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Abstract

Existing tax schedules are often overly complex and characterized by discontinuities in the marginal tax burden. In this paper we propose a class of progressive smooth functions to replace personal income tax schedules. These functions depend only on three meaningful parameters, and avoid the drawbacks of defining tax schedules through various tax brackets. Based on representative micro data, we derive revenue-neutral parameters for four different types of tax regimes (Austria, Germany, Hungary and Spain). We then analyze possible implications from a hypothetical switch to smoother income tax tariffs. We find that smooth tax functions eliminate the most extreme cases of bracket creep, while the impact on income inequality is mostly negligible, but uniformly reducing.

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[*] The results presented here are based on EUROMOD version H1.0+. EUROMOD is maintained, developed and managed by the Institute for Social and Economic Research (ISER) at the University of Essex, in collaboration with national teams from the EU member states. We are indebted to the many people who have contributed to the development of EUROMOD. The process of extending and updating EUROMOD is financially supported by the European Union Programme for Employment and Social Innovation 'Easi' (2014-2020). We make use of microdata from the EU Statistics on Incomes and Living Conditions (EU-SILC) made available by Eurostat (59/2013-EU-SILC-LFS). The results and their interpretation are the authors' responsibility. We thank Nico Pestel for valuable suggestions.

1 Introduction

Personal income taxation constitutes one of the main tools for income redistribution in developed economies (Joumard et al., 2012). Tax schedules are usually defined in a piecewise manner, consisting of polynomials of order less or equal to two in different tax brackets. This implies non-differentiable and often non-continuous marginal tax rate functions that complicates the computation of an individual's tax liability. Even flat tax regimes with a non-taxed threshold feature a discrete jump at this threshold. This is considered undesirable, as jumps in marginal tax rates (kinks) alter individual behavior compared to a hypothetical situation with smooth functions (Saez, 2010). In addition, the tax liability disproportionally increases particularly at these discontinuities if gross wages increase.

In this contribution, we derive a new class of smooth income tax tariffs and discuss their advantages vis-à-vis current approaches. They are easy to define and could be used instead of the piecewise defined polynomial functions. Defining tax schedules in terms of the marginal tax rate is attractive for policy-makers by making distributional objectives explicit. Particular attention has been devoted to the the top marginal tax rate, both in the policy debate and in the optimal taxation literature (Diamond and Saez, 2011). Our approach, while not being generally defined in marginal tax rates, maintains the top marginal tax rate, along with the general exemption, as a key policy parameter. With smooth tax tariffs, however, it becomes harder to alter the top marginal tax rate without changing the tax function for lower incomes at the same time. This potentially hinders policy-makers from targeting specific income groups when implementing tax reforms. Finally, we apply a microsimulation model to demonstrate distributional effects for four countries (Austria, Germany, Hungary, Spain), each representing a different type of tax schedule. As a general lesson, the theoretical merits of smooth tax functions can be achieved by both maintaining tax revenue and the level of income inequality.

The paper is organized as follows. After classifying existing tax schedules (Section 2), we derive a new class of tax functions in Section 3. In Section 4, we present how to determine the parameters for these tax functions. Section 5 discusses the distributional effects of introducing smooth tax functions in four countries, before Section 6 concludes.

2 Typology of Existing Tax Tariffs

Notation In what follows, we will consider the annual taxable income $x > 0$, i.e. net of exemptions and allowances. For this variable, we consider the income tax liability $T(x)$ and the effective (average) income tax rate $E(x)$, i.e.

$$T(x) := E(x) \cdot x, \quad \text{or} \quad E(x) := \frac{T(x)}{x}.$$

The derivative of T, corresponding to the marginal tax rate, is therefore defined by

$$M(x) := T'(x) = E'(x) \cdot x + E(x).$$

Existent tax schedules in the EU can be broadly classified into three categories, as shown in Table 1.[1]

[1] Throughout the paper, we refer to the general shape of the nationwide personal income tax schedule. We do not consider further taxes imposed on personal income, such as social security contributions,

Table 1: Different types of definitions for the functions $E(x)$, $T(x)$, $M(x)$

Type	*Prescribed function*	*Resulting functions*		*Examples*
Flat	$M(x)$	$T(x) := M \cdot x$	$E \ := M$	Hungary, Latvia, Czech Republic
C-Prog	$M(x)$	$T(x) := \int_0^x M(\xi)\, d\xi$,	$E(x) \ := T(x)/x$	Austria, France, Italy, UK, U.S.
G-Prog	$T(x)$	$E(x) := T(x)/x$,	$M(x) := T'(x)$	Germany
N-Prog	$E(x)$	$T(x) := E(x) \cdot x$,	$M(x) := T'(x)$	

Source: (OECD, 2017).

Flat : (Flat rate tax) The same constant tax rate M applies to every taxpayer regardless of income x. Consequently, $T(x) = M \cdot x$ is linear and $E = M$.

C-Prog : (Common progressive tax) For almost all countries with progressive tariff, $M(x)$ is defined by piecewise constant functions $M_i(x) = M_i$ for $i = 0, 1, \ldots, n$. Consequently, $T(x)$ is a polygonal line defined piecewise by functions of the type

$$T_i(x) = M_i x + r_i$$

and $E(x)$ consists of piecewise defined functions with a rational summand, i.e.

$$E_i(x) = M_i + \frac{r_i}{x}.$$

G-Prog : (German progressive tax) German tax law defines a continuous $T(x)$ by piecewise considered polynomials of degree up to 2, i.e.

$$T_i(x) = a_i x^2 + b_i x + c_i, \quad i = 0, 1, \ldots, n.$$

Consequently, $M(x)$ consist of piecewise defined polynomial of degree up to 1

$$M_i(x) = 2a_i x + b_i,$$

and $E(x)$ is piecewise defined by

$$E_i(x) = a_i x + b_i + \frac{c_i}{x},$$

i.e. also in this case we obtain a rational summand.

N-Prog : (New type of progressive tax) For our purposes, we introduce tax functions defining smooth functions for $E(x)$, such that $T(x)$ and $M(x)$ result accordingly to Table 1 and no piecewise definition is necessary.

regional tax schedules (as in Spain), separate taxation of capital gains (e. g., Austria, Germany) or other top-up taxes. Beyond, we take differences in the extent of tax deductions as given. Interactions of the counterfactual tax schedule with other elements of the tax-benefit system will be however accounted for in the distributional analysis.

3 Derivation of Smooth Tax Tariffs

3.1 Parameters

The main common idea of the functions that will be discussed is to assume that $E(x)$ is a strictly increasing nonlinear saturation function. Consequently, $T(x)$ and $M(x)$ result straight forward. Moreover, all functions will depend only on three parameters (E_{max}, x_0, x_h). First, we denote the upper bound for both effective and marginal tax rates with E_{max}, such that

$$0 \leq E(x) \leq E_{max}, \qquad \lim_{x \to \infty} E(x) = E_{max}, \tag{1}$$

$$0 \leq M(x) \leq E_{max}, \qquad \lim_{x \to \infty} M(x) = E_{max}. \tag{2}$$

Second, all existing tax tariffs feature a basic threshold which leaves income below this threshold untaxed. We denote this with x_0, i. e.

$$\lim_{x \to x_0} E(x) = 0, \quad \text{for} \quad x > x_0.$$

Tax tariffs usually kick in with a positive marginal tax rate $M(x)$, thus creating a first kink in the tax schedule. We will also discuss the special case for a tariff starting with a zero marginal tax rate, i.e.

$$\lim_{x \to x_0} M(x) = 0, \quad \text{for} \quad x > x_0.$$

The third parameter $x_h > x_0$ denotes the taxable income for which the effective tax rate equals half the maximal rate:

$$E(x_h) = \frac{E_{max}}{2}. \tag{3}$$

Technical and scientific applications offer a variety of saturation functions that could be used in our context which satisfy the general requirements of tax schedules (non-decreasing, positive) and are continuous. Some examples are given in Table 2.

