}/(\beta^{1/2}+1)$,or 1. Further, $\pi < \mu_1$ if and only if $\pi \in (0, \beta^{1/2}/(\beta^{1/2}+1))$.

(b) $\pi = \mu_3$ if and only if $\pi = 0, 1/2$, or 1, and $\mu_3 > \pi$ if and only if $\pi \in (0, 1/2)$

(c) $\mu_4 = \pi$ if and only if $\pi = 0, 1/2, 1$, and $\mu_4 > \pi$ if and only if $\pi \in (0, 1/2)$

(d) For any fixed π, $0 < \pi < \beta^{1/2}/(\beta^{1/2}+1)$, then $\mu_3 > \mu_4 > \mu_1 > \mu_5 = \pi$.

(e) For any fixed π, $\beta^{1/2}/(\beta^{1/2}+1) < \pi < 0.5$, then $\mu_3 > \mu_4 > \mu_5 > \mu_1$.

(f) For any fixed π, $1 > \pi > 0.5$, $\mu_5 > \mu_4 > \mu_3 > \mu_1$.

(h) $lim_{\beta \to 1} \mu_3 \to \mu_4$ and $lim_{\beta \to 1} \mu_1 \to \mu_4$.

Using these relationships it is possible to establish multiple equilibria can exist. As a complete analysis would be tedious and add few new insights, we shall only present two examples to illustrate the basic principles at work.

>From (d) and (e) above it follows that $\mu_4 > \mu_5 = \pi$ for all $\pi < 1/2$. From (2), $A_{m4}(x_G) > A_{m5}(x_G)$ if $\pi < 1/2$. It is now simple to show that if $\pi < 1/2$ there always exist parameter values which generate (a) an equilibrium where all marry the first person they meet (a type 5 equilibrium) and (b) and equilibrium where only G/G and B/B marriages form and no woman has a child before marriage (a type 4 equilibrium). This is illustrated by the shaded area in Figure 7.

The reasoning behind this multiple equilibria is simple to explain. Suppose $\pi < 1/2$. When all believe that Gs marry the first person of the opposite sex they meet, then the proportion of Gs of either sex participating is π. However, if all believe that Gs reject Bs and women don't have babies before marriage, then the steady-state proportion of Gs is larger; equal to $\mu_4 > \pi$. Hence, in the first case the proportion of Gs is small enough that Gs are willing to marry Bs, whereas in the second case, the proportion of Gs is large enough that Gs reject Bs. The sorting externality generates the possibility of multiple equilibria. Of course, if $\pi > 1/2$, then $A_{m4}(x_G) < A_{m5}(x_G)$ and a type 4 and 5 equilibrium cannot co-exist.

The problem is much more complicated when considering if a type 1 and 5 equilibrium can co-exist for the same parameter values. There are two reasons for this. First, although μ_1 and μ_5 (implicitly defined in Table 1) can be written as explicit function of π and β, these expressions, unfortunately, are horrendously long and complicated. Second, unlike when comparing equilibria type 4 and 5, the equilibrium arrival rate facing childless women, α_w, is different in these types of equilibria. Due to these complexities, we merely illustrate the situation.

Suppose $\Omega = 1$, $b = 1$ $\pi = 0.3$, $\beta = 0.2$ and $r = .05$. Calculation establishes that (to two decimal places) $A_{w5}(x_G) = (0.86)x_G$, $A_{w1}(x_G) = (0.89)x_G$, $C_5(1) = 1.46$, $C_1(1) = 1.40$, and $D_1(1) = 1.32$ This is illustrated in Figure 8. In Figure 8 the combinations of x_G and x_B that generate a type 1 and 5 equilibria are illustrated by the thicker lines. The shaded area indicates where both types of equilibria can exists for the same parameter values. In this shaded area, if all believe all marry the first person the meet, then the best action of Gs is to marry the first person they meet which, of course, generates a type 5 equilibrium. However, if all believe that Gs will not marry Bs and that B women have children by G men if they make contact, then (after some calculation) it can be shown all expect there is a greater probability of contacting a G than in the type 5 equilibrium. Two factors are at work. First, $\mu_1 > \mu_5$, (given the assumed parameter values) which implies on any contact there is a greater probability a G is contacted . Second, the arrival rate of contacts when only G/G and B/B marriages form and some B women have a child is not that much smaller than when all marry the first person they meet. This implies in a type 1 equilibrium that the arrival rate of Gs of the opposite sex, ($\alpha_w\lambda_m = \alpha_m\lambda_w = 0.388$) is greater than in a type 5 equilibrium ($\alpha_w\lambda_m = \alpha_m\lambda_w = 0.300$). This difference in arrival rates of Gs implies in a type 1 equilibrium Gs reject Bs when they make contact, whereas in a type 5 equilibrium they accept Bs.

