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Equilibrium policy simulation with random utility models of labor supply

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Equilibrium policy simulation with random utility models of labor supply[1*]

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Abstract

Many microeconometric models of discrete labor supply include alternative-specific constants meant to account for (possibly besides other factors) the density or accessibility of particular types of jobs (e.g. part-time jobs vs. full-time jobs). The most common use of these models is the simulation of tax-transfer reforms. The simulation is usually interpreted as a comparative static exercise, i.e. the comparison of different *equilibria* induced by different policy regimes. The simulation procedure, however, typically keeps fixed the estimated alternative-specific constants. In this note we argue that this procedure is not consistent with the comparative statics

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interpretation. Since the constants reflect the number of jobs and since the number of people willing to work change as a response to the change in tax-transfer regime, it follows that the constants should also change. A structural interpretation of the alternative-specific constants leads to the development of a simulation procedure consistent with the comparative static interpretation. The procedure is illustrated with an empirical example.

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1. Introduction

A common practice in the specification of models of labor supply based on the discrete choice approach consists of introducing alternative-specific constants, which should account for a number of factors such as the different density or accessibility of different types of jobs, search or fixed costs and systematic utility components otherwise not accounted for. In the basic framework, the agent chooses among a set Ω of alternatives or "job" types *j*, including non-market "jobs" (non-participation). Let $V_i(j;w_i,T)+\varepsilon_{ij}$ denote the utility attained by agent *i* if a job of type *j* is chosen, given wage rate w_i and tax-transfer regime *T*, where $V_i(j;w_i,T)$ is the systematic part (containing observed variables) and ε_{ij} is a random component. By assuming that ε_{ij} is i.i.d. Type I extreme value, we get the familiar Multinomial Logit expression for the probability that a job of type *j* is chosen:

$$P_i(j;w_i,T)=\frac{\exp\{V_i(j;w_i,T)\}}{\sum_{k\in\Omega}\exp\{V_i(k;w_i,T)\}} \tag{1}$$

Model (1) usually does not fit the data very well. A common experience is that that the model largely over-predicts the number of people working part-time. Certain types of jobs might differ according to a number of systematic factors that are not accounted for by the observed variables contained in *V*: (a) availability or density of job-types**;** (b) fixed costs; (c) search costs; (d) systematic utility components. What might be called the "dummies refinement" is a simple way to account for those factors. Let us define subsets $\{S_\ell\}$ of Ω and the corresponding dummies $\{D(j\in S_\ell)\}$ such that *D(e)* = 1 iff *e* is true. Clearly the definition of the subsets should reflect some hypothesis upon the differences among the job types with respect to the factors (a) – (b) mentioned above. Now we specify the choice probability as follows:

$$P_i(j;w_i,T)=\frac{\exp\left\{V_i(j;w_i,T)+\sum_\ell \mu_\ell D(j\in S_\ell)\right\}}{\sum_{k\in\Omega}\exp\left\{V_i(k;w_i,T)+\sum_\ell \mu_\ell D_\ell(k\in S_\ell)\right\}} \tag{2}$$

Many papers – although with differing focus and motivation – have adopted a similar procedure, e.g.: Van Soest (1995), Aaberge et al. (1995, 1999), Kalb (2000), Aaberge and Colombino (2006, 2010, 2011), Dagsvik and Strøm (2006), Kornstad et al. (2007) and Colombino et al. (2010). The main use of microeconometric models of labor supply consists of the simulation of tax-transfer reforms. The standard simulation proceeds as follows. Once *V*( ) and the $\{\mu_\ell\}$ are estimated, the current tax regime *T* is replaced by a "reform" *R* and a new distribution of choices is simulated using expression (2). The policy simulation is most commonly interpreted as a comparative statics exercise, where different *equilibria* – induced by different tax-transfer regimes – are compared. All the authors adopting the "dummies refinement" so far have performed the simulations while leaving the $\{\mu_\ell\}$ unchanged. We claim that this procedure in general is not consistent with the comparative statics interpretation. According to a basic notion of equilibrium, the number of people willing to work must equal the number of available jobs. Since the $\{\mu_\ell\}$ reflect – at least in part, depending on the interpretations – the number and the composition of available jobs, and since the number of people willing to work and their distribution across different job types in general change as a consequence of the reforms, it follows that in general the $\{\mu_\ell\}$ must also change. A series of papers by Aaberge et al. (1995, 1999) and by Aaberge and Colombino (2006, 2011), building on a matching model developed by Dagsvik (1994, 2000) extend the basic random utility approach to include a random choice set and provide a structural interpretation of the "dummies refinement" that leads very naturally to a simulation procedure consistent with comparative statics.[2] The procedure is explained in Sections 2 and 3. Section 4 introduces an empirical example and Section 5 illustrates and discusses the results.

