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The Cost of Job Loss

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Non-technical summary

Modern labour markets are characterised by a large degree of worker turnover. This turnover is typically considered a result of the process through which workers and firms find suitable job matches. However, it may also come at a cost to some workers. It has been extensively documented that workers who lose their jobs may also face large and persistent wage losses in the future. Several explanations for why we observe these wage losses have been put forward. Wage losses can occur because jobless workers do not accumulate and potentially depreciate their human capital. They can also occur through the loss of firm specific payments, the interruption of workers' on-the-job search or because some workers or jobs may be more stable than others.

In this paper we present a framework that captures all these features. We use this framework to quantitatively assess the role of human capital, on-the-job search and worker heterogeneity in explaining the observed long-term wage losses suffered after a job loss. We estimate the wage losses due to job separation for young workers in the UK and show that our calibrated model fits the observed patterns very well. We use the model to evaluate the importance of each of the components that affect the cost of job loss.

Human capital losses exert a strong negative and permanent effect on future wages. The effects of workers' on-the-job search and firms' tenure contracts, although temporary and smaller in size, are long-lasting. It takes workers around 10 years to recover wages through these channels. Human capital losses play a more important role in explaining the extent and persistence of wage losses among low skilled workers. Among high skilled workers, on-the-job search implies re-employment wages start recovering sooner.

The Cost of Job Loss*

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Abstract

In this paper we develop and quantitatively assess a tractable equilibrium search model of the labour market to analyse the long-term wage costs of a job loss. In our framework, these costs occur due to losses in workers' human capital and firm specific compensation, interruptions to workers' on-the-job search and due to turnover heterogeneity. A key feature is that firms post wage-tenure contracts as an optimal response to their employees' search behaviour and human capital accumulation. We estimate the wage losses due to job separation for young workers in the UK and show that our calibrated model fits the observed patterns very well. We use the model to evaluate the importance of each of the components that affect the cost of job loss. Human capital losses exert a strong negative and permanent effect on future wages. The effects of workers' on-the-job search and firms' tenure contracts, although temporary and smaller in size, are long-lasting. It takes workers around 10 years to recover wages through these channels. Human capital losses play a more important role in explaining the extent and persistence of wage losses among low skilled workers. Among high skilled workers, on-the-job search implies re-employment wages start recovering sooner.

Keywords: Job search, human capital accumulation, job loss.
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1 Introduction

Modern labour markets are characterised by a large degree of worker turnover. In the UK, for example, on average 4.5% of the labour force separates from their jobs and 4.3% of the labour force start a new job every quarter.[1] This turnover is typically considered a result of the process through which workers and firms find suitable job matches. However, it may also come at a cost to some workers. In particular, since Jacobson, Lalonde and Sullivan (1993), it has been extensively documented that workers who lose their jobs may also face large and persistent wage losses in the future. Several explanations for why we observe these wage losses have been put forward. Wage losses can occur because jobless workers do not accumulate and potentially depreciate their human capital (Ljungqvist and Sargent, 1998). They can also occur through the loss of firm specific payments (Topel, 1991), the interruption of workers' on-the-job search (Manning, 2003) or because some workers or jobs may be more stable than others (Steward, 2007). In this paper we present a tractable theory that captures all these features.[2] We use this theory to quantitatively assess the role of human capital, on-the-job search and worker heterogeneity in explaining the observed long-term wage losses suffered after a job loss.

Our theoretical framework generalises the models of Burdett and Coles (2003) and Burdett, Carrillo-Tudela and Coles (2011). In particular, we study labour market equilibrium in a balance growth path setting in which risk averse workers search for a job when unemployed and for a better one when employed. Further, workers face two forms of involuntary mobility: a job loss and a job reallocation shock. The former renders workers unemployed, while the latter keeps workers in employment but moves them to another (possibly worse) job. We assume employed workers accumulate general human capital through learning-by-doing and lose part of it when unemployed. These processes generate a rich set of worker job turnover dynamics and endogenous human capital heterogeneity.