Of course, it is possible to construct examples in this case where there exists no pure strategy equilibrium for a given set of parameter values. In this case either the men or the women used a mixed strategy.

7 Conclusions

The paper has presented a simple equilibrium model of sorting in a marriage market with frictions. Given the restrictions made, it has been shown for particular parameter values that there exists an equilibrium where the less desirable women choose to have a child before marriage if the opportunity arises. Women who select such an option and have a child before marriage take a longer time to find a husband. An equilibrium cannot exist where only the more desirable women choose to have a child before marriage. In equilibrium, if more desirable women choose to have child before marriage, so will the less desirable women. Indeed, if the discount rate is large enough, then the more desirable women will never have a child outside marriage.

In an equilibrium where only the less desirable (B) women choose to have a child outside marriage if the opportunity arises, two factors play a major role. First, the flow payoff to marrying a less desirable man (x_B) is not much greater than the flow payoff to raising a child alone (b). Second, the flow payoff to a desirable man from marrying a desirable woman (x_G) must be sufficiently larger than what he obtains if he married a less desirable woman (x_B). Of course, a reduction in the cost of having a child outside marriage, i.e., any increase in β[11], increases the parameter set which generate this type of equilibria.

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APPENDIX

Steady-States

Before considering the various situations below we specify the steady-state flow of men given only G/G and B/B marriages form. The relevant equations are:

$$\pi g = \alpha_m \lambda_w \mu N$$

and

$$(1-\pi)g = \alpha_m(1-\lambda_w)(1-\mu)N$$

Hence, any steady-state where only B/B and G/G marriages form must satisfy

$$\lambda_w = \frac{(1-\mu)\pi}{\pi+\mu(1-2\pi)} \tag{A1}$$

This result will prove a useful in what follows.

The first goal is to calculate the steady-state values of the endogenous variables that holds when a type k equilibrium exists. The parameters of the model are (π, g, β, Ω). Given these parameters and the assumed behavior of a type k equilibrium it is possible to calculate the steady-state values of each endogenous variables (μ_k, $\lambda_w(k)$, $\lambda_m(k)$, $\alpha_m(k)$, $\alpha_w(k)$, $N(k)$, $U^B(k)$, $S^B(k)$, $U^G(k)$, $S^G(k)$), where μ_k is the steady-state proportion of G-people in the marriage market, which must be the same for both sexes.

Type 1 Equilibrium

Assume now a type 1 equilibrium exists. As only G/G and B/B marriages form with this type of equilibrium, equation (A1) must be satisfied. Further, as G women do not have babies outside marriage the steady-state flow of G women in this case can be written as

$$\pi g = \alpha_w \mu^2 N$$

This equation and the steady-state flow of G men imply

$$\alpha_w \mu = \alpha_m \lambda_w \tag{A2}$$

Consider now B women. It follows that

$$U^B + S^B = (1-\mu)N$$

The steady-state flows into and out of these states can be written as

$$\alpha_w U^B = (1-\pi)g$$

and

$$(1-\mu)\alpha_w\beta S^B = \mu\alpha_w U^B$$

This implies the steady-state number of single women with children are

$$S^B S^B = \frac{\mu(1-\mu)N}{(1-\mu)\beta+\mu},$$

whereas the number of single women without children can be written as

$$U^B = \frac{(1-\mu)^2\beta N}{(1-\mu)\beta+\mu}$$

As $N_w = \mu N + U^B + \beta S^B$, substitution implies

$$\frac{N_w}{N} = \frac{\beta(1-\mu^2)+\mu^2}{(1-\mu)\beta+\mu} \tag{A3}$$

Hence, the probability a man encounters a G woman, given a contact is made $(\mu N/N_w)$ can be written as

$$\lambda_w = \frac{(1-\mu)\beta+\mu}{\beta(1-\mu^2)+\mu^2}\mu > \mu \tag{A4}$$

Using (A1) and (A4) it follows that in a steady state $\mu = \mu_1$, where μ_1 is implicitly by

$$\pi = \frac{\mu_1^2(\mu_1+\beta(1-\mu_1))}{\mu_1^3(1-\beta)+2\beta\mu_1^2-2\beta\mu_1+\beta} \tag{A5}$$

Given a solution for μ_1 and β it is possible to solve for $N_{w1}/N(1)$ via (A3). This result, (13) and (14) imply solutions to $\alpha_w(1)$ and $\alpha_m(1)$. It is now straightforward to solve for the other steady-state variables. The results are shown in Table 1.