2. **A structural interpretation of the "dummies refinement".**

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[2] A different procedure for equilibrium simulation of microeconometric models– which however would not be appropriate with the "dummies refinement" – has been proposed by Creedy and Duncan (2001).

We consider here a single individual. The generalization to couples is developed in Colombino (2010). Letting $\delta(j)$ denote the density of available jobs of type *j*, under appropriate assumptions the probability that individual *i* is matched to a job of type *j* turns out to be (e.g. Aaberge et al. 1999):[3]

$$P_i(j;w_i,T) = \frac{\exp\{V_i(j;w_i,T)\}\delta(j)}{\sum_{k\in\Omega}\exp\{V_i(k;w_i,T)\}\delta(k)}. \tag{3}$$

The opportunity density $\delta(j)$ can be interpreted as reflecting the demand side. By assuming that $\delta(j)$ is uniform except for peaks at jobs of type *g = 1,,…, G*, we end up with the following expression:

$$P_i(j;w_i,T) = \frac{\exp\left\{V_i(j;w_i,T) + \mu_0 D_0(j\in M) + \sum_{g=1}^{G}\mu_g D_g(j\in M_g)\right\}}{\sum_{k\in\Omega}\exp\left\{V_i(k;w_i,T) + \mu_0 D_0(k\in M) + \sum_{g=1}^{G}\mu_g D_g(k\in M_g)\right\}} \tag{4}$$

where *M* is the subset of market job-types and $M_1,...,M_G$ are *G* subsets of *M*. It can then be shown that the coefficients of the dummy variables have the following interpretation:

$$\mu_0 = \ln\left(\frac{J}{H}\right) \tag{5}$$

and

$$\mu_g = \ln\left(\frac{J_g/J}{A_g}\right) \tag{6}$$

where *J* = number of jobs of type $j \in M$ (i.e. number of market jobs), *H* = number of "jobs" of type $j \notin M$ (i.e. the number of non-market "jobs"), J_g = number of jobs of type $j \in M_g$ and A_g = number of types in M_g.[4] The presence of factors other than jobs density (such as search or fixed costs) is

[3] The opportunity density can be specified as individual-specific but in this illustration we assume it to be common to everyone for the sake of simplicity.
[4] For the derivation of expressions (3) – (6) see Aaberge et al. (1999) and Aaberge and Colombino (2011).

not incompatible with expressions (5) and (6): indeed H and A_g can be more generally interpreted as normalizing constants that include the effect of those other factors.

3. Equilibrium conditions

We further simplify the exposition by assuming that the model contains only one dummy $D_0(j \in M)$.[5]

As a simple illustration, let us assume that the number of available pre-reform jobs J depends on the average wage rate $\overline{w}$

$$J = J(\overline{w}). \tag{7}$$

Using (5) and (7) we can write:

$$\mu_0 = \mu_0(\overline{w}). \tag{8}$$

We then define $\pi_i(T, \overline{w}_T, \mu_0(\overline{w}_T))$ as the probability that individual i is working given tax-transfer regime T and average wage rate $\overline{w}_T$:

$$\pi_i(T, \overline{w}_T, \mu_0(\overline{w}_T)) \equiv \sum_{j \in M} \frac{\exp\left\{V(j; \overline{w}_T + u_i, T) + \mu_0(\overline{w}_T) D_0(j \in M)\right\}}{\sum_{k \in \Omega} \exp\left\{V(k; \overline{w}_T + u_i, T) + \mu_0(\overline{w}_T) D_0(k \in M)\right\}} \tag{9}$$

where $\overline{w}_T + u_i = w_{iT}$. Assuming that the observed (or simulated) choices under the current tax-transfer regime T corresponds to an equilibrium, we must have:

$$\sum_i \pi_i(T, \overline{w}_T, \mu_0(\overline{w}_T)) = J(\overline{w}_T). \tag{10}$$

In a comparative statics perspective, an analogous condition must hold under the “reform” R:

$$\sum_i \pi_i(R, \overline{w}_R, \mu_0(\overline{w}_R)) = J(\overline{w}_R) \tag{11}$$

where $\overline{w}_R$ denotes the new average equilibrium wage.

[5] The general case with many types of jobs is treated in Colombino (2010).