Firms commit to a sequence of wages that maximise the expected profit per new hire, taking into account its applicants' human capital levels and risk aversion, its current workers' on-the-job search and human capital accumulation, and the aggregate state of the economy. To solve the firm's problem, we focus on a "timeless" equilibrium (see Woodford, 2003). In such an equilibrium we show that the optimal contract is independent of applicants' initial human capital levels and the aggregate state of the economy. The nature of its solution is similar to Burdett and Coles (2003): firms' incentives to backload compensation and reduce workers' quit probability is tempered by workers' desire to smooth consumption over time. We show that in equilibrium such wage contracts can be expressed in terms of a sequence of piece rates (wages per unit of human capital) that continuously increase with a worker's tenure in the firm.

Even though our model has a very rich structure, it remains tractable and the equilibrium can be fully characterised. For example, we show that the endogenous distribution of workers'

[1] These statistics are calculated using the UK Labour Force Survey for the period 1993-2013. See Carrillo-Tudela, Hobijn, She and Visschers (2015) for details on the characteristics of worker reallocation in the UK labour market.

[2] We abstract from other potential sources of wage loss that also have been put forward in the literature such as adverse selection (Lookwood, 1991) or the loss of match-specific payments (Den Haan, Ramey and Watson, 2000).

productivities is log-normal. Observed wages are then a convolution of the latter distribution and the equilibrium distribution of piece rates paid by firms. Unlike Burdett and Coles (2003), the resulting wage distribution no longer has an increasing and convex density, but has similar properties to the empirical wage distribution. The combination of human capital accumulation, employer-to-employer transitions and tenure effects further implies that displaced workers' wage losses have a "permanent" and "transitory" components. The permanent component is due to human capital loss relative to those workers that did not lose their jobs. The temporary component is due to workers' on-the-job search. Once re-employed, workers can recover part of their losses through voluntary employer-to-employer transitions and positive returns to firm tenure. Job reallocation shocks also imply that involuntary employer-to-employer transitions speed up wage convergence.

We use these insights to quantitatively assess the cost of job loss. To do so, we calibrate the model using simulation methods of moments by matching salient features of the labour market histories of young workers in the UK. We focus our analysis on young workers as it is precisely at this stage of workers' labour market history that job mobility is most common and a model like the one developed here is more relevant. Further, Kletzer and Fairlie (2003) find that large and persistent job separation losses are also present among young workers and not confined to older workers. Following Burdett, Carrillo-Tudela and Coles (2014) we divide the sample into two education (skill) categories: low and high skill.[3] An important reason for analysing these workers separately is because they exhibit very different job turnover patterns. Low skilled workers face a much higher job loss probability, a much lower probability of changing jobs directly through an employer-to-employer transition and a much lower probability of regaining employment than high skilled workers. Overall, our model fits the data well. The model is able to reproduce workers' turnover patterns, their returns to labour market experience and to firm tenure, as well as the observed amount of frictional wage dispersion as measured by Hornstein, Krusell and Violante's (2011) Mm ratio and initial re-employment wage losses.

We estimate the long-term wage losses of job separation using longitudinal data from the British Household Panel Survey (BHPS) for the 1991-2004 period. We do so following the econometric approach proposed by Jacobson et al. (1993) and used by Kletzer and Fairlie (2003), Couch and Placzek (2010) and Davis and von Wachter (2011), among many others. Under this approach one estimates a distributed-lag equation to evaluate the wage path some years before and some years after job separation relative to a control group. We find that young workers experience large and persistent wage losses and these losses start occurring prior to the job separation episode. Moreover, the difference between the average wages of those workers who lost their jobs and those workers in the control group do not seem to recover over time. Even after 8 years after the job loss event this difference is 20 percent of average pre-separation wages. We find, however, that their is some recovery of the loss in terms of wage growth.[4]

[3]These categories are defined by whether individuals managed to achieve basic qualifications during their secondary education.

