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Combining Microsimulation and Optimization to Identify Optimal Flexible Tax-transfer Rules

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Combining microsimulation and optimization to identify optimal flexible tax-transfer rules*

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Abstract

We use a behavioural microsimulation model embedded in a numerical optimization procedure in order to identify optimal (social welfare maximizing) tax-transfer rules. We consider the class of tax-transfer rules consisting of a universal basic income and a tax defined by a 4th degree polynomial. The rule is applied to total taxable household income. A microeconometric model of household, which simulates household labour supply decisions, is embedded into a numerical routine in order to identify – within the class defined above – the tax-transfer rule that maximizes a social welfare function. We present the results for five European countries: France, Italy, Luxembourg, Spain and United Kingdom. For most values of the inequality aversion parameter, the optimized rules provide a higher social welfare than the current rule, with the exception of Luxembourg. In France, Italy and Luxembourg the optimized rules are significantly different from the current ones and are close to a Negative Income Tax or a Universal basic income with a flat tax rate. In Spain and the UK, the optimized rules are instead close to the current rule. With the exception of Spain, the optimal rules are slightly disequalizing and the social welfare gains are due to efficiency gains. Nonetheless, the poverty gap index tends to be lower under the optimized regime.

JEL: H21, C18

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1. Introduction

Two main approaches to empirical optimal income taxation have been adopted so far.

The Analytical approach produces a general formula for optimal taxes. The seminal contribution is Mirrlees 1971, where the problem is solved by optimal control techniques. The so-called “sufficient statistics” (Saez 2001, 2002) approach permits to obtain a local solution simply by “perturbing” the optimum. Heathcote et al. (2015) develop a dynamic stochastic programming with a representative agent and a parametric tax rule.

The Computational approach locates the optimum within a class of tax rules using numerical procedures. Aaberge & Colombino (2006, 2012, 2013) identify optimal tax rule in Norway and Italy by iterating the simulation of a microeconometric model of labour supply over a tax parameter range. Colombino et al. (2010, 2013) and Colombino (2013, 2015) focussing on income support mechanisms, use microsimulation to scan the performance of alternative configurations of tax-transfer parameters. Islam and Colombino (2018) develop a systematic procedure that combines microeconometrics, microsimulation and numerical optimization in order to identify optimal rules within the Negative Income Tax + Flat Tax class.

In this paper we adopt the simulation approach. The class of tax-transfer rules (TTRs) is strictly speaking parametric (a 4^{th} degree polynomial). However, it is flexible enough to be judged close to a non-parametric rule. As a matter of fact, the actual “effective” tax rule presents a rather simple shape. Looking for an optimal rule, it seems reasonable to adopt a polynomial approximation, since it can approximately encompass the current rules and possible also significant departures from them.

2. Analytical approaches

2.1. Mirrlees

The analytical approach pioneered by Mirrlees (1971), can be summarized as follows. It considers a population of agents that differ only with respect to skill or productivity *n* with distribution function *F(n)* and probability density function *f(n)*. The agent's preferences are represented by a utility function *U(C, e)*, where *C* = income and *e* = "effort" or labour supply. The Government (i.e. the "principal") solves

$$
\begin{aligned}
&\max_{T(.)} \int_0^\infty S\left(U\left(ne_n - T(n), e_n\right)\right) f(n)dn \\
&s.t. \\
&\int_0^\infty T(n) f(n)dn \geq R \\
&e_n = \arg\max_e u\left(ne - T(n), e\right)
\end{aligned}
\tag{1}
$$

The first constraint is the public budget constraint. The second one – the so-called Incentive Compatibility Constraint – says that e_w is the labour supply level that maximizes the utility of the agent with productivity *n*. *S*(.) is a social welfare transformation of the individual utility levels, *T*(.) is a TTR to be determined optimally, *R* is the average tax revenue to be collected. As a simple example, by assuming a quasi-linear *U*(.) – i.e. no income effects – one can obtain:

$$
\begin{aligned}
&\frac{T'(n)}{1-T'(n)} = \left(1+\frac{1}{\eta}\right)\left(\frac{1-F(n)}{nf(n)}\right)\left(1-G(n)\right) \\
&T_0 = \int_0^\infty T(n)dF(n) - R
\end{aligned}
\tag{2}
$$

where *T'(n)* is the marginal tax rate (MTR) applied to agents with productivity *n* (who have income we_w), *G(n)* is a social weight – that depends on *S*( ) and *U*( ) – assigned to agents with productivity greater than or equal to *n* and η is the elasticity of *e* with respect to *n*. T_0 is a lump-sum paid to agents with no income. In this literature, it is common to label *U*(.), *S*(.), *F*(.), *f*(.), η and *R* as the "primitives" (or the basic characteristics of the economy). Therefore, to a given configuration of "primitives" there correspond an optimal TTR. The empirical application consists of computing optimal policies using theoretical formulas such as expression (2) – or generalizations of it – with imputed or calibrated "primitives" (e.g. Mirrlees 1971 and Tuomala 2010). In Mirrlees' original formulation, n and e are not directly observed by the planner. When it comes to empirical applications, n might be equated to the wage rate or recovered with a calibration procedure (Brewer et al. 2008). By assuming an explicit utility function and using $e_n = \arg\max_e u\left(ne - T(n), e\right)$ one can compute the gross income ne_n and express the solution (2) in terms of gross income.

2.2. The "sufficient statistics" approach.

Saez (2001, 2002) advocates an approach – labelled the "sufficient statistics" approach in Chetty (2009) – where the expressions for optimal TTR can be expressed solely in terms of directly observed variables and non-parametrically estimable parameters. However, those expressions are "snapshots" of the optimal solution and – except for special cases – do not permit to compute directly the optimal taxes. In Saez (2001) the following expression is obtained:

$$\frac{T'(z)}{1-T'(z)} = \left(\frac{1}{\eta_z}\right)\left(\frac{1-H(z)}{zh(z)}\right)\left(1-\Pi(z)\right) \qquad (3)$$

where z denotes taxable income, $h(z)$ and $H(z)$ are the density and distribution functions, $\Pi(z)$ is a social weight assigned to people with income greater than or equal to z and η_z is the elasticity of z with respect to $(1-T'(z))$. However, the optimal z and its distribution (and possibly also η_z) depend on the optimal tax function $T(.)$. Therefore, in order to be able to compute the optimal taxes we must specify how z, $H(z)$ and $h(z)$ depend on $T(.)$. In other words, we must go back to Mirrlees (1971) as in Saez (2001) and Brewer et al. (2008) or introduce some ad hoc assumptions as in Saez (2002). An interesting special case concerns the top marginal tax rate. There is evidence that in the range of the highest income levels, say above $\bar{z}$, the term $\frac{1-H(z)}{zh(z)}$ is approximately constant. Therefore, given an estimate of η_z, the top marginal tax rate $T'(\bar{z})$ can be compute as a function of the exogenous social welfare weight $\Pi(\bar{z})$.

3. The microsimulation-optimization approach

The analytical approach is a fundamental contribution since it sets the basic problem to be solved. Its empirical applications can also indicate promising directions of solution. However, in our view, it can be usefully complemented by adopting an approach that combines microeconometric modelling, microsimulation and numerical optimization in a consistent way. The background of our approach is represented by a series of papers where a numerical approach to optimal taxation is adopted. Islam and Colombino (2018) identify optimal tax-transfer rules within the NIT+FT class in eight European countries. Aaberge and Colombino (2006, 2013) identify optimal taxes for Norway within the class of 9-parameter piece-wise linear TTRs. Aaberge and Colombino (2012) perform a similar exercise for Italy. Aaberge and Flood (2008) and Ericson and Flood (2012) address the optimal reform of tax-transfer system in Sweden with particular focus on tax-credit policies. Blundell and Shephard (2012) design an optimal tax-transfer systems for lone mother in the UK. Closely related contributions are Fortin et al. (1993) and Sefton and Van de Ven (2009). Our methodology can be summarized as follows. First, we estimate a microeconometric model of household labour supply. The model accounts for both singles and couples, extensive and intensive responses, multidimensional source of welfare, heterogeneous preferences and quantity constraints. Second, given a flexible class of tax-transfer rules, we simulate the new household choices based on the estimated household preferences and compute the attained value of a Social Welfare function. This step replaces the Incentive-Compatibility Constraint of the Analytical Approach. We then apply a maximization algorithm that iterates step two in order to identify the optimal TTR belonging to that class. This step replaces the analytical characterization of the optimal solution. Sections 3.1, 3.3 and 3.4 hereafter follow the presentation provided in Islam and Colombino (2018).

3.1 The empirical model of household labour supply

We model the households as agents who can choose within an opportunity set Ω containing jobs or activities characterized by hours of work *h*, wage rate *w* and sector of market job *s* (wage employment or self-employment) and other characteristics (observed by the household but not by us). We define **h** and **w** as vectors with one element for the singles and two elements for the couples, $\mathbf{h} = \begin{pmatrix} h_F \\ h_M \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_F \\ w_M \end{pmatrix}$, where the subscripts F and M refer to the female and the male partner respectively. Analogously, in the case of couples, $\mathbf{s} = \begin{pmatrix} s_F \\ s_M \end{pmatrix}$. The above notation assumes that each household member can work only in one sector. We write the utility function of the i-th household at a (h, s) job as follows (Coda Moscarola et al. 2014):

$$U_i(\mathbf{h},\mathbf{s},\varepsilon;\boldsymbol{\tau}) = \mathbf{Y}_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \varepsilon \tag{4}$$

where:

$\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ are parameters to be estimated;

$\mathbf{Y}_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau})$ is a vector including

- $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s}; \boldsymbol{\tau})$ = household disposable income on a (**h,s**) job given the tax-benefit parameters $\boldsymbol{\tau}$;
- the square of the household disposable income $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s}; \boldsymbol{\tau})$ defined above;
- the product of disposable income $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s}; \boldsymbol{\tau})$ and household size N (interaction term);

$L_i(\mathbf{h})$ is a row vector including

- the leisure time (defined as the total number of available weekly hours (80) minus the hours of work h) of the two partners (for a couple) or of the individual (for a single): $L_{ig} = 80 - h_{ig}$, where $g = F, M$.
- the square of leisure time(s), $(L_{ig})^2$;
- the interaction(s) of leisure time(s) with household disposable income ($L_{ig} \times C_i$), with age of the couple's partners of the single, age square and three dummy variables indicating presence of children of different age range (any age, 0-6, 7-10);

ε is a random variable that measures the effect of unobserved (by the analyst) characteristics of the job-household match.