Table 2: Examples for strictly increasing saturation functions

	$E(x \mid E_{max}, x_0, x_h)$
Rational function	$E_{max} \cdot \frac{x - x_0}{x - 2x_0 + x_h}$
Exponential function	$E_{max} \cdot \left(1 - 0.5^{\frac{x - x_0}{x_h - x_0}}\right)$
Arcus tangens	$E_{max} \cdot \frac{2}{\pi} \cdot \arctan\left(\tan(0.5) \frac{x - x_0}{x_h - x_0}\right)$
Tangens hyperbolicus	$E_{max} \cdot \tanh\left(\operatorname{arctanh}(0.5) \frac{x - x_0}{x_h - x_0}\right)$
Composed function	$E_{max} \cdot 0.5^{\frac{x_h - x_0}{x - x_0}}$

For the sake of clarity, we will discuss only the two most straightforward functions $E(x \mid E_{max}, x_0, x_h)$ from Table 2 in detail, i. e. the rational and the composed function. The corresponding $T(x)$, $M(x)$ result from the definition, see Section 2.

3.2 Definition with a rational function: $E_r(x)$

As emphasized in Section 2, $E(x)$ involves a rational expression for progressive tariffs. For this first approach, we therefore consider a simple rational function such that both the numerator and the denominator are polynomials of degree one and (1)-(3) are fulfilled. Therefore, we define

$$E_r(x) := \begin{cases} 0, & \text{for} \quad 0 < x \leq x_0, \\ E_{max} \cdot \frac{x - x_0}{x - 2x_0 + x_h}, & \text{for} \quad x > x_0, \end{cases}$$

leading to

$$\begin{aligned} T_r(x) &= \begin{cases} 0, & \text{for} \quad 0 < x \leq x_0, \\ E_{max} \cdot \frac{x - x_0}{x - 2x_0 + x_h} \cdot x, & \text{for} \quad x > x_0, \end{cases} \\ M_r(x) &= \begin{cases} 0, & \text{for} \quad 0 < x \leq x_0, \\ E_{max} \cdot \frac{x^2 - 4\,x\,x_0 + 2\,x_h\,x + 2\,{x_0}^2 - x_h\,x_0}{(x - 2\,x_0 + x_h)^2}, & \text{for} \quad x > x_0, \end{cases} \end{aligned}$$

and consequently

$$M_r(x_0) = E_{max} \cdot \frac{x_0}{x_h - x_0}.$$

This means that for positive tax exemptions $x_0 > 0$ the function $M_r(x)$ is not continuous at x_0 and $T_r(x)$ is not smooth at x_0, keeping a kink at the exemption.[2] In fact, from the functions listed in Table 2, only the last one permits a definition of smooth functions $E(x)$ and $T(x)$ at x_0. Therefore, this will be the second function we discuss in detail.

3.3 Definition with a composed function: $E_s(x)$

In Estévez Schwarz (2017), a smooth function E_s is introduced deducing a composed function (an exponential function with rational exponent) as solution of the linear differential equation

$$E'(x) = \frac{k}{(x - x_0)^2} E(x), \quad k > 0, \quad x > x_0$$

in order to approximate the German income tax function. The resulting function reads

$$E_s(x) := \begin{cases} 0, & \text{for} \quad 0 < x \leq x_0, \\ E_{max} \cdot 0.5^{\frac{x_h - x_0}{x - x_0}}, & \text{for} \quad x > x_0, \end{cases}$$

leading to

$$\begin{aligned} T_s(x) &= \begin{cases} 0, & \text{for} \quad 0 < x \leq x_0, \\ E_{max} \cdot 0.5^{\frac{x_h - x_0}{x - x_0}} \cdot x, & \text{for} \quad x > x_0, \end{cases} \\ M_s(x) &= \begin{cases} 0, & \text{for} \quad 0 < x \leq x_0, \\ E_{max} \cdot 0.5^{\frac{x_h - x_0}{x - x_0}} \left(1 - \frac{\ln(0.5) \cdot (x_h - x_0)\,x}{(x - x_0)^2}\right) & \text{for} \quad x > x_0, \end{cases} \end{aligned}$$

and consequently

$$\lim_{x \to x_0} M_s(x) = 0.$$

In fact, $E_s(x)$, $T_s(x)$ and $M_s(x)$ result to be endlessly continuous differentiable in $\mathbb{R}$. Therefore, we will denote T_s as the fully smooth tariff in the following.

[2] Arguably, the exemption threshold represents a social norm. Maintaining this kink could hence be socially desirable (Kleven, 2016).

4 Country-wise determination of parameters

In order to derive a smooth tax schedule for a specific country, the three parameters described above need to be determined. As the top marginal tax rate E_{max} embodies a redistributional objective, we fix it at today's level. Alternatively, one could determine this parameter on the basis of sufficient statistics (Piketty and Saez, 2013). We obtain the remaining parameters (x_0, x_h) such that a specific tax revenue is collected. To this end, we estimate the tax revenue obtained with a tax function by the scalar product

$$\langle P, T(X) \rangle = \sum_{i=1}^{k} p_i T(x_i).$$

$X = (x_1, x_2, \ldots, x_k)$ denotes taxable incomes in steps of 250 €. $T(X) = (T(x_1), T(x_2), \ldots, T(x_k))$ represents the tax liability and $P = (p_1, p_2, \ldots, p_k)$ the number of tax cases for a given value of x_i.

P and $T(X)$ stem from EUROMOD, the only tax-benefit microsimulation model covering all EU countries (Sutherland and Figari, 2013). EUROMOD enables us to conduct a comparative analysis of different tax tariffs consistently in a common framework. It is based on nationally representative micro-data from the EU-SILC, collected by Eurostat. They provide information on household characteristics, market income from various sources and a comprehensive set of tax-benefit rules, e. g. unemployment benefits, social assistance, social security contributions and personal income taxation. Based on these data, along with a replication of the core elements of the tax-benefit rules, household disposable income can be calculated. We rely on the most recent available EU-SILC survey from 2015, to which the tax-benefit rules from 2017 are applied. The accuracy of the survey data is inferior to administrative data due their lower sample size and due to income misreporting. On the other hand, survey data provide detailed information on e. g., family composition and labor market behavior. This allows us to assess the consequences of counterfactual tax tariffs, also taking into account interactions with other elements of the tax-benefit system. Examples include the tax treatment of social security contributions or eligibility to public transfers. Another reason for relying on survey data is their representativeness for the population as a whole. In order to scale the survey households to the full population, we make use of sample weights. P hence denotes the sum of sample weights across bins of taxable income. The associated distributions of taxpayers are presented in Figure 1.

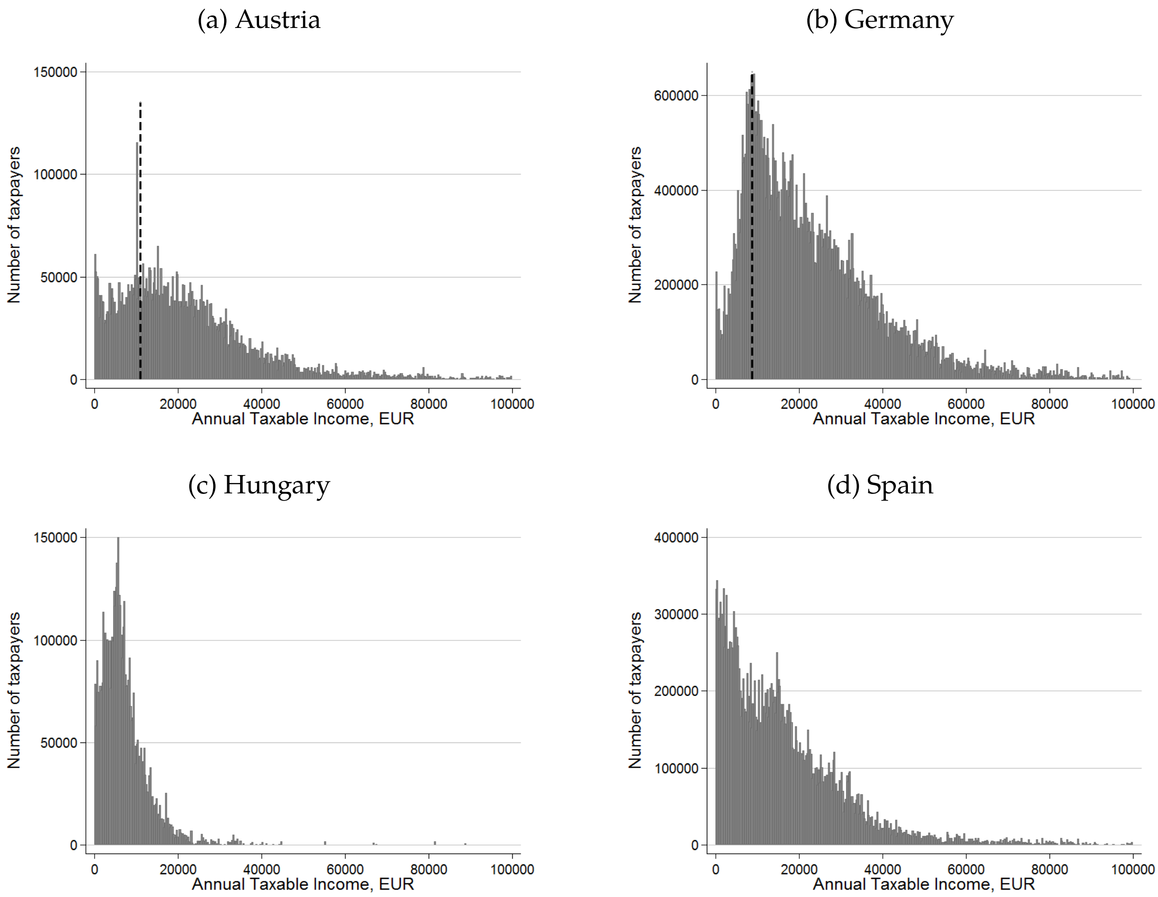
To take into account the basic exemption, we suppose that $E_{min} < \frac{E_{max}}{2}$ is the minimal effective tax-rate that will in practice be charged and that x_{min} with $x_0 < x_{min} < x_h$ is the value that satisfies