[4]These results echo the findings of Kletzer and Fairlie (2003) for young workers in the US. Our estimates also complement the existing evidence on the cost of job loss for the UK. In particular, Arulampalam (2001) uses the BHPS for the period 1991-1997 and estimates a standard mincer wage equation augmented by variables that capture the effects of a non-employment spell on wages during the re-entry employment spell. She finds that 4

To evaluate the predictions of our theory relative to the data, we estimate the same distributed-lag equation using simulated data. The model's estimates replicate the data very well, reproducing the main features of the empirical wage losses described above. When decomposing these wage losses by skill group, our model shows that low skilled workers are the ones that experience the largest and more persistent wage losses. It is these workers' wage losses which do not seem to recover, while the wage losses of high skilled workers start recovering two years after displacement. We also find that human capital accumulation on its own predicts much larger and persistent wage losses than the one observed in the data, more so for low skilled workers. In contrast, when evaluating on-the-job search on its own we find that it generates much lower wage losses than the ones observe in the data. However, its temporary effects are long-lasting: it takes around 10 years after the job separation for wages to recover to their pre-separation levels. Taken together, these results suggest that the strong permanent effects induced by the loss of human capital are tempered by workers' on-the-job and firms' optimal responses to worker turnover. Further, differences in worker turnover patterns across skill groups imply that human capital losses play an more important part in explaining the extent and persistence of wage losses among low skilled workers. On-the-job search exerts a strong moderating force that allows the wages of high skilled workers to start recovering sooner.

Our paper contributes to the recent literature that uses search models to evaluate the cost of job loss. Davis and von Wachter (2011) evaluate a search model based on Mortensen and Pissarides (1994) augmented to allow workers' on-the-job search. They argue that this model has trouble matching the observed losses of displaced workers in the US. Using this framework, Krolikowski (2014) brings the model closer to the data by adding a fixed quality component to the job and assuming that all hires from unemployment start at the bottom of the quality ladder. Jung and Kuhn (2014) explain the observed job losses by further introducing general human capital accumulation and life-cycle considerations. Our theoretical framework is very different from the one presented in those papers. In particular, here we allow firms to post optimal long-term wage contracts in response to workers' on-the-job search instead of assuming Nash Bargaining. Further, we emphasise the interaction between general human capital accumulation/depreciation and optimal wage-tenure contracts as important force in shaping the wage losses of displaced workers.

Jarosh (2014) also examines the cost of job loss by proposing a version of the Bagger, Fontaine, Postel-Vinay and Robin (2014) model in which firms differ in their job destruction rates as modelled by Pinheiro and Visschers (2014). In addition to on-the-job search and general human capital accumulation, his approach stresses the role of firm specific heterogeneity through difference in job stability.[5] In contrast, our approach stresses differences in worker turnover across skill groups. Further, Jarosh (2014) considers a framework in which firms offer optimal wage-tenure contracts based on the sequential auction mechanism proposed by Postel-Vinay and

years after re-employment, workers face a reduction of 11% in log wages. See also Gregory and Jukes (2001) for evidence on the impact of the duration of unemployment on re-employment wages.

[5] Pesoa de Araujo (2014) estimates the framework of Pinheiro and Visschers (2014) on Brazilian data, but do not consider optimal contracts as in our paper.

Robin (2002).[6] Here we assume that firms do not counteroffer their employees outside offers. Although offer-matching arises in certain markets where firms have access to a fair amount of information about job applicants, such as the academic market for economists, this may not be a good description of behaviour in other labour markets. Therefore, it seems worthwhile exploring alternative wage determination mechanisms and their effects on the interaction between human capital and search frictions to explain the cost of job loss.

Our paper also contributes to the theoretical analysis of equilibrium models with on-the-job search, proposed by Burdett and Mortensen (1998). In that framework each firm commits to a singleton, a time-invariant wage, and a timeless equilibrium requires each wage strategy is a best response to the market steady state. Similarly Moscarini and Postel-Vinay (2013) can be interpreted as a timeless equilibrium with aggregate shocks where each firm's wage policy is a best response to the market's stationary dynamics. Here we extend those approaches to the optimal contract structure first considered in Stevens (2004), Burdett and Coles (2003): that wage payments are also conditioned on tenure. In this version, however, we simplify by assuming no aggregate shocks and so the timeless equilibrium characterises the balanced growth path.