The opportunity set each individual can choose among is $\Omega = \{(0,0),(h_1,s),(h_2,s),(h_3,s)\}$, where (0,0) denotes a non-market "job" or activity (non-participation), h_1, h_2, h_3 are values drawn from the observed distribution of hours in each hour interval 1-26 (part time), 27-52 (full time), 52-80 (extra time) and sector indicator s is equal to 1 (wage employment) or 2 (self-employment).

A **(h,s)** job is "available" to household i with p.d.f. $f_i(\mathbf{h},\mathbf{s})$, which we call "opportunity density".

We estimate the labour supply models of couples and singles separately. In the case of singles, we have 7 alternatives, while in the case of couples, who make joint labour-supply decision, we combine the choice alternatives of two partners, thus getting 49 alternatives.

When computing the earnings of any particular job **(h, s)** we face the problem that the wage rates of sector *s* are observed only for those who work in sector *s*. Moreover, for individuals who are not working we do not observe any wage rate. To deal with this issue, we follow a two-stage procedure presented in Dagsvik and Strøm (2006) and also adopted in Coda-Moscarola et al. (2014). The procedure is analogous to the well-known Heckman correction for selectivity but is specifically appropriate for distribution assumed for ε .

By assuming the ε is i.i.d. Type I extreme value we obtain the following expression for the probability that household *i* holds a $(\mathbf{h}_i, \mathbf{s}_i)$ job (e.g. Aaberge and Colombino 2013)

$$P_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau}) = \frac{\exp\left\{\mathbf{Y}_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h}_i)'\boldsymbol{\lambda} + \ln f_i(\mathbf{h}_i, \mathbf{s}_i)\right\}}{\sum_{\mathbf{s}} \sum_{\mathbf{h} \in \Omega} \exp\left\{\mathbf{Y}_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \ln f_i(\mathbf{h}, \mathbf{s})\right\}} \tag{5}$$

By choosing a convenient (uniform with peaks") specification for the opportunity density *f(.,.)* it turns out that expression (5) can be rewritten as follows (e.g. Aaberge and Colombino 2013, Colombino 2013),

$$P_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau}) = \frac{\exp\left\{\mathbf{Y}_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h}_i)'\boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{h}_i, \mathbf{s}_i)'\boldsymbol{\delta}\right\}}{\sum_{\mathbf{s}} \sum_{\mathbf{h} \in \Omega} \exp\left\{\mathbf{Y}_i(\mathbf{h}, \mathbf{s}; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \mathbf{D}_i(\mathbf{h}, \mathbf{s})'\boldsymbol{\delta}\right\}} \tag{6}$$

where, for a single household, $\mathbf{D}_i$ is the vector (with 1[.] denoting the indicator function)

$$\begin{aligned}
D_{1,0} &= 1\left[s = 1, h > 0\right], \\
D_{1,1} &= 1\left[s = 1, 1 \le h \le 26\right], \\
D_{1,2} &= 1\left[s = 1, 27 \le h \le 52\right], \\
D_{2,0} &= 1\left[s = 2, h > 0\right], \\
D_{2,1} &= 1\left[s = 2, 1 \le h \le 26\right], \\
D_{2,2} &= 1\left[s = 2, 27 \le h \le 52\right].
\end{aligned} \tag{7}$$

and $\boldsymbol{\delta}$ is vector of parameters to be estimated. For couples, $\mathbf{D}_i$ contains two analogous sets of variables, one for each partner.

The model is a simplified version of the so-called RURO model (Aaberge and Colombino 2014). The main simplification concerns the wage rates. In the most general versions of the RURO model (e.g. Aaberge and Colombino 2013) the wage rates densities are estimated simultaneously with the preference parameters and the hours' opportunity density. In this paper we use instead pre-estimated wage densities.

The estimates of $(\gamma, \lambda, \delta)$ for couples (32 parameters), singles females (17 parameters) and single males (17 parameters) in all the eight countries are reported in Appendix D.

The datasets used in the analysis are the EUROMOD input data based on the European Union Statistics on Income and Living Conditions (EU-SILC) for the year 2015 in France, Italy, Luxembourg, Spain and the United Kingdom. The input data provide all required information on demographic characteristics and human capital, employment and wages of household members, as well as information about various sources of non-labour income. We apply common sample selection criteria for all countries under study by selecting individuals in the age range 18-55 who are not retired or disabled. EUROMOD[1] is used for two different operations. First, for every household in the sample computes the net available income under the current TTR at each of the 49 (7) alternatives available to the couples (singles). The net available incomes are used in the estimation of the labour supply model. Second, for each household, it computes the gross income at each alternative. Gross incomes are used in the simulation and optimization steps, where EUROMOD is not used anymore and new values of net available incomes are generated by applying the new TTRs to the gross incomes.

The estimates for the labour supply model for couples and singles in France, Italy, Luxembourg, Spain and the United Kingdom are reported in the Appendix.

3.2 Polynomial tax-transfer rule

We look for optimal tax-transfer rules within the class of rules defined as a polynomial functions of total taxable income $y_i = \mathbf{w}_i^{'}\mathbf{h}_i + I_i - \mathrm{S}_i$ where S_i denotes social security contributions:

$$C_i = \tau_0\sqrt{N_i} + \tau_1 y_i + \tau_2 y_i^2 + \tau_3 y_i^3 + \tau_4 y_i^4$$

[1] EUROMOD is a large-scale pan-European tax-benefit static micro-simulation engine (e.g. Sutherland and Figari, 2013). It covers the tax-benefit schemes of the majority of European countries and allows computation of predicted household disposable income, on the basis of gross earnings, employment and other household characteristics.

where y_i (= household total taxable income) and N_i = household size. The parameter τ_0 is constrained to be greater than or equal to zero (lump-sum taxes are ruled-out). A pure flat tax rule is the special case $C_i = \tau_1 y_i$. A negative income tax matched with a flat tax corresponds to $C_i = \tau_0\sqrt{N_i} + \tau_1 y_i$. In general, the rule can be interpreted as a negative income tax or a basic income matched with a generic non-linear tax rule. In the former case $\tau_0\sqrt{N_i}$ is the guaranteed minimum income when $y_i = 0$, in the latter case it is a basic income. The term $\sqrt{N_i}$ rescales the guaranteed minimum income or the basic income according to the household size (square root rule). The rule is sufficiently flexible to represent many alternative versions of non-linear tax rules. The tax, the marginal tax rate and the average tax rate are, respectively:

$$T(y;\boldsymbol{\tau}) = y - \tau_0\sqrt{N} - \tau_1 y - \tau_2 y^2 - \tau_3 y^3 - \tau_4 y^4$$

$$MT(y;\boldsymbol{\tau}) = \frac{\partial T(y;\boldsymbol{\tau})}{\partial y} = 1 - \tau_1 - 2\tau_2 y \ - 3\tau_3 y^2 - 4\tau_4 y^3$$

$$AT(y;\boldsymbol{\tau}) = \frac{T(y;\boldsymbol{\tau})}{y} = 1 - \frac{\tau_0\sqrt{N}}{y} - \tau_1 - \tau_2 y \ - \tau_3 y^2 - \tau_4 y^3$$

3.3 Comparable Money-metric Utility

Based on the estimated model described in Section 3, we define hereafter the Comparable Money-metric Utility (CMU). This index transforms the household utility level into an inter-household comparable monetary measure that will enter as argument of the Social Welfare function (to be described in Section 4.2). First, we calculate the expected maximum utility attained by household i under tax-transfer regime $\boldsymbol{\tau}_i$ (e.g. McFadden 1978):

$$E(\max U_i \setminus \boldsymbol{\tau}) = \ln\left(\sum_{\mathbf{s}}\sum_{\mathbf{h}\in\Omega} \exp\left\{\mathbf{Y}_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda}\right\}\right) \tag{8}$$

Analogously, we define

$$E(\max U_R \setminus \boldsymbol{\tau}_R) = \ln\left(\sum_{\mathbf{s}}\sum_{\mathbf{h}\in\Omega} \exp\left\{\mathbf{Y}_R(\mathbf{h},\mathbf{s};\boldsymbol{\tau}_R)'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda}\right\}\right) \tag{9}$$

as the expected maximum utility attained by the "reference" household R under the "reference" tax-transfer regime $\boldsymbol{\tau}_R$. The reference household is the couple household at the median value of the distribution of $E(\max U \setminus \boldsymbol{\tau}_R)$. The CMU of household i under tax regime $\boldsymbol{\tau}$, $\mu_i(\boldsymbol{\tau})$, is defined as the gross income that a reference household under a reference tax-transfer regime $\boldsymbol{\tau}_R$ would need in order to attain an expected maximum utility equal to $E(\max U_i \setminus \boldsymbol{\tau})$. The CMU is analogous to the "equivalent income" defined by King (1983).[2] Although the choice of the reference household is essentially arbitrary, some choices make more sense than others. Our choice of the median household as reference household can be justified in terms of representativeness or centrality of its preferences. Aaberge and Colombino (2006, 2013) adopt a related, although not identical, procedure that consists of using a common utility function as argument of the social welfare function (Deaton and Muelbauer, 1980). A significant portion of the empirical policy evaluation literature is silent upon the issue of interpersonal preference comparability. Theoretical models or general equilibrium models typically assume identical preferences or a representative individual, so that the problem is absent by construction. In the empirical literature based on microdata and micro-modelling, frequently either income is interpreted as an index of welfare or the utility levels are directly used, maybe under the assumption that the solution of the comparability problem is somehow implicitly accounted for by the social welfare function. We follow here the tradition of Deaton and Muellbauer (1980) and King (1983), which, in our view, is both practical and theoretically sound.