$$E(x_{min}) = E_{min}.$$

For all countries, we use the actual values for x_{min} and set $E_{min} = 0.001\%$, with the goal of determining x_0, c. f. Appendix. Put differently, x_0 will be determined such that the tax liability becomes relevant at x_{min}. In order to avoid distributional discussions, we consider a revenue-neutral reform, i. e. $\langle P, T(X) \rangle$ corresponds to the current value $\mathrm{Revenue}$, as simulated in EUROMOD for 2017.

Summarizing, for each country we determine x_0 and x_h numerically for both tax

Figure 1: Distribution of taxpayers



(a) Austria (b) Germany (c) Hungary (d) Spain

Note: Own presentation for the 2017 tax regimes, based on EU-SILC data from 2015 (Germany: 2014). In Germany, married partners enter twice with half the couple's taxable income. The vertical line indicates the statutory exemption in Germany and Austria.

functions such that

$$\sum_{i=1}^{k} p_i T(x_i) = \text{Revenue}, \tag{4}$$

$$E(x_{min}) = E_{min}, \tag{5}$$

is fulfilled. The solvability of (4)-(5) is verified in the Appendix.

4.1 Germany

As of 2017, the German Personal Income Tax Tariff is defined as follows:

$$T^{\text{DE}}(x) = \begin{cases} T_0(x) = & 0, & \text{for} & 1 \le x \le 8820, \\ T_1(x) = & \left(1007.27 \cdot \frac{x-8820}{10000} + 1400\right) \cdot \frac{x-8820}{10000}, & \text{for} & 8821 \le x \le 13769, \\ T_2(x) = & \left(223.76 \cdot \frac{x-13769}{10000} + 2397\right) \cdot \frac{x-13769}{10000} & & \\ & +939.57, & \text{for} & 13770 \le x \le 54057, \\ T_3(x) = & 0.42 \cdot x - 8475.44, & \text{for} & 54058 \le x \le 256303, \\ T_4(x) = & 0.45 \cdot x - 16164.53, & \text{for} & x \ge 256304. \end{cases}$$

The other component of the personal income tax is the solidarity surcharge (*Solidaritätszuschlag*), which amounts to 5.5% of the income tax due. We disregard this tax for simplicity and instead regard $E_{max} = 0.45$ as fixed. For married taxpayers A and B with income x_A and x_B filing jointly, the tax is twice the amount of applying the formula to half of the married couple's joint taxable income. The resulting benefit is called splitting effect S:

$$S(x_A, x_B) := T^{DE}(x_A) + T^{DE}(x_B) - 2 \cdot T^{DE}\left(\frac{x_A + x_B}{2}\right). \tag{6}$$

In Germany, no income tax is charged on incomes below the basic allowance of 8820 € for unmarried persons. Consequently, to determine x_0 and x_h for a function that is continuous up to rounding effects, we assume $x_{min} = 8821$. Moreover, we set $E(x) = 0$, $T(x) = 0$, $M(x) = 0$ for $x < x_{min}$. Solving equations (4)-(5) leads to the following respective tariff functions:

$$T_r(x) = \begin{cases} 0, & \text{for} \quad 0 < x \leq 8820, \\ 0.45 \cdot \frac{x-8820}{x+22433} \cdot x, & \text{for} \quad x \geq 8821. \end{cases}$$

$$T_s(x) = \begin{cases} 0, & \text{for} \quad 0 < x \leq 8820, \\ 0.45 \cdot 0.5^{\frac{30293}{x-6861}} \cdot x, & \text{for} \quad x \geq 8821. \end{cases}$$

It becomes apparent that with these functions, individuals can compute their tax liability more easily than with the current schedule.

4.2 Flat Tax: Hungary

We choose Hungary as an example for a Flat Tax Regime as implemented in a number of New EU Member States (Table 1). Flat Tax regimes obviously do not suffer from complexity. For the sake of completeness, we demonstrate the implications of our approach also for such regimes. Hungary taxes all income at a 16% rate. There is no general exemption; families are granted a tax allowance that increases with the number of children.

$$T^{HU}(x) = 0.16 \cdot x \text{ for } x > 0.$$

For singles, this already constitutes a smooth tax tariff.

Sticking to $E_{max} = 0.16$ would not deliver a solution for a progressive tariff, as tax revenue would be strictly lower than in the status quo. We instead set the new top marginal tax rate to 20%. The resulting functions for $x_{min} = 0$ are

$$T_r(x) = 0.2 \cdot \frac{x}{x + 2086} \cdot x, \quad T_s(x) = 0.2 \cdot 0.5^{\frac{2586}{x}} \cdot x \quad \text{for} \quad x > 0.$$

4.3 Common Progressive Tax: Austria

The Austrian tax schedule is a typical example for tariff that is determined in brackets of constant marginal tax rates:

$$M(x) := \begin{cases} M_0 = & 0, & \text{for} & 0 \leq x \leq 11000, \\ M_1 = & 0.25, & \text{for} & 11000 < x \leq 18000, \\ M_2 = & 0.35, & \text{for} & 18000 < x \leq 31000, \\ M_3 = & 0.42, & \text{for} & 31000 < x \leq 60000, \\ M_4 = & 0.48, & \text{for} & 60000 < x \leq 90000, \\ M_5 = & 0.50, & \text{for} & 90000 < x \leq 1000000, \\ M_6 = & 0.55, & \text{for} & x > 1000000. \end{cases}$$

Piecewise integration according to the definition from C-Prog in Table 1 leads to

$$T^{AT}(x) = \begin{cases} T_0(x) = & 0, & \text{for} & 0 \leq x \leq 11000, \\ T_1(x) = & 0.25 \cdot (x - 11000), & \text{for} & 11000 < x \leq 18000, \\ T_2(x) = & 1750 + 0.35 \cdot (x - 18000), & \text{for} & 18000 < x \leq 31000, \\ T_3(x) = & 6300 + 0.42 \cdot (x - 31000), & \text{for} & 31000 < x \leq 60000, \\ T_4(x) = & 18480 + 0.48 \cdot (x - 60000), & \text{for} & 60000 < x \leq 90000, \\ T_5(x) = & 32880 + 0.50 \cdot (x - 90000), & \text{for} & 90000 < x \leq 1000000, \\ T_6(x) = & 487880 + 0.55 \cdot (x - 1000000), & \text{for} & x \geq 1000000. \end{cases}$$

Our micro data do not contain taxpayers above 1 million €. We hence set $E_{max} = 0.5$ for the derivation of the smooth functions. For $x_{min} = 11001$, we obtain

$$T_r(x) = \begin{cases} 0, & \text{for} \quad 0 < x \leq 11000, \\ 0.5 \cdot \frac{x-11000}{x+17432} \cdot x, & \text{for} \quad x \geq 10^6. \end{cases}$$

$$T_s(x) = \begin{cases} 0, & \text{for} \quad 0 < x \leq 11000, \\ 0.5 \cdot 0.5^{\frac{27414}{x-9245}} \cdot x, & \text{for} \quad x \geq 10^6. \end{cases}$$

Accounting for incomes above 1 million € would require the application of administrative (tax return) data. Alternatively, one could ignore this 'tax for millionaires', as a smooth increase towards $E_{max} = 0.55$ would lead to higher marginal tax rates for a substantial share of the population below 1 million €.