The rest of the paper is outlined as follows. Section 2 describes the model and characterises a timeless equilibrium. Section 3 describes the data, the estimation procedure and the fit of the model. Section 4 presents estimates the cost of job loss for the UK and compares it to the one generated by the model. Section 5 concludes by further discussing our findings in relation to the literature that analysis the cost of job loss in the UK. We relegate to the Appendix tedious proofs and a description of the data, simulations and the estimation procedure of the wage loss equation.

2 The Model

Time is continuous with an infinite horizon. There is a continuum of both firms and workers, each of measure one. All agents are infinitely lived and discount the future at rate $r > 0$. Firms are indexed by $j \in [0, 1]$, are equally productive and have a constant returns to scale technology. Workers are heterogenous with general human capital $k \in (0, \infty)$. A worker type k generates revenue flow $Ak > 0$ while employed and home production flow bAk while unemployed where $b \in [0, 1)$. $A > 0$ is an aggregate productivity parameter which grows at exogenous rate $\gamma_A \geq 0$.

Unemployed workers receive job offers at exogenous Poisson rate $\lambda_0 > 0$, employed workers receive outside offers at rate $\lambda_1 > 0$. Job search is random in that any job offer is considered a random draw from the set of all firms in the market. There are also exogenous quits: at rate $\lambda_q \geq 0$ an employed worker receives a random outside offer and automatically accepts it.

Learning-by-doing implies a worker's human capital grows at rate $\rho > 0$ while employed. At exogenous rate $\delta > 0$, a worker-firm match separates and the worker becomes unemployed.[7] Rather than assume the worker's human capital falls by ϕk in the event of such a shock, however,

[6] Krolikowski (2014) also considers the contracting approach of Postel-Vinay and Robin (2002) as an alternative wage determination process to Nash Bargaining.

[7] We interpret such separations as a reallocation shock which forces the worker to seek new employment and possibly in a new occupation.

it is more convenient to suppose that human capital declines at constant rate ϕ while unemployed. Then $\phi/(\phi+\lambda_0)$ describes the expected fall in human capital due to a separation shock.

Throughout we assume unemployment equals its steady state value $u=\delta/(\delta+\lambda_0)$, such that $\gamma_K=(1-u)\rho-u\phi$ will describe the average growth rate of human capital in the economy and we assume $\gamma_K\geq 0$. Throughout we require sufficient discounting that values are bounded - the sufficient condition on r is obtained below.

Although human capital shocks imply workers have a precautionary motive to save, for tractability we ignore this issue and assume worker consumption equals earnings at all points in time. We further assume a flow utility function with constant relative risk aversion; i.e. $u(w)=w^{1-\sigma}/(1-\sigma)$ with $\sigma>0$. Firms must always pay a positive wage $w\geq 0$ but, for simplicity, we only consider equilibria where the non-negative wage constraint is never binding on the optimal contract (assuming $\sigma\geq 1$ is sufficient as that implies $u'(0)=-\infty$).

Firms post wage contracts where equal treatment (no discrimination) requires that should a firm hire equally productive workers on the same day, those workers are thereafter paid the same. The equal treatment assumption rules out the offer-matching approach as considered in Postel-Vinay and Robin (2002). Following Burdett and Coles (2003) firms may offer contracts where wages vary with tenure and so reward worker loyalty.[8] Thus we assume each firm $j\in[0,1]$ can, in general, pre-commit to a wage tenure contract which, at any future date t, promises to pay wage $w=w_{jt}(\tau,k,A)$ to employees with tenure τ, human capital k with aggregate productivity A. We assume anti-slavery legislation, where bonding arrangements which tie the worker to the firm are non-enforceable, implies workers can always freely quit.

As is well-known, the initial value problem in dynamic games with policy pre-commitment generates uninteresting non-stationary dynamics; e.g. Woodford (2003). We focus attention on "timeless" equilibria where each firm $j\in[0,1]$ pre-commits to a wage contract $w=w_j(\tau,k,A)$ which does not explicitly depend on time.

Firms are risk neutral and each firm j offers a timeless wage tenure contract $w_j(\tau,k,A)$ which maximizes expected discounted profit along the balanced growth path. Given random outside offers, an employee compares the value of remaining at his/her current firm on contract $w(.)$ with current tenure $\tau\geq 0$ or switching to the outside offer by firm $j\in[0,1]$, a wage tenure contract $w_j(\cdot)$ with zero tenure. Assuming no recall of rejected employment offers, the worker accepts employment at the firm whose (continuation) contract offers greatest expected discounted utility.