[2] The basic idea is using the preferences of the "reference household" in the same way as reference prices are used in computing equivalent or compensating variations for comparing utility levels attained under different budget sets.

3.4 Social Welfare function

We choose Kolm (1976) Social Welfare index, which can be defined as:

$$W = \bar{\mu} - \frac{1}{k}\ln\left[\sum_{\mathrm{i}} \frac{\exp\left\{-k\left(\mu_{\mathrm{i}} - \bar{\mu}\right)\right\}}{\mathrm{N}}\right] \tag{10}$$

where

$\bar{\mu} = \frac{1}{N}\sum_i \mu_i$ is an index of Efficiency ,

$\frac{1}{k}\ln\left[\sum_{\mathrm{i}} \frac{\exp\left\{-k\left(\mu_{\mathrm{i}} - \bar{\mu}\right)\right\}}{\mathrm{N}}\right]$ = Kolm Inequality Index,

k = Inequality Aversion parameter,

μ_i = comparable money-metric utility of household i (defined in Section 4.1).

W has limit $\bar{\mu}$ as $k \to 0$ and $\min\left\{\mu_1,...,\mu_N\right\}$ as $k \to \infty$.

The meaning of k might be clarified by the following example. Let us take two individuals with $\mu_2 - \mu_1 = 1$. Given the social marginal evaluation of μ_i, $\frac{\partial W}{\partial \mu_i} = \frac{e^{-k\mu_i}}{e^{-k\mu_1} + e^{-k\mu_2}}$, we get the social marginal rate of substitution: $SMRS_{1,2} = e^{k(\mu_2 - \mu_1)} = e^k$. Now let us consider a (small) transfer $\tau < 1$ from individual 2 to individual 1 in order to reduce the inequality. Note that the social planner would be willing to take $\exp\left\{k\right\}\tau$ from individual 2 in order to give τ to individual 1. Since $\exp\left\{k\right\} \geq 1$, $\exp\left\{k\right\} - 1$ measures (approximately) the "excess willingness to pay" for a "inequality reducing" transfer from individual 2 to individual 1:

k	0.05	0.10	0.25	0.50
$\exp\left\{k\right\} - 1$	0.051	0.105	0.284	0.649

Kolm Inequality Index is an absolute index, meaning that it is invariant with respect to translations (i.e. to adding a constant to every μ_i). Absolute indexes are less popular than relative indexes (e.g. Gini's or Atkinson's), although there is no strict logical or economic motivation for preferring one to the other.[3] Blundell and Shephard (2012) adopt a social welfare index which turns out to be very close to Kolm's index. Their main motivation for their index seems to be the computational convenience, since it handles negative numbers (random utility levels, in their case). Our motivation in choosing Kolm's index is analogous. In our case, μ_i is a monetary measure, yet it can happen to be negative when the utility level of household i is very far from the utility level of the reference household. Kolm's index handles negative arguments. Alternatively, it is also possible to shift the μ_i-s by adding a constant (which would not be allowed with a relative index).

3.5 Identifying the optimal policies

In order to highlight both the differences and the analogies between the microeconometric-computational approach with respect to the Mirrlees-Saez tradition, we start with a formulation of the former as close as possible to Mirrlees' problem (1):

$$\begin{aligned}&\max_{\boldsymbol{\tau}} W\left(\mu_1(\boldsymbol{\tau}),...,\mu_H(\boldsymbol{\tau})\right)\\&\text{s.t.}\\&\sum_{i=1}^{H}\sum_{\mathbf{s}}\sum_{\mathbf{h}\in\Omega}\left[P_i\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}\right)T\left(\mathbf{w}_i'\mathbf{h};\boldsymbol{\tau}\right)\right]\geq R\end{aligned} \tag{11}$$

The constraint requires that the total expected net tax revenue must be greater than (or equal to) a given amount *R*. Note that problem (11) assumes that the households are maximizing their utility functions, since the arguments of W are the (money-metric) maximized utilities.

The solution process proceeds as follows.

1. Start with an initial $\boldsymbol{\tau}^0$
2. Compute the comparable money metric measures $\mu_1(\boldsymbol{\tau}^0),...,\mu_H(\boldsymbol{\tau}^0)$

[3] Atkinson and Brandolini (2010) provide a discussion of relative indexes, absolute indexes and intermediate cases.

3. Compute $W\left(\mu_1(\boldsymbol{\tau}^0),\ldots,\mu_H(\boldsymbol{\tau}^0)\right)$

4. Compute $\sum_{i=1}^{H}\sum_{\mathbf{s}}\sum_{\mathbf{h}\in\Omega}\left[P_i\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}^0\right)T\left(\mathbf{w}_\mathbf{i}^{'}\mathbf{h};\boldsymbol{\tau}^0\right)\right]$

5. Iterate until W is maximized and $\sum_{i=1}^{H}\sum_{\mathbf{s}}\sum_{\mathbf{h}\in\Omega}\left[P_i\left(\mathbf{h},\mathbf{s};\boldsymbol{\tau}\right)T\left(\mathbf{w}_\mathbf{i}^{'}\mathbf{h};\boldsymbol{\tau}\right)\right]\geq R$ is satisfied.

4. Results and concluding remarks.

The main results are reported in Tables 1-5. For each of the five countries France, Italy, Luxembourg, Spain and the United Kingdom and for different values of the inequality aversion parameter *k* (0, 0.05, 0.075, 0.10, 0.125, 0.15) the Tables shows the tax-transfer parameters $\tau_0,\ldots,\tau_4$, various economic results (gross income, taxable income, disposable income, employment, hours worked, poverty gap index) and social welfare components (welfare, components, inequality). Below each component of social welfare, we also report the corresponding values for the current system. The column "Approximated Current" reports the parameters of 4th degree polynomial to the actual current system. Note that The "Approximated Current" τ_0 is not strictly comparable to the optimized values since the last ones are universal transfers to be received with certainty, while the former is an expected value across the population. In the same column, however, the results are the real current ones. For most values of *k*, the optimal polynomial rules provide a higher social welfare than the current rule, with the exception of Luxembourg. With the exception of Spain, the optimal rules are slightly disequalizing and the social welfare gains are due to efficiency gains. Nonetheless, the poverty gap index tends to be lower under the optimized regime.

The Graphs 1-5 show the optimal and the approximated current tax-transfer rules and the corresponding marginal tax rates (MTRs). The differences are best highlighted by the marginal tax rates. In Spain and the United Kingdom, the profiles of the optimal MTRs are very similar to the (approximated) current ones, while they are extremely different in France, Italy and Luxembourg. Overall, in these last three countries, the optimal tax profiles are similar and much flatter than the current one. The current systems in France and Luxembourg appear to envisage relatively generous income support policies at low or zero income followed by very high implicit marginal benefit reduction rates. The optimal rules suggest less generous income support and a longer and smoother phase-out. Under this perspective, Italy seems to represent as *unicum*, where current MTRs are first steeply increasing up to taxable incomes around 100000 and then decreasing. An explanation of these difference among countries requires to identify a general relationship between the basic ("primitive") characteristics of the economies and the features of the optimal tax-transfer rules. This type of analysis is just sketched and exemplified in Islam and Colombino (2018). A full analysis on a large sample of countries is left for future work.

Table 1. Tax-transfer parameters of current (polynomial approximation) system, optimal systems and main effects. France

	Approximated Current	Optimal, k = .00	Optimal, k = .05	Optimal, k= .075	Optimal, k = .10	Optimal, k =.125	Optimal, k = .15
τ_0	603.27	61.43	181.72	265.57	367.97	453.77	592.02
τ_1	0.521	0.930	0.877	0.839	0.795	0.759	0.700
τ_2	3.01×10^{-6}	-0.0000×10^{-6}	0.0106×10^{-6}	0.013×10^{-6}	0.011×10^{-6}	0.0098×10^{-6}	0.0097×10^{-6}
τ_3	-1.51×10^{-11}	0.0004×10^{-11}	0.0062×10^{-11}	0.015×10^{-11}	0.005×10^{-11}	0.0064×10^{-11}	0.0053×10^{-11}
τ_4	0.20×10^{-16}	0.0009×10^{-16}	0.008×10^{-16}	0.014×10^{-16}	0.017×10^{-16}	0.0098×10^{-16}	0.0051×10^{-16}
Gross income	6087.26	6232.77	6200.95	6177.67	6146.37	6120.34	6077.91
Taxable income	3848.67	3937.91	3918.52	3904.33	3885.23	3869.33	3843.42
Disposable income	3612.08	3755.70	3725.04	3699.48	3667.52	3644.35	3604.65
Weekly hours	35.98	36.54	36.41	36.32	36.20	36.10	35.93
Employment %	0.92	0.92	0.92	0.92	0.92	0.92	0.92
Poverty gap %	0.04	0.08	0.07	0.06	0.05	0.04	0.03
Optimized Welfare	-	7986.23	7755.07	7645.56	7540.40	7451.46	7354.70
Current Welfare	-	7827.00	7665.44	7592.39	7524.03	7460.06	7400.17
Optimized Efficiency	-	7986.23	7936.94	7900.82	7855.77	7820.26	7761.23
Current Efficiency	-	7827.00	7827.00	7827.00	7827.00	7827.00	7827.00
Optimized Inequality	-	0	181.87	255.26	315.37	368.80	406.53
Current Inequality	-	0	161.56	234.62	302.97	366.94	426.83