4.4 Modified Common Progressive Tax: Spain

We take a closer look to the Spanish tax system in order to illustrate the difficulties that may arise if the introduced approach is applied to a tax system that does not completely fit into the types discussed in Section 2 and Table 1.

4.4.1 Income Taxation in Spain

We disregard differences between the autonomous regions and consider, in a first step, the nationwide schedule

$$M(x) = \begin{cases} M_1 = & 0.19, & \text{for} & 0 \leq x \leq 12450, \\ M_2 = & 0.24, & \text{for} & 12450 < x \leq 20200, \\ M_3 = & 0.30, & \text{for} & 20200 < x \leq 35200, \\ M_4 = & 0.37, & \text{for} & 35200 < x \leq 60000, \\ M_5 = & 0.45, & \text{for} & x > 60000. \end{cases}$$

Integrating $M(x)$ piecewise yields

$$R^{ES}(x) = \begin{cases} R_1(x) = & 0.19 \cdot x, & \text{for} & 0 \leq x \leq 12450, \\ R_2(x) = & 2365.50 + 0.24 \cdot (x - 12450), & \text{for} & 12450 \leq x \leq 20200, \\ R_3(x) = & 4225.50 + 0.30 \cdot (x - 20200), & \text{for} & 20200 \leq x \leq 35200, \\ R_4(x) = & 8725.50 + 0.37 \cdot (x - 35200), & \text{for} & 35200 \leq x \leq 60000, \\ R_5(x) = & 17901.50 + 0.45 \cdot (x - 60000), & \text{for} & x \geq 60000. \end{cases}$$

There are substantial allowances for recipients of earnings and self-employment income that is subtracted before applying the tax schedule.[3]

[3] As an example, every individual receiving less than € 11250 of earnings and less than € 6500 of self-employment income can claim an allowance (*Reducciones por rendimientos del trabajo*) of up to € 5700. Beyond, pension expenditures can be partly deducted and additional allowances are granted for families.

The consideration of the family situation involves two aspects (Agencia Tributaria, 2017). On the one hand, incomes below certain thresholds x_t, depending on the household context, are tax-exempt. These thresholds x_t are also taken into account in the limitation that guarantees the effective tax rate does not exceed 43%. For simplicity, we neglect this fact and assume $x_t = 0$. On the other hand, in contrast to German and Austrian tax functions, that are shifted along the axis of abscissae in order to guarantee that the exemption is untaxed ($T(x_0) = 0$ for a continuous function T), the Spanish tax function is lifted along the axis of ordinates by means of

$$T^{ES}(x, x_f) = \begin{cases} T_0(x, x_f) = \quad 0, & \text{for} \quad x \leq x_f, \\ T_{1-5}(x) = \quad R^{ES}(x) - R^{ES}(x_f), & \text{for} \quad x > x_f \end{cases}$$

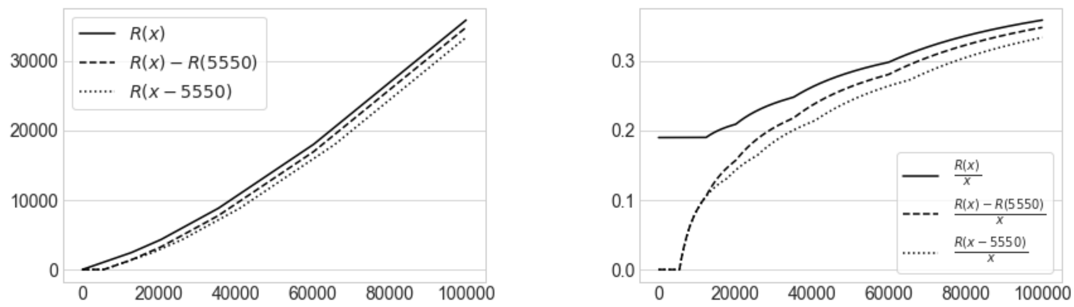
whereas x_f for singles is fixed by $x_f = 5550$. x_f increases if elderly parents or children are present in the household (*mínimo personal y familiar*).

Formally, this means that in fact for $x > x_f$, the tax liability equals the area beneath the function M between x_f and x, i. e.

$$T^{ES}(x, x_f) := \int_{x_f}^{x} M(\xi)\, d\xi, \quad E^{ES}(x, x_f) := \frac{1}{x} \int_{x_f}^{x} M(\xi)\, d\xi,$$

is considered instead of the definition from C-Prog in Table 1, cf. Figure 2.

Figure 2: Spanish Tax Schedule: Comparison of the shift along the axis of abscissae vs. shift along the axis of ordinates for R^{ES}.



4.4.2 Deriving smooth tax functions for Spain

The above described shift along the axis of ordinates makes it more complicated to apply the approach introduced in Section 3. If for a new tax function R_* with $R_*(x) < R^{ES}(x)$ for small incomes, this would lower the deduction from the tax credit $R(x_f)$. We therefore decide to consider tax tariff and tax deductions apart from each other and assume, for simplicity, that the treatment of the deductions from tax credit remains unchanged, i.e. we aim at computing R_r, R_s such that we may define

$$T_{r/s}(x, x_f) = \begin{cases} 0, & \text{for} \quad R_{r/s}(x) \leq R^{ES}(x_f), \\ R_{r/s}(x) - R^{ES}(x_f), & \text{else}\ . \end{cases}$$

In order to obtain revenue-neutral parameters for the smooth tax tariffs, we have to take into account both the distributions of x and x_f. As before, $X = (x_1, x_2, \ldots, x_k)$

denotes taxable incomes x in steps of 500 €. $\hat{X} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_{\hat{k}})$ in turn, denotes the sum of granted tax credits x_f. Then

$$T_{r/s}(x_i, \hat{x}_j) = \max\left(0, R_{r/s}(x_i) - R^{ES}(\hat{x}_j)\right)$$

represents the tax liability for an income x_i and a tax credit $\hat{x}_j$. Accordingly, we consider a matrix $P \in \mathbb{R}^{k \times \hat{k}}$, with $p_{i,j}$ representing the number of tax cases with taxable income x_i and tax credits $\hat{x}_j$.

In order to obtain $R_{r/s}$ for Spain, instead of Equations (4)–(5) we solve

$$\begin{aligned} \sum_{i=1}^{k}\sum_{j=1}^{\hat{k}} p_{i,j} \cdot T_{r/s}(x_i, \hat{x}_j) &= \sum_{i=1}^{k}\sum_{j=1}^{\hat{k}} p_{i,j} \cdot T^{ES}(x_i, \hat{x}_j), \qquad (7)\\ E_{r/s}(x_{min}) &= E_{min}, \qquad (8) \end{aligned}$$

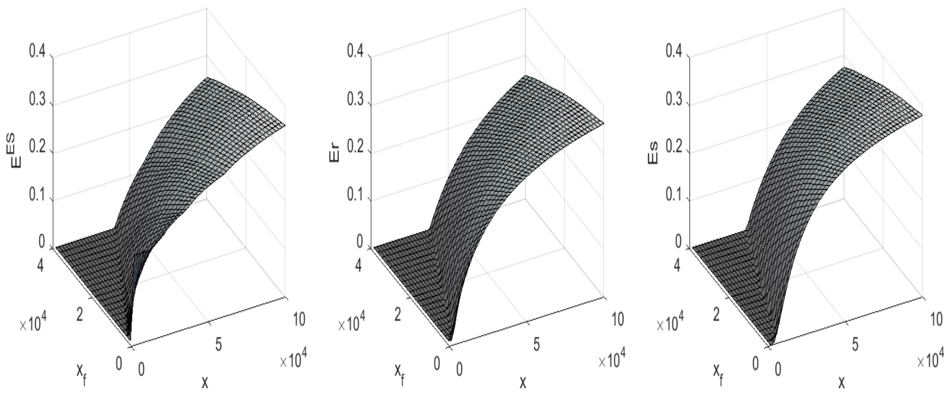
for $x_{min} = 0$.