Type 2 Equilibrium

Again, the steady-state flows of men must satisfy (A1). As single B women do not have babies. the steady-state flows of B women in this case satisfy

$$(1-\pi)g = \alpha_w(1-\mu)^2 N$$

This implies

$$\alpha_w\mu - \alpha_m\lambda_w = \alpha_w - \alpha_m \qquad \text{A6}$$

As some G women have a child before marriage

$$U^G + S^G = \mu N$$

$$S^G = \frac{(1-\mu)\mu N}{(1-\mu)+\beta\mu}$$

and

$$U^G = \frac{\beta\mu^2 N}{(1-\mu)+\beta\mu}$$

These imply the effective number of participating women in this steady-state, $N_w = (1-\mu)N + \beta S^G + U^G$, can be written as

$$\frac{N_w}{N} = \frac{1+(1-\beta)\mu^2 - 2\mu(1-\beta)}{(1-\mu)+\beta\mu} \tag{A7}$$

Hence, the probability a man contacts a G woman, given a contact is made ($\lambda_w = (\beta S^G + U^G)N_w$), can be written as

$$\lambda_w = \frac{\beta\mu}{1+(1-\beta)\mu^2 - 2\mu(1-\beta)} < \mu \tag{A8}$$

Using (A1) and (A8) it follows that in a steady-state $\mu = \mu_3$ where μ_3 is implicitly defined in Table 1. *In* a similar fashion to that used when considering a type 1 equilibrium it is now possible to establish the other relevant steady-state values.

Type 3 Equilibrium

In this case the effective number of G women in this case can be written as

$$U^G + \beta S^G = \frac{\beta\mu N}{(1-\mu)+\beta\mu} \tag{A9}$$

whereas the effective number of B women is

$$U^B + \beta S^B = \frac{\beta(1-\mu)N}{(1-\mu)\beta+\mu} \tag{A10}$$

Manipulating (A9) and (A10) implies the efffective number of women, $N_w = U^G + U^B + \beta(S^G + S^B)$, can be written as

$$\frac{N_w}{N} = \frac{[2\beta\mu(1-\mu)+\mu^2+(1-\mu)^2]\beta}{[(1-\mu)\beta+\mu][(1-\mu)+\beta\mu]} \tag{A11}$$

Hence, the probability a man encounters a G women, given and contact is made is

$$\lambda_w = \frac{\mu((1-\mu)\beta + \mu)}{2\beta\mu(1-\mu) + \mu^2 + (1-\mu)^2} \tag{A12}$$

Using (A1) and (A12) it follows that with this type of steady state $\mu = \mu_3$, where μ_3 is implicitly defined in Table 1.

Type 4 Equilibrium

As no women have babies outside marriage, and therefore women faced the same encounter rate as men, i.e., $\lambda_w(4) = \lambda_m(4) = \mu_4$. This implies that the steady-state flow of G and B women into and out of the market can be written as

$$\pi g = \alpha_w \mu^2 N = \alpha_m \mu^2 N$$

and

$$(1-\pi)g = \alpha_w(1-\mu)^2 N = \alpha_m(1-\mu)^2 N$$

It follows that $\alpha_w(4) = \alpha_m(4) = \alpha$ and (13) implies $\alpha = \Omega$.

Using (A1) and $\lambda_w(4) = \mu_4$ implies the unique steady-state proportion of G men and women is as written in Table 1.

Type 5 Equilibrium

In this case women and men of both types act the same. Further, as women do not have babies before marriage, $\lambda_w(5) = \mu_5$. This implies the steady-state flows of G men and G women can be written as

$$\pi g = \alpha_w \mu N = \alpha_m \mu N,$$

whereas the steady-state flows of B men and B women can be written as

$$(1-\pi)g = \alpha_w(1-\mu)N = \alpha_m(1-\mu)N$$

Hence, the encounter rate faced by all participants in this case is the same, i.e., $\alpha_w(5) = \alpha_m(5) = \alpha$. Finally, the matching function specified above implies $e/N = \alpha = \Omega$. All this implies that in any steady-state where all marry the first person they meet we have $\mu_5 = \pi$ and $N(5) = g/\Omega$.