Notes to the Table:

- The parameters of the "Current system" are computed with a polynomial approximation. The "Current" τ_0 is not strictly comparable to the optimized values since the last ones are universal transfers to be received with certainty, while the former is an expected value across the population.
- All income variables are the monthly per household average
- Hours and employment are individual averages
- Poverty Gap is referred to households

Table 2. Tax-transfer parameters of current (polynomial approximation) system, optimal systems and main effects. Italy

	Approximated Current	Optimal, k = .00	Optimal, k = .05	Optimal, k= .075	Optimal, k = .10	Optimal, k =.125	Optimal, k = .15
τ_0	217.24	98.99	177.28	236.69	270.67	350.15	417.72
τ_1	0.745	0.752	0.698	0.655	0.631	0.575	0.526
τ_2	-1.98×10^{-6}	-0.02×10^{-6}	-0.01×10^{-6}	0.002×10^{-6}	0.005×10^{-6}	0.0002×10^{-6}	0.0008×10^{-6}
τ_3	0.69×10^{-11}	0.04×10^{-11}	0.01×10^{-11}	0.004×10^{-11}	0.004×10^{-11}	0.0000×10^{-11}	-0.0004×10^{-11}
τ_4	-0.07×10^{-16}	-0.02×10^{-16}	-0.01×10^{-16}	-0.0000×10^{-16}	0.003×10^{-16}	0.0005×10^{-16}	0.0025×10^{-16}
Gross income	3215.89	3238.67	3224.09	3213.70	3207.96	3191.92	3179.07
Taxable income	2279.76	2296.48	2286.13	2278.90	2274.96	2263.3	2254.26
Disposable income	1851.96	1874.84	1860.22	1849.77	1844.00	1827.97	1814.94
Weekly hours	28.68	28.77	28.66	28.59	28.54	28.44	28.34
Employment %	79.58	79.60	79.41	79.27	79.18	78.98	78.81
Poverty gap %	19.00	19.63	17.01	15.06	13.96	11.42	9.33
Optimized Welfare	-	4227.57	3742.16	3503.35	3278.49	3050.10	2831.62
Current Welfare	-	4154.67	3696.23	3474.95	3259.85	3051.45	2850.16
Optimized Efficiency	-	4227.57	4203.60	4186.12	4176.28	4153.19	4132.96
Current Efficiency	-	4154.67	4154.67	4154.67	4154.67	4154.67	4154.67
Optimized Inequality	-	0	461.44	682.77	897.79	1103.09	1301.34
Current Inequality	-	0	458.44	679.71	894.82	1103.22	1304.50

Notes to the Table:

- The parameters of the "Current system" are computed with a polynomial approximation. The "Current" τ_0 is not strictly comparable to the optimized values since the last ones are universal transfers to be received with certainty, while the former is an expected value across the population.
- All income variables are the monthly per household average
- Hours and employment are individual averages
- Poverty Gap is referred to households

Table 3. Tax-transfer parameters of current (polynomial approximation) system, optimal systems and main effects. Luxembourg

	Approximated Current	Optimal, k = .00	Optimal, k = .05	Optimal, k= .075	Optimal, k = .10	Optimal, k =.125	Optimal, k = .15
τ_0	1469.68	615.73	680.78	746.64	809.23	858.74	926.17
τ_1	0.316	0.761	0.75	0.717	0.706	0.692	0.676
τ_2	4.12×10^{-6}	0.228×10^{-6}	0.24×10^{-6}	0.23×10^{-6}	0.245×10^{-6}	0.227×10^{-6}	0.223×10^{-6}
τ_3	-1.869×10^{-11}	0.019×10^{-11}	0.022×10^{-11}	0.014×10^{-11}	0.051×10^{-11}	0.041×10^{-11}	0.013×10^{-11}
τ_4	0.25×10^{-16}	0.07×10^{-16}	0.017×10^{-16}	0.075×10^{-16}	0.008×10^{-16}	0.009×10^{-16}	-0.004×10^{-16}
Gross income	6111.06	6275.14	6247.83	6242.82	6216.56	6201.31	6175.85
Taxable income	4819.60	4955.11	4932.66	4929.36	4907.74	4895.52	4874.90
Disposable income	4733.65	4897.22	4870.26	4865.09	4839.08	4823.82	4798.50
Weekly hours	34.20	35.07	34.93	34.88	34.75	34.66	34.52
Employment %	0.88	0.89	0.89	0.89	0.89	0.89	0.88
Poverty gap %	0.04	0.07	0.06	0.06	0.05	0.05	0.04
Optimized Welfare	-	5910.04	4376.92	3650.13	3012.46	2420.53	1892.11
Current Welfare	-	5907.98	4412.55	3718.48	3075.09	2488.07	1959.05
Optimized Efficiency	-	5910.04	5914.17	5882.67	5891.79	5880.70	5868.98
Current Efficiency	-	5907.98	5907.98	5907.98	5907.98	5907.98	5907.98
Optimized Inequality	-	0	1537.25	2232.54	2879.34	3460.17	3976.87
Current Inequality	-	0	1495.43	2189.50	2832.89	3419.91	3948.93

- The parameters of the "Current system" are computed with a polynomial approximation. The "Current" τ_0 is not strictly comparable to the optimized values since the last ones are universal transfers to be received with certainty, while the former is an expected value across the population.
- All income variables are the monthly per household average
- Hours and employment are individual averages
- Poverty Gap is referred to households

Table 4. Tax-transfer parameters of current (polynomial approximation) system, optimal systems and main effects. Spain

	Approximated Current	Optimal, k = .00	Optimal, k = .05	Optimal, k= .075	Optimal, k = .10	Optimal, k =.125	Optimal, k = .15
τ_0	196.42	247.44	248.46	359.21	391.92	470.85	584.95
τ_1	0.969	0.93	0.93	0.85	0.83	0.77	0.68
τ_2	-4.236×10^{-6}	-4.237×10^{-6}	-4.243×10^{-6}	-4.256×10^{-6}	-4.279×10^{-6}	-4.311×10^{-6}	-4.315×10^{-6}
τ_3	2.051×10^{-11}	2.049×10^{-11}	2.051×10^{-11}	2.052×10^{-11}	2.040×10^{-11}	2.030×10^{-11}	2.034×10^{-11}
τ_4	-0.361×10^{-16}	-0.369×10^{-16}	-0.376×10^{-16}	-0.427×10^{-16}	-0.483×10^{-16}	-0.697×10^{-16}	-0.721×10^{-16}
Gross income	3165.55	3142.89	3142.27	3099.01	3084.44	3047.95	3002.39
Taxable income	2326.88	2313.85	2313.36	2280.73	2269.61	2241.71	2207.66
Disposable income	2233.42	2211.22	2210.65	2167.36	2153.15	2115.96	2070.48
Weekly hours	31.68	31.48	31.48	31.19	31.09	30.86	30.55
Employment %	0.82	0.81	0.81	0.81	0.80	0.80	0.79
Poverty gap %	0.11	0.08	0.08	0.06	0.05	0.03	0.01
Optimized Welfare	-	9044.29	8948.40	8909.59	8866.79	8825.64	8786.96
Current Welfare	-	9050.96	8946.08	8894.10	8842.45	8791.12	8740.13
Optimized Efficiency	-	9044.29	9044.41	9048.84	9050.32	9050.04	9052.01
Current Efficiency	-	9050.96	9050.96	9050.96	9050.96	9050.96	9050.96
Optimized Inequality	-	0	96.01	139.25	183.53	224.41	265.05
Current Inequality	-	0	104.88	156.86	208.52	259.85	310.83

Notes to the Table:

- The parameters of the "Current system" are computed with a polynomial approximation. The "Current" τ_0 is not strictly comparable to the optimized values since the last ones are universal transfers to be received with certainty, while the former is an expected value across the population.
- All income variables are the monthly per household average
- Hours and employment are individual averages
- Poverty Gap is referred to households

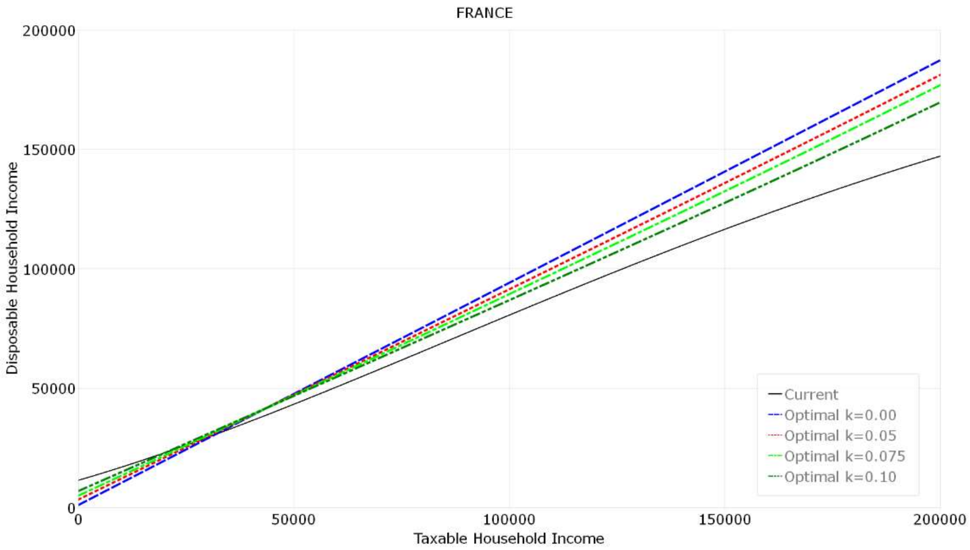
Table 5. Tax-transfer parameters of current (polynomial approximation) system, optimal systems and main effects. United Kingdom