Solving Equations (7)–(8) yields the self financing alternative tax functions

$$\begin{aligned} T_r(x, x_f) &:= \max(0, 0.45 \cdot \frac{x}{x + 23897} \cdot x - R^{ES}(x_f)),\\ T_s(x, x_f) &:= \max(0, 0.45 \cdot 0.5^{\frac{24228}{x}} \cdot x - R^{ES}(x_f)). \end{aligned}$$

It is worth emphasizing that $T_{r/s}$ and $E_{r/s}$ result to be positive only for x with $R_{r/s}(x) > R^{ES}(x_f)$, such that $T_{r/s}(x) = 0$ and $E_{r/s}(x) = 0$ becomes possible for some $x > x_f$, see Figure 3. Moreover, the smoothness advantages of $E_s(x)$ would be nullified for $x_f > 0$, since a kink is introduced by the shift along the ordinate.

Figure 3: Effect of $R^{ES}(x_f)$ on E^{ES}, E_r^{ES} and E_s^{ES} (for $x_t = 0$).



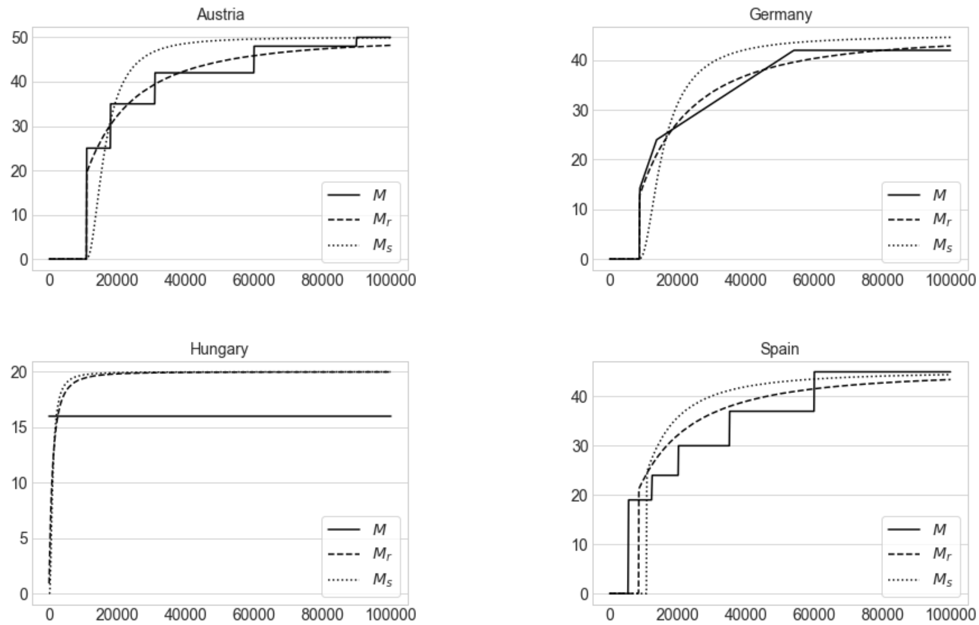
In summary, the introduction of a smooth tariff for Spain should go along with a new concept for computing the tax credit deduction $R(x_f)$ related to the family situation. A shift along the axis of abscissae seems to be a reasonable possibility. However, this would be a considerable modification.

For the comparisons from the next sections we will discuss only the special case $x_f = 5550$ for simplicity, as this is the minimum value every taxpayer can claim.

4.5 Visualization

In Figures 4 and 5, marginal and effective tax rates of the smoother tax schedules are depicted along with current schedules. The rational function is closely aligned with the current schedules for Austria and Germany, while the fully smooth function always deviates from the current one. In all countries, $M_s < M_r$ for low incomes, but the fully smooth function is comparably steep and reaches values close to E_{max} for moderate income values. For Hungary, both counterfactual tariffs necessarily constitute a substantial deviation from the status quo, as it (re-)introduces progression into the system. The remarkable similarity of the marginal tax rates induced by the rational function to existing ones in Austria and Germany suggests that the rational function is suited to reflect current redistributional preferences in these countries.

Figure 4: Marginal Tax Rates



Note: Own presentation. Graphs for Spain assume $x_f = 5550$.

5 Implications of Smooth Tax Schedules

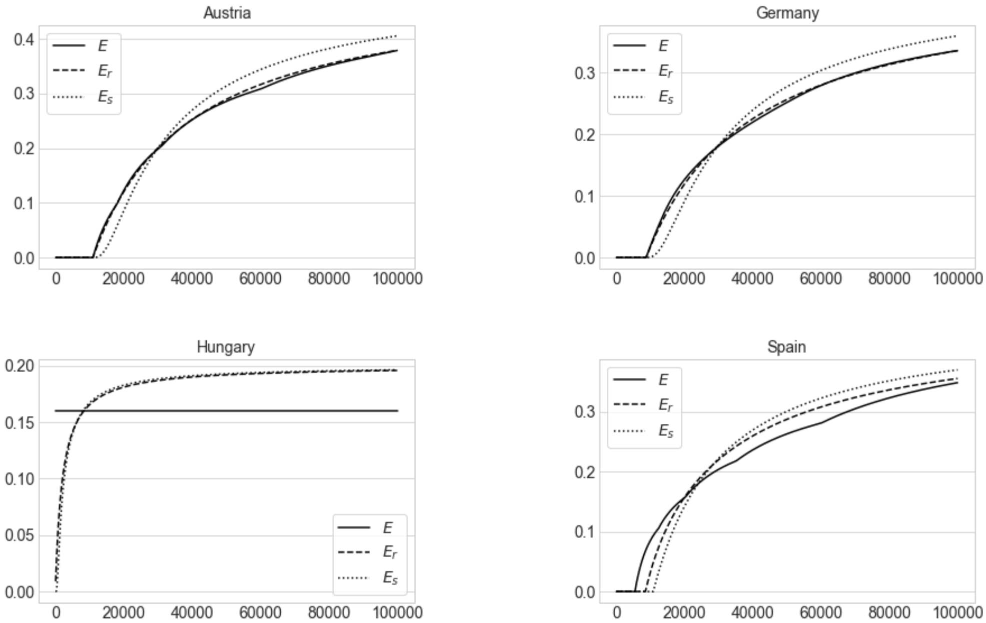
So far, we discussed only the theoretical virtues of the suggested class of tax functions. These would need to be weighed against potential side-effects in the real world.

5.1 Bracket Creep Impact

Bracket creep describes a feature of any progressive tax schedule in which nominal increases in taxable incomes may lead to disproportionate increases in the income tax burden (Immervoll, 2005). If tax policies are fixed in nominal terms, wage growth alone can lead to increasing overall tax ratios. This raises equity concerns if the effective tax rate increases even if the relative income position remains unchanged, thus questioning the proper application of the ability to pay principle.

Bracket creep can effectively be cushioned by indexing tax and benefit policy parameters to price changes. This is commonly done by the change in the consumption price index. Alternatively, one could rely on changes in average earnings. Uprating practices differ substantially across countries and policies (Sutherland et al., 2008). While some

Figure 5: Effective Tax Rates



Note: Own presentation. Graphs for Spain assume $x_f = 5550$.