Table 1

k	$\boldsymbol{\pi}(\boldsymbol{\mu}_k, \boldsymbol{\beta})$	$\mathbf{N}_{wk}/\mathbf{N}(\mathbf{k})$	$\boldsymbol{\alpha}_m(\mathbf{k})$	$\boldsymbol{\alpha}_w(\mathbf{k})$
1	$\frac{{\mu_1}^2(\mu_1+\beta(1-\mu_1))}{{\mu_1}^3(1-\beta)+2\beta{\mu_1}^2-2\beta\mu_1+\beta}$	$\frac{\beta(1-\mu_1^2)+\mu_1^2}{(1-\mu_1)\beta+\mu_1}$	$\Omega[\frac{N_{w1}}{N(1)}]^{0.5}$	$\Omega[\frac{N_{w1}}{N(1)}]^{-0.5}$
2	$\frac{{\mu_2}^2\beta}{1-{\mu_2}^3(1-\beta)+{\mu_2}^2(3-\beta)-\mu_2(3-\beta)}$	$\frac{1+(1-\beta)\mu_2^2-2\mu_2(1-\beta)}{(1-\mu_2)+\beta\mu_2}$	$\Omega[\frac{N_{w2}}{N(2)}]^{0.5}$	$\Omega[\frac{N_{w2}}{N(2)}]^{-0.5}$
3	$\frac{{\mu_3}^2(\beta+\mu_3)(1-\beta)}{1-\mu_3(1-\mu_3)(3-\beta)}$	$\frac{[2\beta\mu_3(1-\mu_3)+\mu_3^2+(1-\mu_3)^2]\beta}{[(1-\mu_3)\beta+\mu_3][(1-\mu_3)+\beta\mu_3]}$	$\Omega[\frac{N_{w3}}{N(3)}]^{0.5}$	$\Omega[\frac{N_{w3}}{N(3)}]^{-0.5}$
4	$\frac{\mu_4^2}{2\mu_4^2-2\mu_4+1}$	1	Ω	Ω
5	μ_5	1	Ω	Ω

k	$\boldsymbol{\lambda}_w(\mathbf{k})$	$\boldsymbol{\lambda}_m(\mathbf{k})$	$\mathbf{U}^B(\mathbf{k})$	$\mathbf{S}^B(\mathbf{k})$	$\mathbf{U}^G(\mathbf{k})$	$\mathbf{S}^G(\mathbf{k})$
1	$\frac{\pi(1-\mu_1)}{\pi+\mu_1(1-2\pi)}$	μ_1	$\frac{(1-\pi)g}{\alpha_w(1)}$	$\frac{\mu_1 U^B(1)}{(1-\mu_1)\beta}$	$\frac{\pi g}{\alpha_w(1)\mu_1^2}$	0
2	$\frac{\pi(1-\mu_2)}{\pi+\mu_2(1-2\pi)}$	μ_2	$\frac{(1-\pi)g}{\alpha_w(2)(1-\mu_2)^2}$	0	$\frac{\pi g}{\alpha_w(2)}$	$\frac{(1-\mu_1)U^B(2)}{\mu_1\beta}$
3	$\frac{\pi(1-\mu_3)}{\pi+\mu_3(1-2\pi)}$	μ_3	$\frac{(1-\pi)g}{\alpha_w(3)}$	$\frac{\mu_1 U^B(3)}{(1-\mu_3)\beta}$	$\frac{\pi g}{\alpha_w(3)}$	$\frac{(1-\mu_3)U^B(3)}{\mu_3\beta}$
4	μ_4	μ_4	$\frac{(1-\pi)g}{\Omega(1-\mu_4)}$	0	$\frac{\pi g}{\Omega\mu_4}$	0
5	μ_5	μ_5	$\frac{(1-\pi)g}{\Omega(1-\mu_5)}$	0	$\frac{\pi g}{\Omega\mu_5}$	0