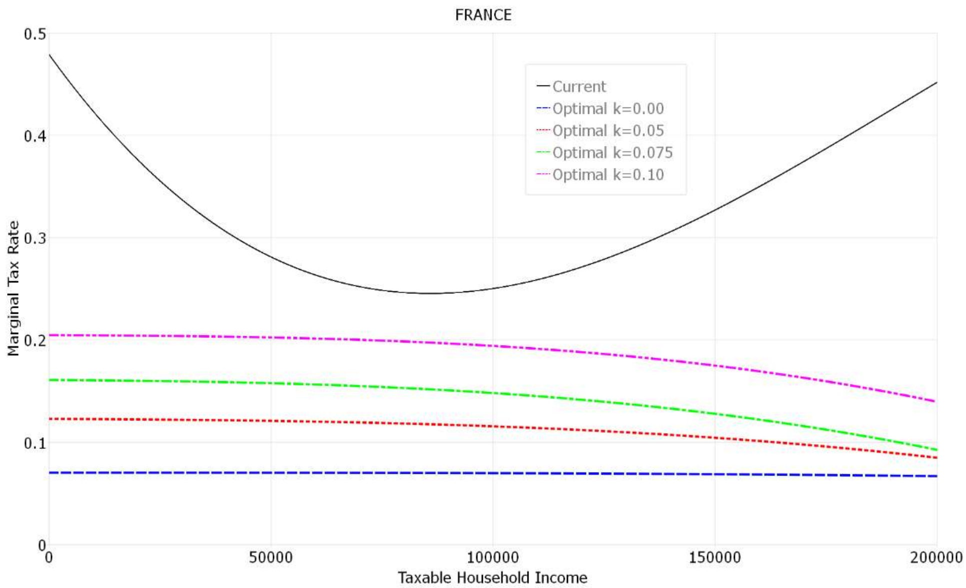
	Approximated Current	Optimal, k = .00	Optimal, k = .05	Optimal, k= .075	Optimal, k = .10	Optimal, k =.125	Optimal, k = .15
τ_0	455.51	265.79	534.69	537.53	608.13	796.91	1181.5
τ_1	0.66	0.77	0.59	0.59	0.55	0.42	0.15
τ_2	1.717×10^{-6}	1.83×10^{-6}	1.84×10^{-6}	1.88×10^{-6}	1.71×10^{-6}	1.64×10^{-6}	1.66×10^{-6}
τ_3	-2.008×10^{-11}	-2.255×10^{-11}	-2.27×10^{-11}	-2.274×10^{-11}	-2.066×10^{-11}	-2.043×10^{-11}	-2.072×10^{-11}
τ_4	0.505×10^{-16}	0.798×10^{-16}	0.771×10^{-16}	0.777×10^{-16}	0.559×10^{-16}	0.401×10^{-16}	0.355×10^{-16}
Gross income	2681.72	2688.59	2683.77	2683.44	2683.07	2678.40	2658.85
Taxable income	2287.05	2292.94	2288.95	2288.66	2288.41	2284.63	2268.54
Disposable income	2273.79	2280.44	2275.37	2275.23	2274.80	2270.32	2251.23
Weekly hours	28.20	28.36	28.23	28.22	28.20	28.11	27.91
Employment %	0.76	0.77	0.76	0.76	0.76	0.76	0.75
Poverty gap %	0.13	0.12	0.05	0.05	0.04	0.002	0.000
Optimized Welfare	-	4711.92	4558.34	4456.57	4371.44	4302.72	4229.72
Current Welfare	-	4740.34	4529.40	4427.32	4327.77	4230.93	4136.92
Optimized Efficiency	-	4711.92	4768.56	4768.57	4782.92	4811.95	4834.59
Current Efficiency	-	4740.34	4740.34	4740.34	4740.34	4740.34	4740.34
Optimized Inequality	-	0	210.22	312.01	411.47	509.24	604.87
Current Inequality	-	0	210.94	313.02	412.57	509.41	603.41

Notes to the Table:

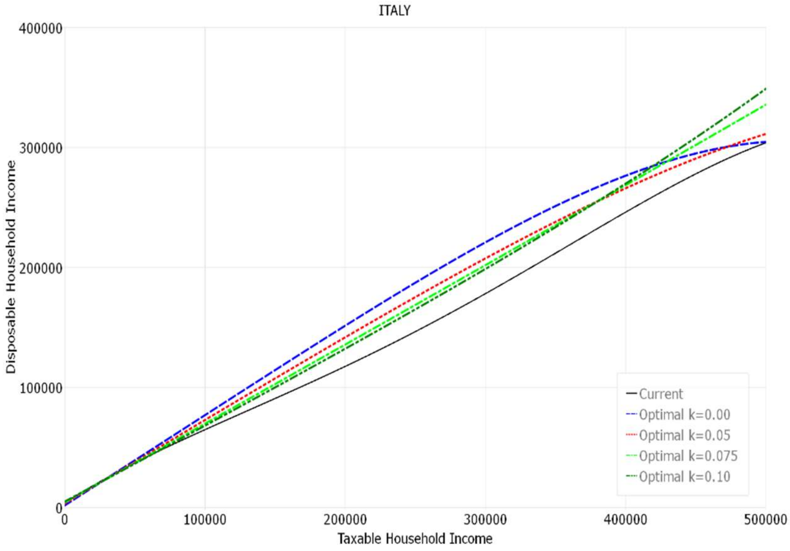
- The parameters of the "Current system" are computed with a polynomial approximation. The "Current" τ_0 is not strictly comparable to the optimized values since the last ones are universal transfers to be received with certainty, while the former is an expected value across the population.
- All income variables are the monthly per household average
- Hours and employment are individual averages
- Poverty Gap is referred to households

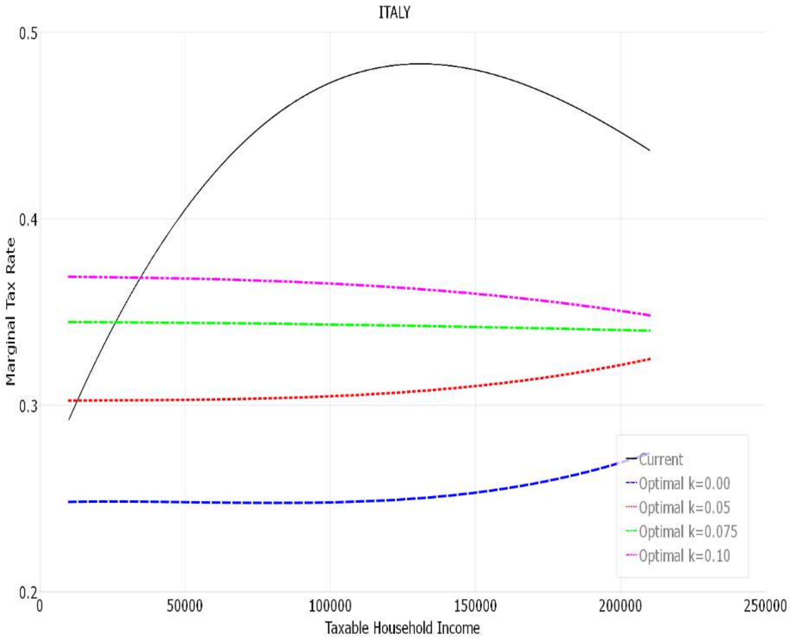
Graph1. Tax- transfer rules and marginal tax rates. France



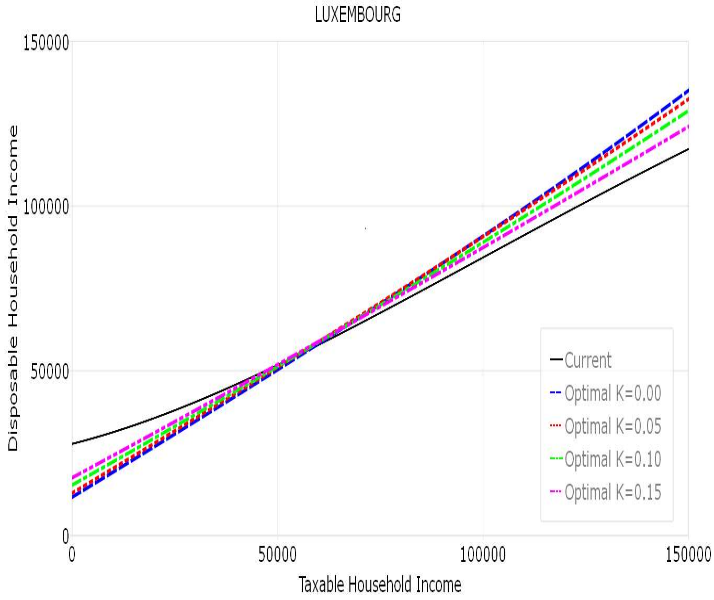


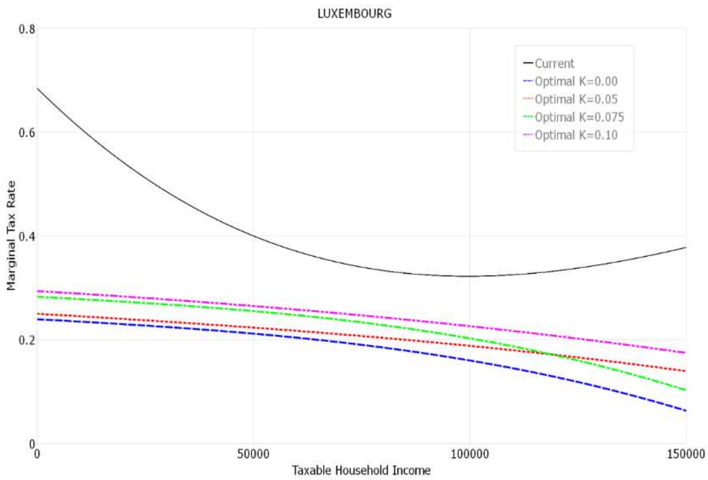
Graph 2. Tax- transfer rules and marginal tax rates. Italy