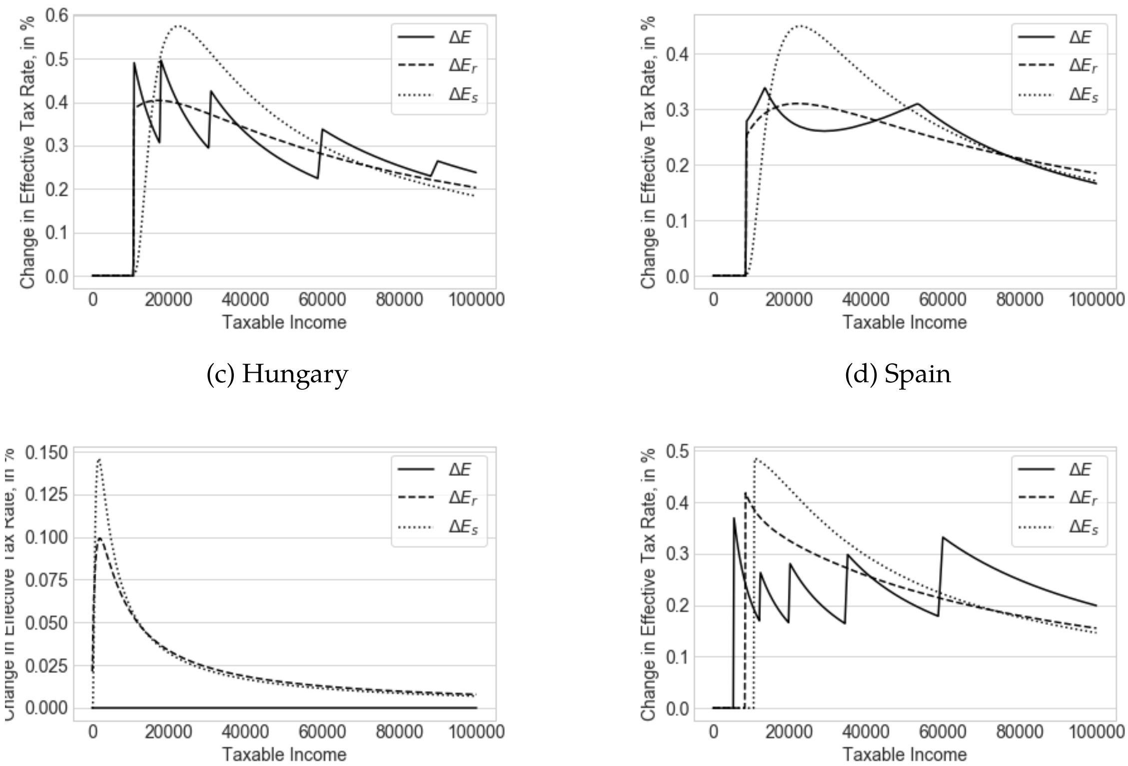
countries have implemented legal requirements for automatic adjustment, other countries uprate parameters on a discretionary basis, if at all. Social benefits and old-age pensions are frequently tied to the price level, while this is less often the case for tax parameters. Among the countries considered here, Germany follows an intermediate strategy. The tax exemption defines the subsistence income level and therefore moves parallel with the social assistance level. Other thresholds of taxable income, in contrast, are adjusted on a discretionary basis. Tax tariff parameters in Spain and Austria are adjusted fully discretionary and have experienced little change over the last couple of years. The Hungarian pure flat tax regime with exemption is not prone to bracket creep as of now.

Figure 6 compares the statutory impact of bracket creep by reform and country, indicating the difference in effective tax rates due to an increase in the taxable incomes by 2%. The solid line indicates the impact for the 2017 tax schedules. The bracketed schedules in Austria and Spain cause particularly high tax increases for those taxpayers jumping into the upper next bracket after due to the wage increase. Introducing a smooth tax schedule as derived here could also be a measure to reduce, although not eliminate, this undesired effect of progressive tax schedules. As can be seen, the counterfactual tariffs smoothen the spikes. The magnitude of the bracket creep impact of the fully smooth tariff exceeds the current one for middle income groups. The magnitude of bracket creep associated with the rational tariff, in contrast, is quantitatively comparable to the status quo schedules, while eliminating the extreme cases. In Spain, the new initial spike is due to the discussed kink caused by the shift along the axis of ordinates. Smoother tax tariffs would introduce inflation-induced tax raises in Hungary, but with a maximum additional burden of 0.14%, which is low compared to the other countries considered here.

The derivation of our class of tax tariffs could be modified to avoid bracket creep altogether. To achieve this, one simply needs to adjust the parameters x_0 and x_h with the inflation rate. We demonstrate this considering $\hat{x}_0 = x_0 \cdot (1 + p\%)$, $\hat{x}_h = x_h \cdot (1 + p\%)$

Figure 6: Bracket Creep Effect on Statutory Tax Due

(a) Austria (b) Germany (c) Hungary (d) Spain



Notes: Own calculations. The graphs show the difference in the effective tax rate $E(x)$ from a 2% increase in taxable income for the three scenarios, i. e. $\Delta E = E(1.02 \cdot x) - E(x)$.

and $\hat{x} = x \cdot (1 + p\%)$ to realize that the original $E_{r/s}$ and the adjusted $\hat{E}_{r/s}$ are identical:

$$\begin{aligned} \hat{E}_r(\hat{x}) &= E_{max} \cdot \frac{\hat{x}-\hat{x}_0}{\hat{x}-2\hat{x}_0+\hat{x}_h} = E_{max} \cdot \frac{(x-x_0)\cdot(1+p\%)}{(x-2x_0+x_h)\cdot(1+p\%)} &= E_r(x), \\ \hat{E}_s(\hat{x}) &= E_{max} \cdot 0.5^{\frac{\hat{x}_h-\hat{x}_0}{\hat{x}-\hat{x}_0}} = E_{max} \cdot 0.5^{\frac{(x_h-x_0)\cdot(1+p\%)}{(x-x_0)\cdot(1+p\%)}} &= E_s(x). \end{aligned}$$

Particularly for the current German tax tariff, an adjustment is considerably more complicated.

5.2 Distributional Impact

In order to gauge the distributional impact of smooth tax functions, we implement the revenue-neutral smooth tax schedules in EUROMOD and compare the resulting income distributions against the current ones. Table 3 presents the changes for three standard measures in income inequality, i. e. the Gini coefficient of disposable incomes, the P90P10 ratio, and the P90P50 ratio. In order to make changes comparable across countries, we present relative changes. As indicated by negative signs, both counterfactual tax schedules exert a uniformly equivalizing impact on the income distribution in all countries. In Austria, Germany and Spain, the rational tariff reduces inequality to a lesser extent than the fully smooth tariff. In Hungary, the rational tariff decreases income inequality slightly stronger. To put these figures into perspective, it has to be noted that Austria, Germany and Hungary exhibit Gini values below 0.3, which clearly qualifies as them as low-inequality countries (OECD, 2015). In absolute terms, the Gini coefficient is simulated to decrease by 0.007 and 0.001 points, respectively, for Austria. At least for Austria and Germany, it is hence fair to conclude that the rational tax tariffs are not only neutral to the government budget, but also to the overall income distribution.

Table 3: Effect on income inequality

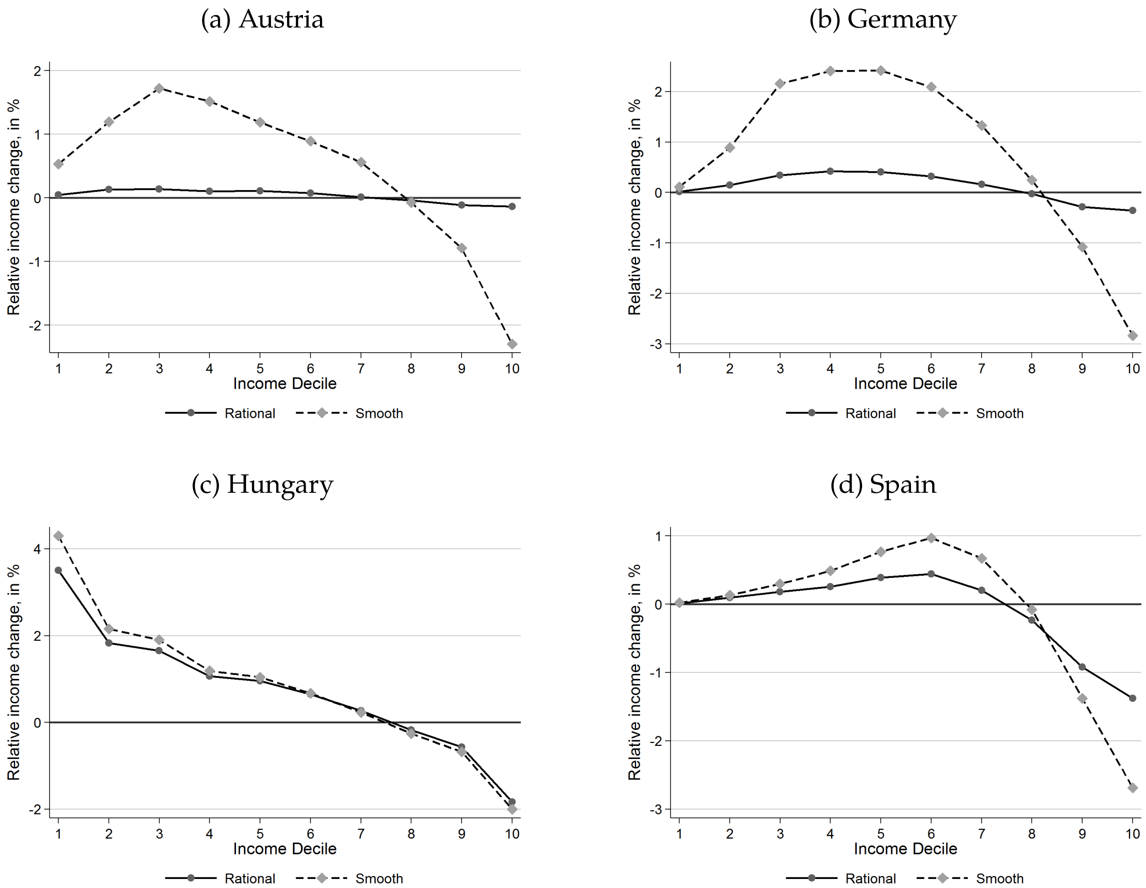
	Baseline	*relative change*		*Baseline*	*relative change*	
		Rational	Smooth		Rational	Smooth
		Austria			Germany	
Gini	0.249	-0.20%	-2.61%	0.278	-0.47%	-2.95%
P90P10	3.026	-0.01%	-2.02%	3.527	-0.28%	-1.55%
P90P50	1.697	-0.05%	-2.43%	1.835	-0.76%	-3.86%
		Hungary			Spain	
Gini	0.294	-2.22%	-2.54%	0.335	-0.88%	-1.61%
P90P10	3.952	-4.31%	-5.35%	5.308	-0.96%	-1.66%
P90P50	1.771	-2.22%	-2.47%	2.053	-1.59%	-2.66%