In the special case of a perfectly rigid demand (zero elasticity), the number of jobs remains fixed but the wage rate must be adjusted so that the number of people willing to work under the new regime is equal to the (fixed) number of jobs:

$$\sum_i \pi_i(R, \overline{w}_R, \mu_0(\overline{w}_R)) = J(\overline{w}_T)\,. \tag{12}$$

When the demand for labor is perfectly elastic, the market is always in equilibrium at the initial wage rate. However, since the number of working people in general will change under a new tax-transfer rule and since the number of jobs in equilibrium must be equal to the number of people willing to work, it follows that the parameter $\mu_0 = \ln\left(J/H\right)$ must change. Let us rewrite expression (5) as $J = He^{\mu_0}$. Then the equilibrium condition can be written as follows:

$$\sum_i \pi_i(R, \overline{w}_T, \mu_{0R}) = He^{\mu_{0R}}. \tag{13}$$

In this case $\overline{w}_T$ remains fixed. Instead μ_{0R} must be directly adjusted so as to fulfill condition (13). The case with fixed wage rate and the demand absorbing any change in supply at that wage, actually corresponds to the scenario implicitly assumed in most tax-transfer simulations. The assumption however is not correct since those simulations do not take condition (13) into account.

4. An empirical illustration

We illustrate the procedure outlined above with a simulation exercise of various hypothetical reforms of income support in Italy with a model of labor supply of couples and singles, estimated using EUROMOD version 27a[6]. The exercise accounts for equilibrium between the total number of jobs and the number of people willing to work (just one dummy as in Section 3). The equilibrium simulation procedure extends expressions (7) – (14) by accounting for behavioral differences between females and males and between singles and couples.[7]

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[6] A description of the EUROMOD version used in this paper can be found in Sutherland (2001).
[7] More details on the model, the simulation procedures, the reforms and the results are provided by Colombino (2011). The EUROMOD project is documented at www.iser.essex.ac.uk/research/euromod.

For gender $S = F, M$ we adopt the following empirical specification for expression:

$$J_S = K_S \overline{w}_S^{-\eta} \tag{14}$$

where K_S is a constant and $-\eta$ is the elasticity of labor demand. Expression (5) turns out to be as follows:

$$\mu_{0S}(\overline{w}_S) = \ln\left(\frac{K_S \overline{w}_S^{-\eta}}{H_S}\right). \tag{15}$$

Given J_S (observed or simulated under the current tax-transfer system), $\overline{w}_S$ (the mean of the estimated wage function), the estimated μ_{0S} and an imputed value of η we can use expressions (14) and (15) to retrieve H_S and K_S. In this exercise we use $\eta = 0, 0.5, 1, \infty$.

Hereafter we succinctly describe the policies. A key parameter is the threshold G defined as fraction (1, 0.75, 0.5) of the Poverty Line (scaled according to the number of household members).

Guaranteed Minimum Income (GMI): each household receives a transfer equal to $G - I$ provided $I < G$, where I denotes individual taxable income.

Universal Basic Income (UBI): each household receives an unconditional transfer equal to G .

Wage Subsidy (WS): each household receives a 10% subsidy on the gross hourly wage and her/his income is not taxed as long as her/his gross income (including the subsidy) does not exceed G.

GMI + WS and **UBI + WS** are mixed mechanisms where the transfer is coupled with the wage subsidy, where G is replaced by $0.5G$.

The policies are assumed to completely replace the current system of income support.[8] For each of the above five types we distinguish: a Flat version (F), in which the income support mechanism is matched with a fixed marginal tax rate t applied to individual incomes above G; a Progressive version (P), in which the income support mechanism is matched with a progressive tax (that replicates the current system but with marginal tax rates proportionally adjusted according to a constant τ) that applies to incomes exceeding G. The parameters t and τ are endogenously

[8] All the reforms have a universal coverage. The current system is based on categorical/local and mean-tested mechanisms and does not entail any general basic or minimum income policy.

determined within the reform simulation so that the total net tax revenue is equal to the one collected under the current tax-transfer system. Altogether we have 5 (types) × 3 (values of *a*) × 2 (tax rules) = 30 reforms.

5. Results

Table 1 illustrates the relevance of the simulation procedure by showing the policies ranked according to a social welfare criterion, for different values of η.[9] The rankings appear to be affected especially when the extreme cases are compared ($\eta = 0$ vs $\eta = \infty$). On the one hand, it is somewhat reassuring that the results of the no-equilibrium simulation are rather close to those obtained with the equilibrium procedure as long as η equals 0.5 or 1 (the more realistic scenarios) On the other hand, it is worthwhile noting that the common practice of not accounting for equilibrium adjustment of the wage rates is usually interpreted as a perfectly elastic demand scenario ($\eta = 0$). This interpretation is not correct: indeed the simulation performed under the correctly specified scenario with perfectly elastic demand produces results that are radically different from those produced by the no-equilibrium simulation.

[9] More detailed results are reported in Colombino (2011).