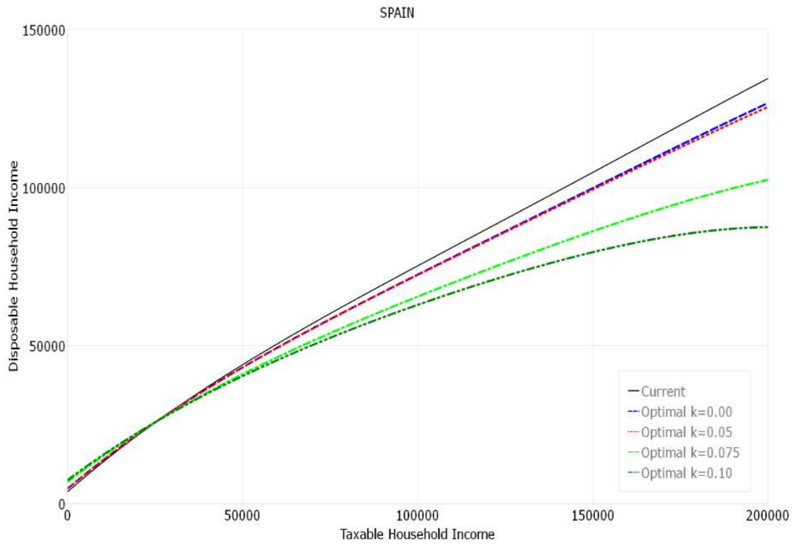


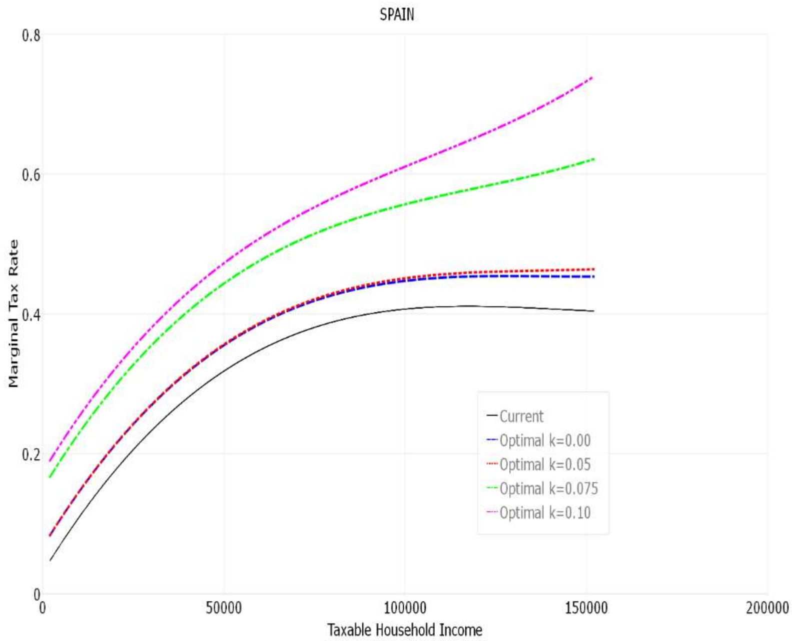
Graph 3. Tax- transfer rules and marginal tax rates. Luxembourg



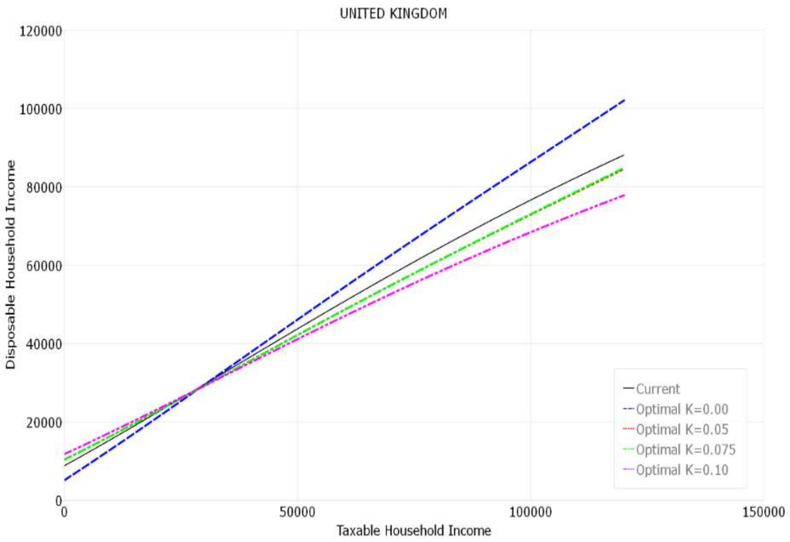


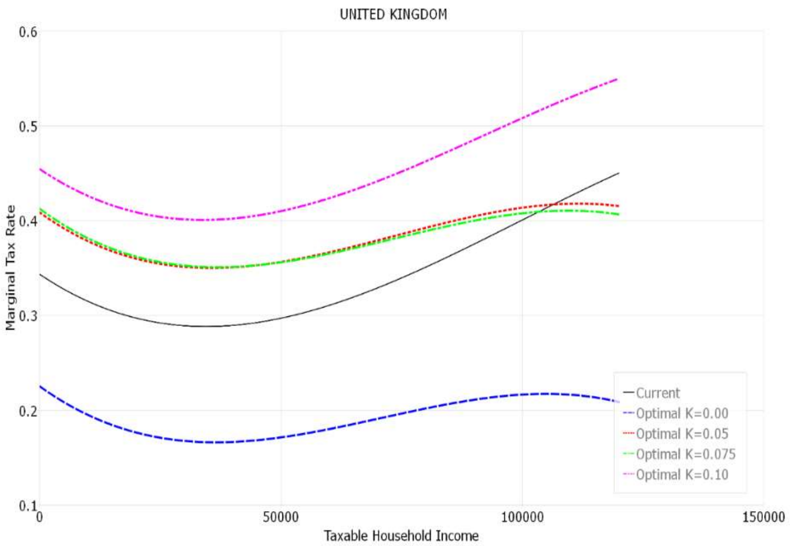
Graph 4. Tax- transfer rules and marginal tax rates. Spain





Graph 5. Tax- transfer rules and marginal tax rates. United Kingdom





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Appendix. Estimates of the Labour Supply Model

Table D1 – Maximum likelihood estimates – couples (France)

Model component	Variable	Coef.	Std. Err.
Opportunity density		δ	
	Employee_Man	0.4761696	0.3665576
	Self-employed_Man	0.2130577	0.3805561
	Employee_Woman	-0.3649212	0.2853927
	Self-employed_Woman	-1.426779	0.3241603
	Part-time_Employee_Man	-0.3805255	0.2414433
	Full-time_Employee_Man	2.83453	0.1249029
	Part-time_Self-employed_Man	-1.841048	0.324269
	Full-time_Self-employed_Man	0.2870089	0.1540075
	Part-time_Employee_Woman	0.6361778	0.2170085
	Full-time_Employee_Woman	2.698627	0.1676399
	Part-time_Self-employed_Woman	-1.014395	0.3149686
	Full-time_Self-employed_Woman	0.6781008	0.2277644
Y vector		γ	
	Household_Disposable_income	0.0003342	0.0001334
	Hosuhold_Disposable_income squared	1.61E-08	6.75E-09
	Household_size X Household_disposable_income	-0.0000513	0.0000175
L vector		λ	
	Leisure_Male	0.1256514	0.0281893
	Leisure_Man squared	0.0000173	0.0001373
	Leisure_Woman	0.163189	0.0255661
	Leisure_Woman squared	-0.000107	0.0001529
	Leisure_Man X Household_disp_income	-7.88E-06	1.02E-06
	Leisure_Woman X Household_disp_income	-1.54E-07	8.04E-07
	Leisure_Man X Age_Man	-0.0059183	0.0011829
	Leisure_Woman X Age_Woman	-0.0085386	0.0009848
	Leisure_Man X Age_Man squared	0.0000742	0.0000138
	Leisure_Woman X Age_Woman squared	0.0001108	0.000012
	Leisure_Man X No. Children	-0.0026133	0.0017907
	Leisure_Woman X No. Children	0.0084813	0.0015617
	Leisure_Man X No. Children0-6	0.0027957	0.0025747
	Leisure_Man X No. Children7-10	0.006507	0.0027523
	Leisure_Woman X No. Children0-6	0.006981	0.0021144
	Leisure_Woman X No. Children7-10	0.0007436	0.002288
	Leisure_Woman X Leisure_Man	0.0001134	0.0000948
Other			
	N. observations (N. couples*49 alternatives)	195804	
	N. couples	3996	
	LR chi2(32)	15140.15	
	Prob > chi2	0	
	Pseudo R2	0.4868	
	Log likelihood	-7981.6412	