The following identifies timeless equilibria in which the optimal wage tenure contract is separable with form $w_j(\tau,k,A)=Ak\widetilde{\theta}_j(\tau)$. In other words firms, in equilibrium, offer piece rate tenure contracts $\widetilde{\theta}_j(.)$.

Definition of Equilibrium: a timeless equilibrium is a set of piece rate tenure contracts $\{\widetilde{\theta}_j(.)\}_{j\in[0,1]}$ such that along the balanced growth path:

(i) wage contract $w_j(\tau,k,A)=Ak\widetilde{\theta}_j(\tau)$ is value maximising for each firm $j\in[0,1]$;

(ii) workers use an optimal job search strategy; and

[8] See Stevens (2004) for a complete discussion on the structure of optimal contracts when workers are risk neutral.

(iii) the distribution of employment, wages and human capital is consistent with equilibrium turnover along the balanced growth path.

As human capital (on average) grows over time we let $H_0(.)$ denote the (normalised) distribution of worker productivities k at some reference date 0. Below we show a timeless equilibrium implies the unemployment rate u is the same for all worker types k. Thus the distribution of skills $H_0(.)$ is the same across the employed and across the unemployed. We will further show a balanced growth path implies H_0 is log-normal with a mean μ which grows at rate γ_K and a standard deviation which also grows at a constant rate. Observed wages are then a convolution of the (expanding) log-normal distribution of productivities and the equilibrium distribution of piece rates paid by firms.

Worker Optimality in a Timeless Equilibrium.

Consider then a timeless equilibrium in which each firm $j \in [0,1]$ posts a piece rate tenure contract with wage paid $w_j(.) = Ak\widetilde{\theta}_j(\tau)$. Along the balanced growth path, let $V = V(\tau, k, A|\theta)$ denote the value enjoyed by an employed worker with tenure τ, human capital k in aggregate state A on representative piece rate tenure contract $\theta(\cdot)$. Let $V^U(k, A)$ denote the value of being unemployed.

Search rents imply firms make strictly positive profit and so a contract which causes its employees to quit into unemployment is never optimal. Given firms offer piece rate tenure contracts $\{\widetilde{\theta}_j(.)\}_{j\in[0,1]}$, an optimal piece rate tenure contract θ in a timeless equilibrium generates employment value $V(\tau, k, A|\theta)$ given by the Bellman equation:

$$\begin{aligned} rV(\tau,k,A|\theta) &= \frac{\theta(\tau)^{1-\sigma}(Ak)^{1-\sigma}}{1-\sigma} + \frac{\partial V}{\partial \tau} + \rho k \frac{\partial V}{\partial k} + \gamma_A A \frac{\partial V}{\partial A} \\ &\quad + \delta[V^U(k,A) - V(\tau,k,A|\theta)] \\ &\quad + \lambda_1 E_j[\max\{V(\tau,k,A|\theta), V(0,k,A|\widetilde{\theta}_j)\} - V(\tau,k,A|\theta)] \\ &\quad + \lambda_q E_j[V(0,k,A|\widetilde{\theta}_j) - V(\tau,k,A|\theta)]. \end{aligned}$$

The flow value of employment varies through time as (i) the piece rate paid varies with tenure (picked up by the $\partial V/\partial \tau$ term), (ii) the worker's productivity increases through learning-by-doing (at rate ρ), (iii) aggregate productivity increases (at rate γ_A), (iv) a separation shock occurs at rate δ, (v) a randomly drawn outside offer $\widetilde{\theta}_j$ is received at rate λ_1 and (vi) an exogenous quit occurs at rate λ_q.