Note: Own calculations based on EUROMOD vH1.0+. The table shows relative changes in the inequality measures, based on equivalized disposable income using the modified OECD equivalence scale.

This finding notwithstanding, a switch to a smoother tax schedule induces winners and losers across the income distribution, as depicted in Figure 7. The relative change in disposable income for each income decile reveals a strictly progressive effect for Hungary, i. e. the lowest income groups benefit strongest while the top income decile loses around 2 per cent. As the other countries exempt low incomes from taxation, the bottom income groups are less affected. In Austria and Germany, the main beneficiaries of the reform are rather found in the middle class, while the top deciles are worse off. Income changes amount to around 2% maximum in these countries. The intuition for the impact of both smooth tax functions becomes apparent when looking at Fig. 4 and 5. For given E_{max}, M_s converges quicker to the top marginal tax rate than M_r, leading to higher effective tax rates already for moderate incomes. In order to achieve a comparable redistributional impact, the introduction of the fully smooth function would need to be coupled with a reconsideration of E_{max} and E_{min}. Overall, the decile composition reveals more redistribution from top to bottom for both types of tariffs. Despite differences in status quo tax schedules and income distributions, our reforms induce a transfer to the bottom 60 per cent in all countries, financed through higher expenses by the top 20 per cent.

The distributional effects presented here ignore behavioral reactions. As labor supply elasticities are typically estimated to be positive, (Bargain et al., 2014), one could expect a net increase in labor supply and employment at least for the fully smooth function with non-negligible income changes.

Figure 7: Income Changes by Deciles

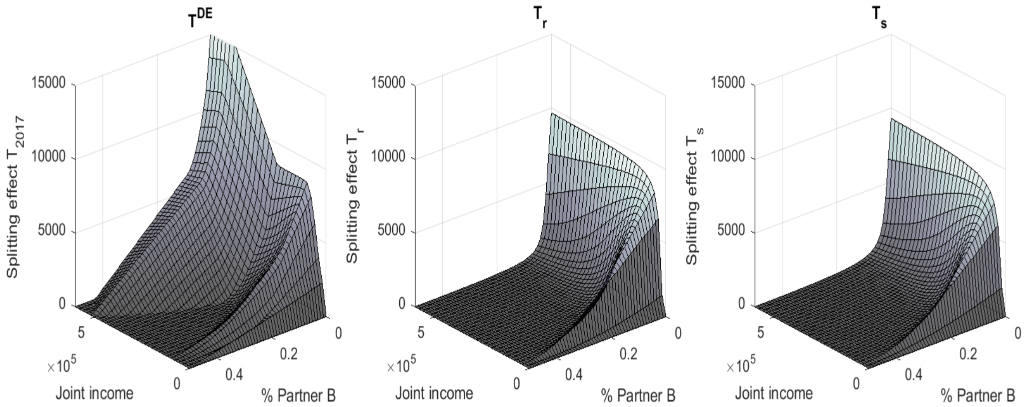


Notes: Own calculations based on EUROMOD H1.0+. Income deciles are based on equivalized disposable income using the modified OECD equivalence scale.

5.3 Splitting Advantage in Germany

One feature of the German tax system deserves special attention (Eq. 6). In Germany, married couples file jointly for the income tax. This way, tax liabilities are lower compared to single-filing. This 'splitting effect' increases with the difference in earnings between both spouses due to the progressive schedule. As becomes apparent in Fig. 8, our counterfactual tax functions lower the maximum tax advantage from income splitting.

Figure 8: Income advantage from splitting for different combinations of joint taxable incomes



6 Conclusion

Existing tax-benefit systems impose plenty of discontinuities in budget constraints. These are predominantly considered undesirable, as they induce behavioral reactions by individuals or firms that would not occur in absence of these discontinuities. Another complaint against existing tax-benefit systems is their complexity, imposing significant compliance costs on taxpayers (Shaw et al., 2010). This concerns both the opaque definition of the tax base with numerous exemptions as well as the shape of the tax schedule itself. This is where our paper steps in by proposing a framework for 'smooth' tax functions which are fully continuous and differentiable. Moreover, they facilitate individuals' calculation of their tax liability for many existing tax regimes. We have shown that, depending on the exact function, a revenue-neutral switch to a smooth tax schedule can be achieved in distributionally-neutral manner. An obvious exception is the replacement of a pure flat tax regime. Beyond, we have shown that smooth tax functions do not aggravate the magnitude of the Bracket Creep problem, but eliminates the undesired spikes. Finally, our tax functions can easily be made robust to bracket creep.

For the two functions considered here in detail, we showed that a rational saturation function of the marginal tax rate results to be a good approximation of the current tariff function for Germany and Austria, while a fully smooth function involves a modest redistribution. For Hungary, we showed how progressive tariffs could be introduced without avoiding the arbitrariness of defining completely new tax brackets. Finally, for Spain we tested a simple possibility to realize a smoother tax functions, but realized that there are other aspects that maybe need to be smoothened first.

Smooth tax functions bear also interesting implications from a polit-economic perspective. A smooth tax function, whatever its exact shape, fixes the redistributional character of the income tax. In this article, the key policy parameters, exemption level and top marginal tax rate, were maintained. The parameters are however interdependent, i. e. the top marginal tax rate cannot be altered without changing the tariff for lower incomes. This makes it harder to target specific income groups via tax reform and could hence serve as a corrective measure to preserve extreme forms of clientelism. In any case, the reduction of the amount of parameters to be determined would generate more transparency.

In a broad context, smooth tariffs represent only one, yet important element of tax-benefit systems. The definition of the tax base and the interaction with other elements of the tax-benefit system may impose further discontinuities in the budget set. However, we are confident that our approach could inspire analogous strategies to avoid them.

Appendix

For completeness, in this appendix we verify in detail that the considered system of equations (4)–(5) has a unique solution.