Table 1. Social Welfare rankings of the policies under different simulation procedures[(a)]

No equilibrium			Equilibrium η = 0			Equilibrium η = 0.5			Equilibrium η = 1			Equilibrium η = oo		
UBI+WS	P	1	GMI	F	1	UBI+WS	P	0.75	UBI+WS	P	0.75	CURRENT		
UBI	P	0.75	GMI	F	0.75	UBI	P	0.5	UBI	P	0.5	UBI+WS	P	0.5
UBI+WS	P	0.75	UBI	F	0.75	WS	P	0.75	UBI	P	0.75	GMI+WS	P	0.5
UBI	P	0.5	GMI	P	1	UBI+WS	P	0.5	WS	P	0.75	UBI	F	0.5
UBI	P	1	UBI	F	0.5	UBI	P	0.75	UBI+WS	P	0.5	UBI	P	0.5
WS	P	1	GMI	P	0.75	WS	P	0.5	WS	P	0.5	WS	P	0.5
UBI+WS	P	0.5	UBI	P	0.5	UBI	P	1	UBI	P	1	WS	F	0.5
WS	P	0.75	UBI+WS	F	1	GMI+WS	P	0.5	GMI+WS	P	0.5	GMI+WS	F	0.5
WS	P	0.5	UBI	F	1	GMI+WS	P	0.75	GMI+WS	P	0.75	UBI+WS	F	0.5
UBI	F	1	GMI	P	0.5	WS	F	1	WS	P	1	GMI	P	0.5
UBI	F	0.75	GMI	F	0.5	WS	P	1	WS	F	1	GMI	F	0.5
GMI+WS	P	1	UBI+WS	F	0.75	UBI	F	1	UBI	F	0.75	UBI+WS	P	0.75
GMI+WS	P	0.75	GMI+WS	F	1	UBI	F	0.75	UBI	F	1	GMI+WS	P	0.75
GMI+WS	P	0.5	UBI	P	0.75	UBI+WS	F	1	UBI+WS	F	1	WS	F	0.75
UBI+WS	F	1	GMI+WS	F	0.75	UBI	F	0.5	UBI	F	0.5	GMI+WS	F	0.75
UBI	F	0.5	UBI+WS	P	0.5	WS	F	0.75	GMI	P	0.5	WS	P	0.75
UBI+WS	F	0.75	UBI+WS	P	0.75	GMI	P	0.5	WS	F	0.75	UBI	F	0.75
GMI+WS	F	1	UBI+WS	F	0.5	UBI+WS	F	0.75	UBI+WS	F	0.75	UBI	P	0.75
GMI	P	0.5	GMI+WS	P	1	GMI+WS	F	1	GMI+WS	F	1	UBI+WS	F	0.75
GMI	P	1	GMI+WS	P	0.75	WS	F	0.5	GMI+WS	F	0.75	GMI	P	0.75
GMI	P	0.75	GMI+WS	P	0.5	GMI	P	0.75	WS	F	0.5	GMI	F	0.75
CURRENT			GMI+WS	F	0.5	UBI+WS	F	0.5	GMI	P	0.75	WS	F	1
UBI+WS	F	0.5	UBI+WS	P	1	GMI+WS	F	0.75	UBI+WS	F	0.5	GMI+WS	F	1
GMI+WS	F	0.75	WS	F	1	GMI+WS	P	1	UBI+WS	P	1	WS	P	1
GMI	F	1	WS	P	1	UBI+WS	P	1	GMI+WS	P	1	UBI+WS	P	1
GMI+WS	F	0.5	WS	P	0.5	GMI+WS	F	0.5	GMI+WS	F	0.5	GMI+WS	P	1
GMI	F	0.75	WS	F	0.75	CURRENT			GMI	F	0.75	UBI	F	1
GMI	F	0.5	WS	F	0.5	GMI	F	1	CURRENT			UBI+WS	F	1
WS	F	1	WS	P	0.75	GMI	F	0.5	GMI	F	1	UBI	P	1
WS	F	0.75	CURRENT			GMI	F	0.75	GMI	F	0.5	GMI	P	1
WS	F	0.5	UBI	P	1	GMI	P	1	GMI	P	1	GMI	F	1

[(a)] The policies are ranked in descending order (best one at the top). Social Welfare is measured as: (Average Individual Welfare) × (1 – Gini index of the distribution of Individual Welfare). Individual Welfare is the money metric maximum expected utility (using as reference the worst-off household). This is similar to the so-called Sen Social Welfare index and it can be rationalized as a member of a rank-dependent social welfare indexes (e.g. Aaberge and Colombino 2011). Each reform is identified by three pieces of information: the income support mechanism (GMI etc.), the F or P tax rule and the value of *a* (0.5, 0.75 or 1). For example, UBI+WS_F_0.75 denotes a policy where the income support mechanism is UBI+WS, the tax rule is Flat and G is 75% of the Poverty line. More details are given in Colombino (2011).

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