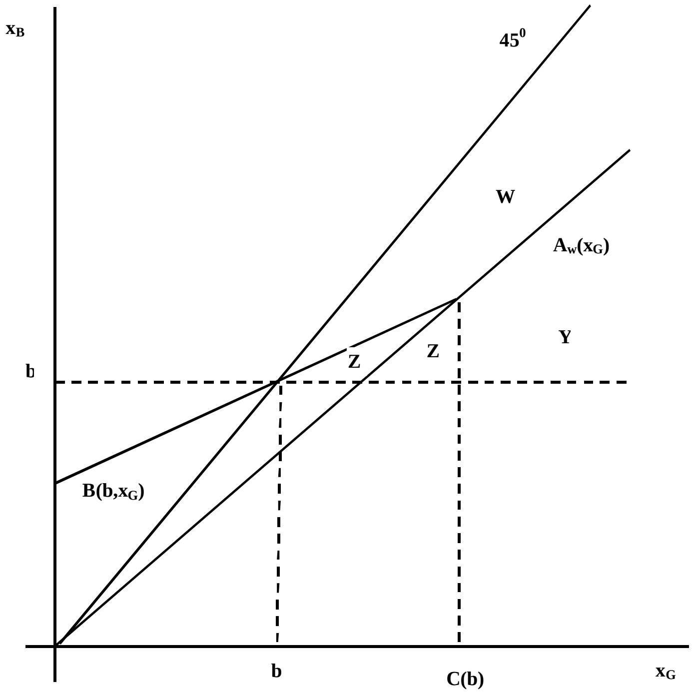

Figure 1: Decisions of G Women

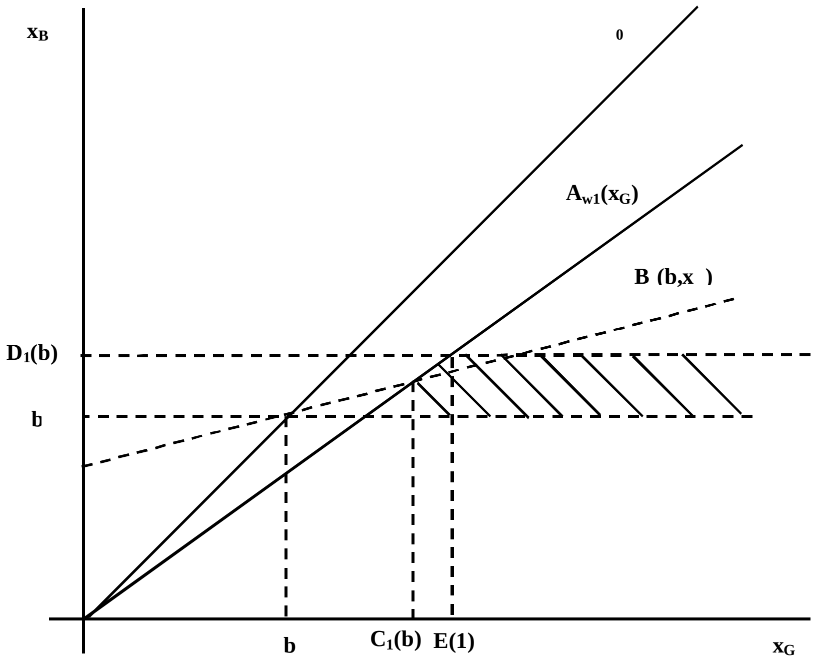

Figure 2: Only B Women Have a Child Outside Marriage

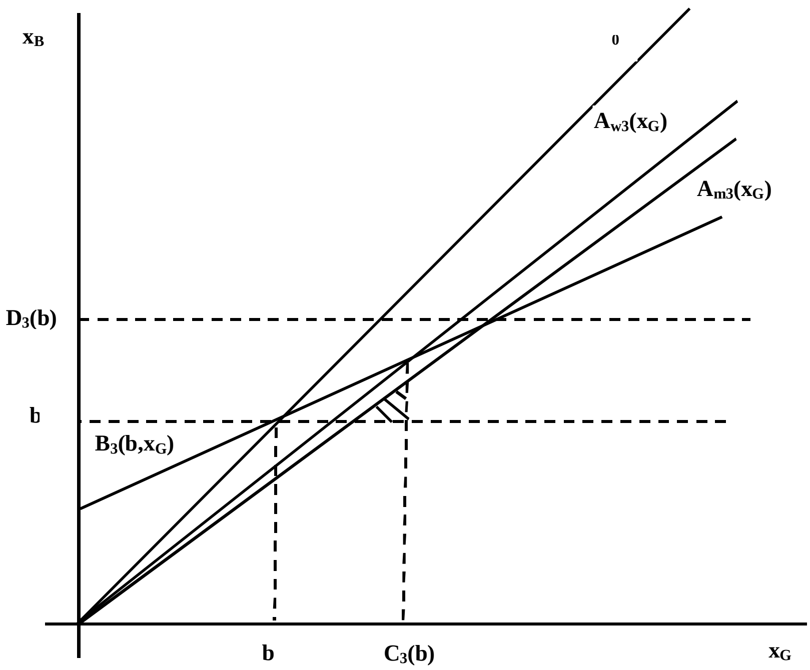