Let us first analyse (5) for the two functions we considered:

- E_r: From

$$E_r(x_{min}) = E_{max} \cdot \frac{x_{min} - x_0}{x_{min} - 2x_0 + x_h} = E_{min}$$

it follows

$$\frac{x_{min} - x_0}{x_{min} - 2x_0 + x_h} = \frac{E_{min}}{E_{max}} =: C < 0.5,$$

i.e.

$$x_0 = \frac{(1-C)x_{min} - Cx_h}{1-2C}.$$

If we now consider fixed C, x_{min} and $x_i > x_{min}$ and analyse E_r as a function of x_h, we obtain

$$\begin{aligned} E_r(x_h) &= E_{max} \cdot \frac{x_i - \frac{(1-C)x_{min} - Cx_h}{1-2C}}{x_i - 2\frac{(1-C)x_{min} - Cx_h}{1-2C} + x_h} \\ &= E_{max} \cdot \frac{C\,x_h + (1-2\,C\,)x_i + (C-1)\,x_{min}}{x_h + (1-2\,C\,)x_i + 2\,(C-1)\,x_{min}}, \end{aligned}$$

that results to be a strictly monotonic decreasing function of x_h,

$$\lim_{x_h \to x_{min}} E_r(x_h) = E_{max}, \quad \lim_{x_h \to \infty} E_r(x_h) = E_{min}.$$

- E_s: From

  $$E_s(x_{min}) = E_{max} \cdot 0.5^{\frac{x_h - x_0}{x_{min} - x_0}} = E_{min}$$

  it follows

  $$\frac{x_h - x_0}{x_{min} - x_0} = \frac{\ln\left(\frac{E_{min}}{E_{max}}\right)}{\ln 0.5} =: C > 1,$$

  i.e.

  $$x_0 = \frac{Cx_{min} - x_h}{C-1}.$$

  If again we consider fixed C, x_{min} and $x_i > x_{min}$ and analyse E_s as a function of x_h, we obtain

  $$E_s(x_h) = E_{max} 0.5^{\frac{x_h - \frac{Cx_{min} - x_h}{C-1}}{x_i - \frac{Cx_{min} - x_h}{C-1}}} = E_{max} 0.5^{\frac{C\,(x_h - x_{min})}{x_h + (C-1)\,x_i - C\,x_{min}}}$$

  that also results to be a strictly monotonic decreasing function of x_h,

  $$\lim_{x_h \to x_{min}} E_s(x_h) = E_{max}, \quad \lim_{x_h \to \infty} E_s(x_h) = E_{min}.$$

Consequently, in both cases the revenue results for given X and P are also continuous strictly monotonic decreasing function of x_h and

$$\text{Revenue}_r(x_h) := \sum_{i=1}^{k} p_i T_r(x_i, x_h), \quad \text{and} \quad \text{Revenue}_s(x_h) := \sum_{i=1}^{k} p_i T_s(x_i, x_h).$$

For our above considerations it follows that $\text{Revenue}_r(x_h) = \text{Revenue}$ and $\text{Revenue}_s(x_h) = \text{Revenue}$ are uniquely solvable with respect to x_h for any Revenue fulfilling

$$E_{min} \sum_{i=1}^{k} p_i \cdot x_i < \text{Revenue} < E_{max} \sum_{i=1}^{k} p_i \cdot x_i.$$

Figure 9: Change in Effective Tax Rates

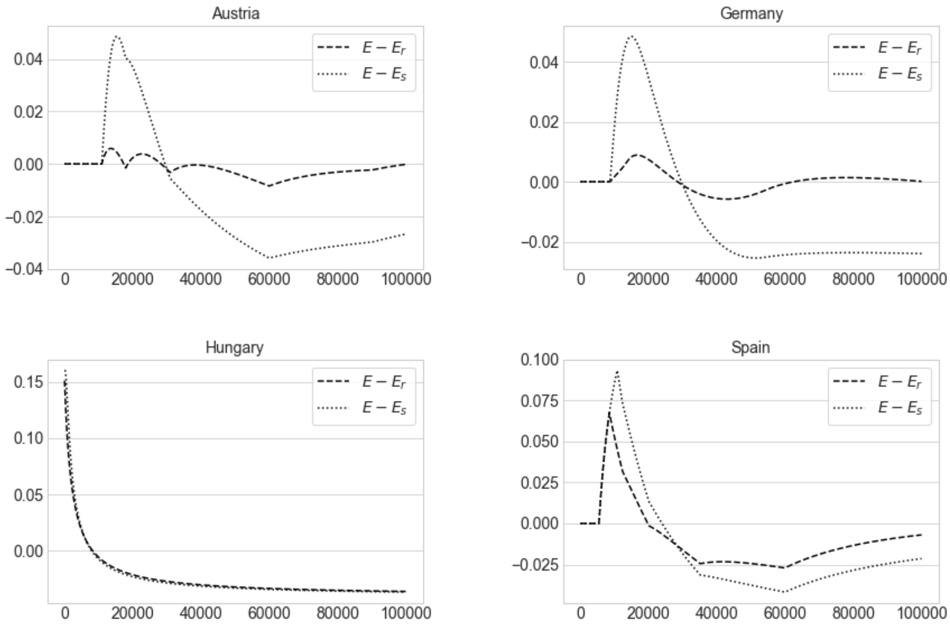


Figure 10: Tax Functions

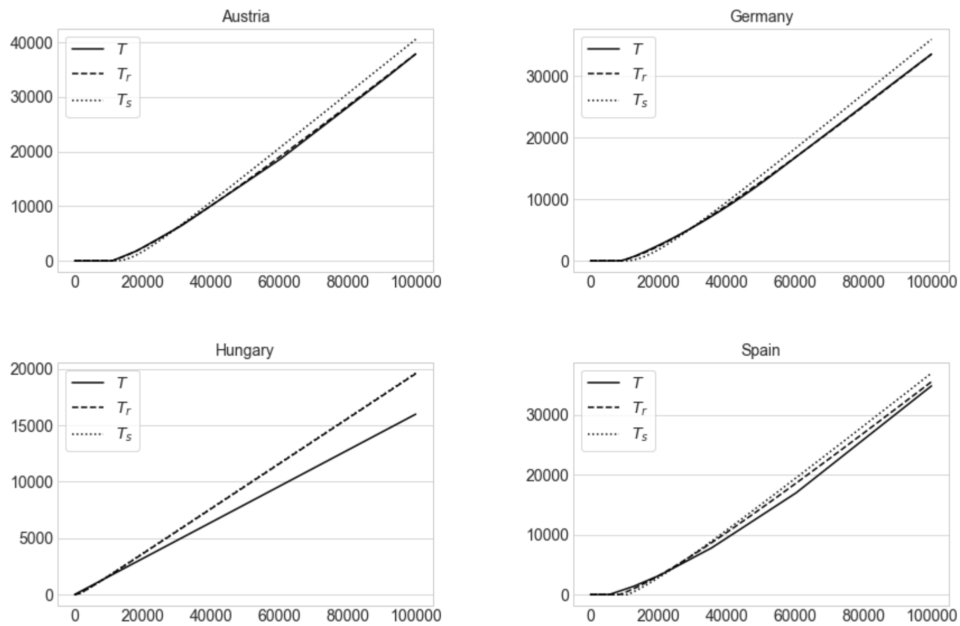
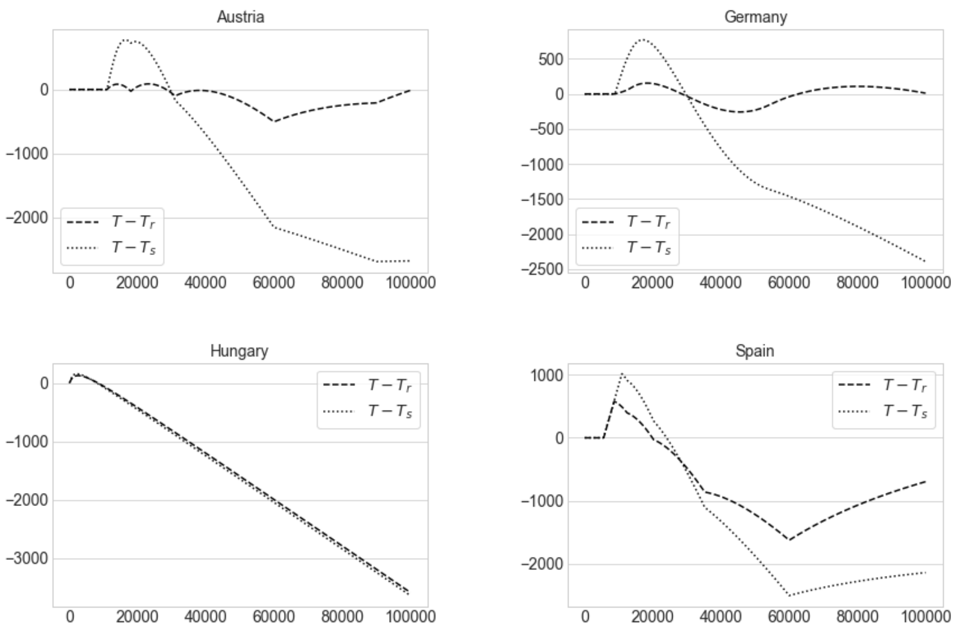


Figure 11: Tax Due Differences



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