Table D2– Maximum likelihood estimates – singles (France)

		Male		Female	
Model component	Variable	Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		δ		δ	
	Employee	0.199478	0.533068	-1.19744	0.470041
	Self_employed	-0.21333	0.593638	-1.68397	0.553979
	Part-time_Employee	-0.69588	0.393698	1.256628	0.352174
	Full-time_Employee	2.213382	0.253926	2.994315	0.267448
	Part-time_Self-employed	-2.70132	0.585303	-1.80785	0.627849
	Full-time_Self-employed	-0.28412	0.336337	0.368914	0.377033
Y vector		λ		λ	
	Disposable income	-0.00012	0.00024	7.55E-05	0.000379
	Disposable income squared	4.53E-08	2.07E-08	6.64E-08	4.13E-08
	Household size X Disp_income	-5.6E-05	4.62E-05	-6.7E-05	6.84E-05
L vector		λ		λ	
	Leisure	0.129227	0.029882	0.15447	0.034747
	Leisure2	-8.1E-05	0.000239	-9.2E-05	0.000254
	Leisure X Disposable income	1.01E-06	2.34E-06	6.43E-07	3.40E-06
	Leisure X Age	-0.00516	0.000963	-0.0075	0.001086
	Leisure X Age squared	6.36E-05	1.21E-05	0.000092	1.34E-05
	Leisure X No. Children	-0.01374	0.005121	0.006768	0.003433
	Leisure X No. Children 0-6	-0.00814	0.019875	0.015925	0.00544
	Leisure X No. Children 7-10	0.011728	0.010413	0.008727	0.004892
Other					
	N. observations (N. single*7 alternatives)	9331		10465	
	N. single	1333		1495	
	LR chi2(17)	2318.15		2657.35	
	Prob > chi2	0		0	
	Pseudo R2	0.4468		0.4567	
	Log likelihood	-1434.83		-1580.46	

Table D3 – Maximum likelihood estimates – couples (Italy)

Model component	Variable	Coef.	Std. Err.
Opportunity density		δ	
	Employee_Man	-2.227042	0.3359151
	Self-employed_Man	-1.793772	0.3327547
	Employee_Woman	-4.205803	0.3711781
	Self-employed_Woman	-3.159583	0.3091701
	Part-time_Employee_Man	1.810835	0.2235256
	Full-time_Employee_Man	3.457804	0.1466732
	Part-time_Self-employed_Man	-1.142189	0.2861769
	Full-time_Self-employed_Man	1.827801	0.1352579
	Part-time_Employee_Woman	3.522802	0.3488772
	Full-time_Employee_Woman	4.233018	0.3257372
	Part-time_Self-employed_Woman	0.2200945	0.3028192
	Full-time_Self-employed_Woman	1.989132	0.2580389
Y vector		γ	
	Household_Disposable_income	0.0005129	0.0001534
	Hosuhold_Disposable_income squared	1.36E-08	7.25E-09
	Household_size X Household_disposable_income	-0.0001608	0.0000251
L vector		λ	
	Leisure_Male	0.0030689	0.05153
	Leisure_Man squared	-0.0000926	0.0001607
	Leisure_Woman	0.2598116	0.0365898
	Leisure_Woman squared	-0.000653	0.0001763
	Leisure_Man X Household_disp_income	4.38E-06	1.43E-06
	Leisure_Woman X Household_disp_income	-5.81E-07	1.01E-06
	Leisure_Man X Age_Man	-0.0015349	0.0025113
	Leisure_Woman X Age_Woman	-0.0097254	0.0016741
	Leisure_Man X Age_Man squared	0.0000135	0.0000318
	Leisure_Woman X Age_Woman squared	0.0001141	0.0000223
	Leisure_Man X No. Children	-0.0081218	0.0022336
	Leisure_Woman X No. Children	0.0078869	0.0017578
	Leisure_Man X No. Children0-6	0.0076125	0.0026554
	Leisure_Man X No. Children7-10	0.0002707	0.0028172
	Leisure_Woman X No. Children0-6	-0.0054445	0.0020634
	Leisure_Woman X No. Children7-10	-0.0009139	0.0020886
	Leisure_Woman X Leisure_Man	0.0003854	0.0000964
Other			
	N. observations (N. couples*49 alternatives)	188405	
	N. couples	3845	
	LR chi2(32)	10209.91	
	Prob > chi2	0	
	Pseudo R2	0.3411	
	Log likelihood	-9859.09	

Table D4– Maximum likelihood estimates – singles (Italy)

		Male		Female	
Model component	Variable	Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		δ		δ	
	Employee	-1.22117	0.331639	-3.43019	0.3787
	Self_employed	-0.47643	0.315555	-2.81075	0.350903
	Part-time_Employee	1.263827	0.268794	3.554593	0.34008
	Full-time_Employee	3.310487	0.207522	4.654217	0.303264
	Part-time_Self-employed	-2.2652	0.32631	0.618142	0.341357
	Full-time_Self-employed	1.473456	0.180946	2.786139	0.266647
Y vector		γ		γ	
	Disposable income	0.000114	0.000145	0.0003	0.000255
	Disposable income squared	5.12E-09	1.08E-08	6.55E-09	3.11E-08
	Household size X Disp_income	-5.5E-05	4.01E-05	-0.00011	4.77E-05
L vector		λ		λ	
	Leisure	0.280595	0.024332	0.312801	0.030346
	Leisure2	0.000164	0.000173	0.000428	0.000198
	Leisure X Disposable income	1.36E-06	1.55E-06	-1.91E-07	2.59E-06
	Leisure X Age	-0.01438	0.001037	-0.01841	0.001297
	Leisure X Age squared	0.000176	1.51E-05	0.000225	1.84E-05
	Leisure X No. Children	-0.0191	0.01175	0.005966	0.003381
	Leisure X No. Children 0-6	0.007813	0.020605	0.00305	0.005703
	Leisure X No. Children 7-10	0.011513	0.022161	-0.00433	0.005772
Other					
	N. observations (N. single*7 alternatives)	22190		18270	
	N. single	3170		2610	
	LR chi2(17)	4055.02		3501.41	
	Prob > chi2	0		0	
	Pseudo R2	0.3287		0.3447	
	Log likelihood	-4141.03		-3328.12	

Table D5– Maximum likelihood estimates – couples (Luxembourg)

Model component	Variable	Coef.	Std. Err.
Opportunity density		δ	
	Employee_Man	2.798179	1.230943
	Self-employed_Man	1.196799	1.218041
	Employee_Woman	-1.670879	0.4877308
	Self-employed_Woman	-3.273727	0.5811094
	Part-time_Employee_Man	-0.9321119	0.5778732
	Full-time_Employee_Man	2.740097	0.2477136
	Part-time_Self-employed_Man	-3.276221	1.176261
	Full-time_Self-employed_Man	0.3923308	0.4062014
	Part-time_Employee_Woman	2.251194	0.381928
	Full-time_Employee_Woman	3.024338	0.2864887
	Part-time_Self-employed_Woman	-0.0916981	0.6417357
	Full-time_Self-employed_Woman	0.9017009	0.4806236
Y vector		γ	
	Household_Disposable_income	0.0001153	0.0001343
	Hosuhold_Disposable_income squared	-2.43E-09	2.07E-09
	Household_sizeÃ—Household_disposable_income	-1.63E-06	0.000023
L vector		λ	
	Leisure_Male	-0.0472945	0.0551945
	Leisure_Man squared	0.0014071	0.0004473
	Leisure_Woman	0.0416601	0.0425495
	Leisure_Woman squared	0.0003121	0.0002634
	Leisure_Man X Household_disp_income	1.64E-06	8.45E-07
	Leisure_Woman X Household_disp_income	1.18E-07	8.47E-07
	Leisure_Man X Age_Man	-0.0038039	0.0021464
	Leisure_Woman X Age_Woman	-0.0059885	0.0016256
	Leisure_Man X Age_Man squared	0.0000479	0.0000254
	Leisure_Woman X Age_Woman squared	0.0000904	0.0000201
	Leisure_Man X No. Children	-0.0067684	0.0038964
	Leisure_Woman X No. Children	0.0069002	0.0027455
	Leisure_Man X No. Children0-6	0.0085339	0.0051382
	Leisure_Man X No. Children7-10	0.002834	0.0060786
	Leisure_Woman X No. Children0-6	0.0088988	0.0034797
	Leisure_Woman X No. Children7-10	0.0022974	0.0039516
	Leisure_Woman X Leisure_Man	0.0002931	0.0001535
Other			
	N. observations (N. couples*49 alternatives)	64435	
	N. couples	1315	
	LR chi2(32)	5058.95	
	Prob > chi2	0	
	Pseudo R2	0.4943	
	Log likelihood	-2588.2705	

Table D6– Maximum likelihood estimates – singles (Luxembourg)

		Male		Female	
Model component	Variable	Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		δ		δ	
	Employee	3.840025	1.467676	-3.83547	0.916069
	Self_employed	3.076828	1.44261	-6.09075	1.124339
	Part-time_Employee	-1.25405	0.727917	3.214104	0.688409
	Full-time_Employee	2.760224	0.389481	3.833989	0.508304
	Part-time_Self-employed	-17.2776	699.2763	2.533271	1.111393
	Full-time_Self-employed	0.139062	0.585945	2.068577	0.878229
Y vector		γ		γ	
	Disposable income	3.53E-05	0.000416	0.00036	0.000262
	Disposable income squared	-8.96E-09	2.75E-08	-8.71E-09	9.30E-09
	Household size X Disp_income	0.000177	0.000082	-4.1E-05	5.76E-05
L vector		λ		λ	
	Leisure	0.083226	0.061447	0.222152	0.066109
	Leisure2	0.00187	0.000632	-0.00012	0.000504
	Leisure X Disposable income	2.07E-07	3.91E-06	3.42E-06	2.72E-06
	Leisure X Age	-0.0096	0.001664	-0.01311	0.00192
	Leisure X Age squared	0.000118	0.000021	0.00016	2.33E-05
	Leisure X No. Children	0.010493	0.008464	0.002518	0.005455
	Leisure X No. Children 0-6	0.006816	0.029223	0.00331	0.010569
	Leisure X No. Children 7-10	0.024772	0.029139	-0.0027	0.009811
Other					
	N. observations (N. single*7 alternatives)	4123		3640	
	N. single	589		520	
	LR chi2(17)	1157.65		951.82	
	Prob > chi2	0		0	
	Pseudo R2	0.505		4703	
	Log likelihood	-567.317		5335.965	