Figure 3: Both Types of Women Have a Child Outside Marriage

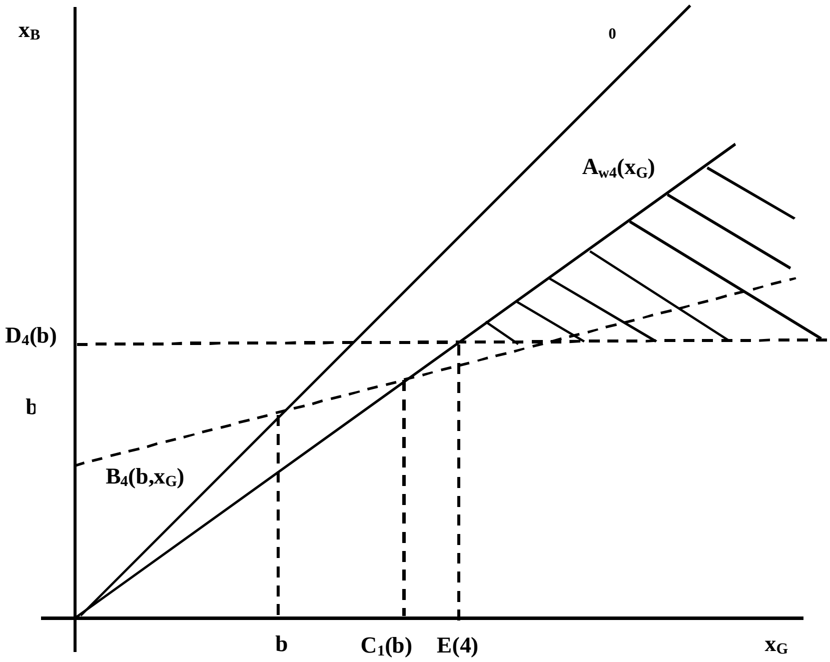

Figure 4: No Children Born Outsuide Marriage

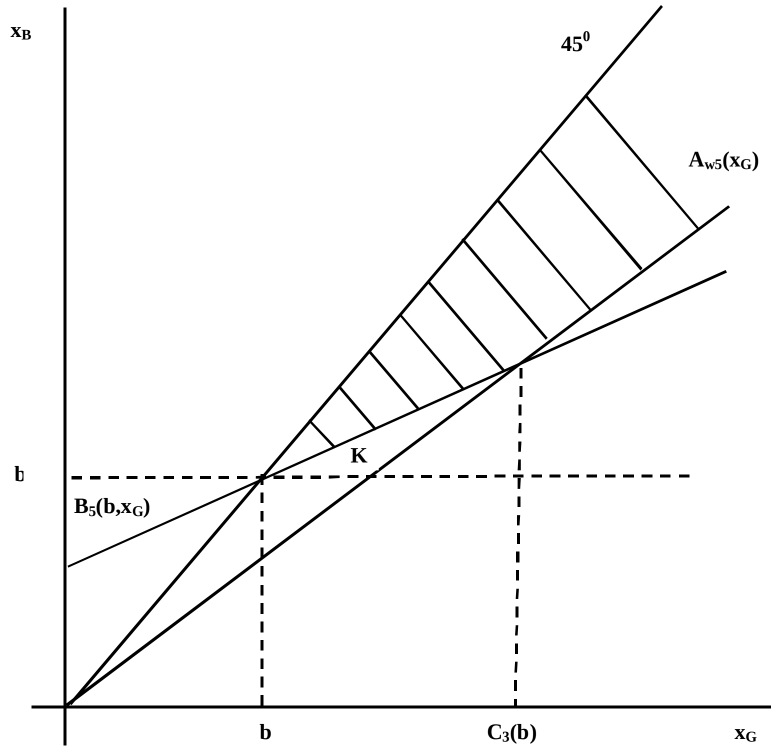

Figure 5: All Women Marry First Person They Meet

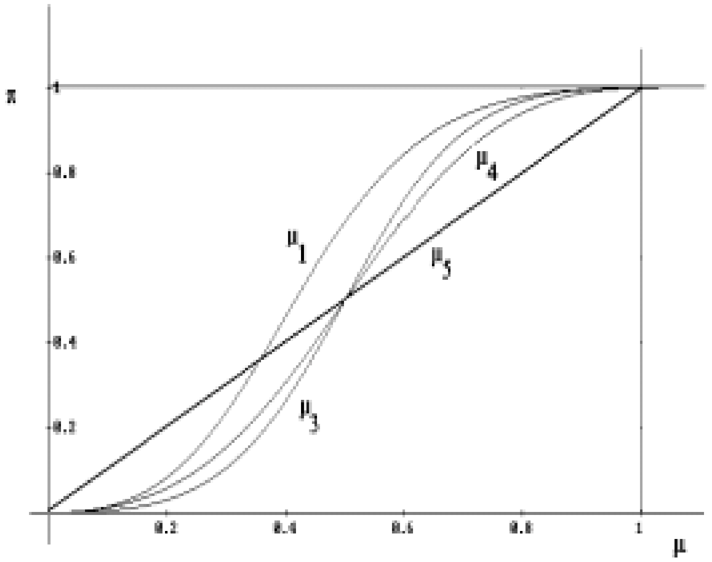