Table D7 – Maximum likelihood estimates – couples (Spain)

Model component	Variable	Coef.	Std. Err.
Opportunity density		δ	
	Employee_Man	-0.2868366	0.2843099
	Self-employed_Man	-0.7698504	0.2931603
	Employee_Woman	-2.403139	0.252017
	Self-employed_Woman	-2.585398	0.2813223
	Part-time_Employee_Man	-0.1938884	0.1981381
	Full-time_Employee_Man	2.390778	0.1070763
	Part-time_Self-employed_Man	-1.089852	0.2490981
	Full-time_Self-employed_Man	0.9119172	0.1280802
	Part-time_Employee_Woman	1.511822	0.2167088
	Full-time_Employee_Woman	2.692898	0.1672168
	Part-time_Self-employed_Woman	-0.3884766	0.2749832
	Full-time_Self-employed_Woman	0.8462308	0.2033327
Y vector		γ	
	Household_Disposable_income	-0.0001841	0.0001271
	Hosuhold_Disposable_income squared	2.51E-08	8.12E-09
	Household_sizeÃ—Household_disposable_income	-0.0000236	0.0000156
L vector		λ	
	Leisure_Male	-0.0294838	0.0239608
	Leisure_Man squared	0.0005993	0.0001227
	Leisure_Woman	0.0991669	0.0229826
	Leisure_Woman squared	-0.0003914	0.0001435
	Leisure_Man X Household_disp_income	5.62E-06	9.76E-07
	Leisure_Woman X Household_disp_income	1.22E-06	7.46E-07
	Leisure_Man X Age_Man	-0.0022498	0.0009384
	Leisure_Woman X Age_Woman	-0.0043625	0.0007961
	Leisure_Man X Age_Man squared	0.0000224	0.0000103
	Leisure_Woman X Age_Woman squared	0.0000589	9.11E-06
	Leisure_Man X No. Children	-0.0015521	0.0013717
	Leisure_Woman X No. Children	0.0036569	0.0011625
	Leisure_Man X No. Children0-6	0.0034279	0.0019022
	Leisure_Man X No. Children7-10	-0.0022426	0.0020741
	Leisure_Woman X No. Children0-6	0.0013244	0.0016922
	Leisure_Woman X No. Children7-10	0.0020022	0.0017517
	Leisure_Woman X Leisure_Man	0.00032	0.0000585
Other			
	N. observations (N. couples*49 alternatives)	244755	
	N. couples	4995	
	LR chi2(32)	13049.94	
	Prob > chi2	0	
	Pseudo R2	0.3357	
	Log likelihood	-12914.672	

Table D8– Maximum likelihood estimates – singles (Spain)

Model component	Variable	Male Coef.	Male Std. Err.	Female Coef.	Female Std. Err.
Opportunity density		δ		δ	
	Employee	-1.30198	0.428921	-1.7268	0.428239
	Self_employed	-1.41084	0.456916	-1.59374	0.505891
	Part-time_Employee	0.607968	0.329195	1.895538	0.346785
	Full-time_Employee	2.496161	0.217392	3.236458	0.246681
	Part-time_Self-employed	-1.10093	0.419394	-0.78624	0.512791
	Full-time_Self-employed	0.757978	0.25528	1.336878	0.331166
Y vector		γ		γ	
	Disposable income	0.000391	0.000202	0.000249	0.000234
	Disposable income squared	-2.86E-08	2.19E-08	-4.02E-09	2.89E-08
	Household size X Disp_income	3.35E-05	4.41E-05	0.000166	5.63E-05
L vector		λ		λ	
	Leisure	0.091073	0.027093	0.060002	0.028202
	Leisure2	7.25E-05	0.000216	0.000661	0.000213
	Leisure X Disposable income	2.92E-08	2.03E-06	2.06E-06	2.36E-06
	Leisure X Age	-0.00465	0.000802	-0.00588	0.000853
	Leisure X Age squared	5.79E-05	9.77E-06	6.94E-05	1.02E-05
	Leisure X No. Children	0.002815	0.0077	0.005045	0.002922
	Leisure X No. Children 0-6	-0.00077	0.015847	0.004662	0.006085
	Leisure X No. Children 7-10	-0.01601	0.021244	0.00168	0.005529
Other					
	N. observations (N. single*7 alternatives)	12530		12194	
	N. single	1790		1742	
	LR chi2(17)	2335.29		2421.22	
	Prob > chi2	0		0	
	Pseudo R2	0.3352		0.3571	
	Log likelihood	-2315.53		-2179.16	

Table D9 – Maximum likelihood estimates – couples (UK)

Model component	Variable	Coef.	Std. Err.
Opportunity density		δ	
	Employee_Man	0.1010326	0.3378421
	Self-employed_Man	-0.6931103	0.3433464
	Employee_Woman	-2.125809	0.2756313
	Self-employed_Woman	-2.693189	0.3039796
	Part-time_Employee_Man	-1.045434	0.2048579
	Full-time_Employee_Man	2.390656	0.1114932
	Part-time_Self-employed_Man	-1.088571	0.24735
	Full-time_Self-employed_Man	1.330872	0.1382285
	Part-time_Employee_Woman	1.85452	0.2366666
	Full-time_Employee_Woman	2.886648	0.2010856
	Part-time_Self-employed_Woman	0.3428018	0.2863139
	Full-time_Self-employed_Woman	0.6378565	0.2530663
Y vector		γ	
	Household_Disposable_income	0.0001122	0.0002534
	Hosuhold_Disposable_income squared	-3.40E-08	1.75E-08
	Household_size X Household_disposable_income	0.0000397	0.000025
L vector		λ	
	Leisure_Male	0.0426497	0.0280003
	Leisure_Man squared	0.0004884	0.0001547
	Leisure_Woman	0.1642487	0.0289328
	Leisure_Woman squared	-0.0006741	0.0001594
	Leisure_Man X Household_disp_income	9.12E-07	1.76E-06
	Leisure_Woman X Household_disp_income	-7.02E-07	1.45E-06
	Leisure_Man X Age_Man	-0.006004	0.0012687
	Leisure_Woman X Age_Woman	-0.0071774	0.0013197
	Leisure_Man X Age_Man squared	0.0000731	0.0000171
	Leisure_Woman X Age_Woman squared	0.0000904	0.0000183
	Leisure_Man X No. Children	0.0023683	0.0016979
	Leisure_Woman X No. Children	0.0169523	0.0016992
	Leisure_Man X No. Children0-6	-0.0029332	0.0017903
	Leisure_Man X No. Children7-10	0.0008209	0.001984
	Leisure_Woman X No. Children0-6	0.0142435	0.0019016
	Leisure_Woman X No. Children7-10	0.0034794	0.0020909
	Leisure_Woman X Leisure_Man	0.0006758	0.0000828
Other			
	N. observations (N. couples*49 alternatives)	220843	
	N. couples	4507	
	LR chi2(32)	12926.1	
	Prob > chi2	0	
	Pseudo R2	0.3685	
	Log likelihood	-11077.385	

Table D10– Maximum likelihood estimates – singles (UK)

		Male		Female	
Model component	Variable	Coef.	Std. Err.	Coef.	Std. Err.
Opportunity density		δ		δ	
	Employee	-0.23213	0.467104	-2.73527	0.381373
	Self_employed	-0.73375	0.479139	-3.96636	0.455567
	Part-time_Employee	-0.48835	0.315075	2.234354	0.311484
	Full-time_Employee	2.423602	0.215381	3.217563	0.257497
	Part-time_Self-employed	-1.83573	0.404768	1.149457	0.429612
	Full-time_Self-employed	0.850365	0.26129	1.745809	0.371801
Y vector		γ		γ	
	Disposable income	-8.1E-05	0.000262	0.001196	0.000448
	Disposable income squared	1.29E-08	3.04E-08	-1.33E-07	6.24E-08
	Household size X Disp_income	1.13E-05	5.03E-05	-1.9E-05	6.18E-05
L vector		λ		λ	
	Leisure	0.122021	0.027423	0.248225	0.030925
	Leisure2	0.00039	0.000239	-0.00026	0.00022
	Leisure X Disposable income	1.27E-06	2.44E-06	-9.32E-06	4.05E-06
	Leisure X Age	-0.0075	0.001001	-0.01234	0.00121
	Leisure X Age squared	0.000102	1.47E-05	0.000166	1.74E-05
	Leisure X No. Children	-0.00517	0.005611	0.015035	0.002616
	Leisure X No. Children 0-6	0.020995	0.012061	0.026352	0.003583
	Leisure X No. Children 7-10	0.013849	0.010819	0.004335	0.003571
Other					
	N. observations (N. single*7 alternatives)	13937		17549	
	N. single	1991		2507	
	LR chi2(17)	2736.7		3775.11	
	Prob > chi2	0		0	
	Pseudo R2	0.3532		0.3869	
	Log likelihood	-2505.96		-2990.84	