Figure 6: Steady-States: $\pi(\beta, \mu_k)$

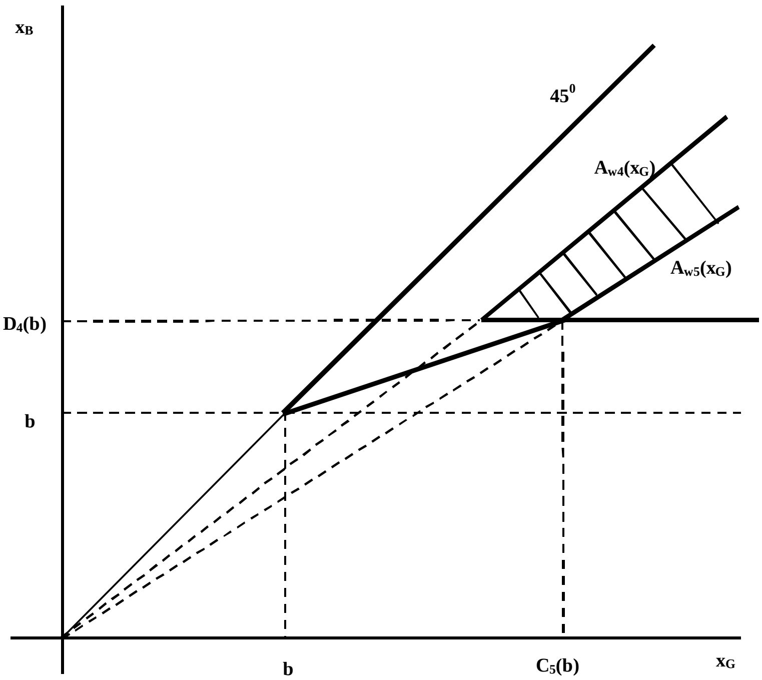

Figure 7: Multiple Equilibria: Types 4 and 5

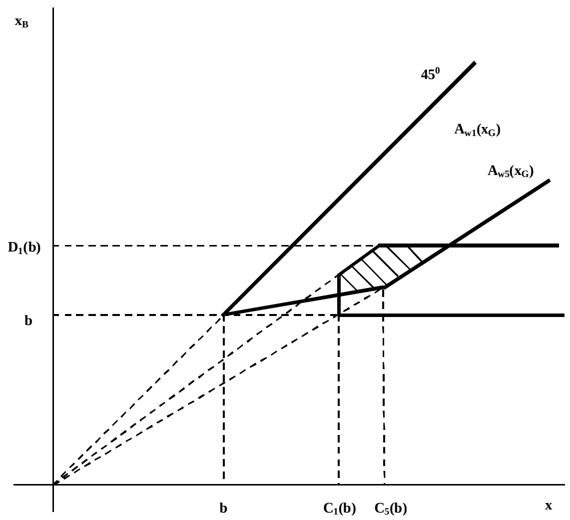

Figure 8: Multiple Equilibria: Types 1 and 5