The value of being unemployed satisfies

$$\begin{aligned} rV^U(k,A) &= \frac{b^{1-\sigma}(Ak)^{1-\sigma}}{1-\sigma} - \phi k \frac{\partial V^U}{\partial k} + \gamma_A A \frac{\partial V^U}{\partial A} \\ &\quad + \lambda_0 E_j[\max\{V^U(k,A), V(0,k,A|\widetilde{\theta}_j)\} - V^U(k,A)]. \end{aligned}$$

Direct inspection establishes these value functions are multiplicatively separable in productivity

Ak; i.e. $V(.)$ and V^U take form

$$\begin{aligned} V(.|\theta) &= (Ak)^{1-\sigma} U(\tau|\theta) \\ V^U(.) &= (Ak)^{1-\sigma} U^U \end{aligned} \tag{1}$$

with $U(\tau|\theta)$, U^U as defined below. In what follows we refer to $U(\tau|\theta)$ as the piece rate value of contract θ (at tenure τ) and U^U as the piece rate value of unemployment.

Let $U_0 = U(0|\theta)$ denote the starting piece rate value of piece rate contract θ. As search is random, let $F(\widetilde{U}_0)$ denote the fraction of offered contracts $\{\widetilde{\theta}_j\}$ whose starting piece rate value is no greater than $\widetilde{U}_0$. Substituting out $V(.|\theta) = (Ak)^{1-\sigma}U(\tau|\theta)$ and $V^U(.) = (Ak)^{1-\sigma}U^U$ in the Bellman equations above yields the following expressions for $U(.|\theta)$ and U^U :

$$[r + \delta + \lambda_q - [\rho + \gamma_A](1 - \sigma)]\, U - \frac{dU}{d\tau} = \frac{\theta(\tau)^{1-\sigma}}{1-\sigma} + \delta U^U + \lambda_1 \int_U^{\overline{U}} [1 - F(U_0)] dU_0 + \lambda_q \int_{\underline{U}}^{\overline{U}} U_0 dF(U_0) \tag{2}$$

$$(r + \phi(1-\sigma) - \gamma_A(1-\sigma))U^U = \frac{b^{1-\sigma}}{1-\sigma} + \lambda_0 \int_{U^U}^{\overline{U}} [U_0 - U^U] dF(U_0) \tag{3}$$

To guarantee bounded solutions exist, we assume r satisfies both $r > [\rho + \gamma_A](1 - \sigma)$ and $r > (\gamma_A - \phi)(1 - \sigma)$. The above expressions now yield the following Claim.

Claim 1: *Optimal job search for any worker k in a timeless equilibrium implies:*
(a) while unemployed, the worker accepts a contract offer $\widetilde{\theta}_j(.)$ if and only if its initial piece rate value $U_0 \geq U^U$;
(b) while employed with current piece rate value $U(\tau|\theta)$, the worker accepts a job offer $\widetilde{\theta}_j$ if and only if it offers greater piece rate value $U(0|\widetilde{\theta}_j) > U(\tau|\theta)$. The worker will quit into unemployment whenever piece rate value $U(\tau|\theta) < U^U$.

Claim 1 implies equilibrium turnover is independent of (k, A). Hence a timeless equilibrium implies the unemployment rate of each type k is the same. Hence the distribution of skills $H(.)$ is the same across both employed and unemployed workers.

Let $G(U)$ denote the fraction of employed workers who enjoy piece rate value no greater than U and let $[\underline{U}, \overline{U}]$ denote its support, noting a strictly positive profit equilibrium implies $\underline{U} \geq U^U$. Claim 1 implies the distribution of piece rate values $G(.)$ is the same for each worker type k.

Optimal Wage Tenure Contracts in a Timeless Equilibrium.

Consider now the optimal wage tenure contract $w_j(\tau, k, A)$ of firm j in a timeless equilibrium. With no loss of generality, the optimal contract $w = w_j(\tau, k, A)$ can always be written as a piece rate contract $\theta_j(.)$ with wage paid $w = Ak\theta_j(\tau, k, A)$. Consider now a representative hire, where k_0 denotes the worker's human capital when first hired and A_0 the aggregate productivity level at that date. As $k = k_0 e^{\rho\tau}$ and $A = A_0 e^{\gamma_A \tau}$ at all future tenures $\tau \geq 0$, there is no further loss in generality by restricting attention to piece rate contracts of the type $\theta_j = \theta_j(\tau|k_0, A_0)$. In other words, firm j potentially discriminates piece rates paid by employee type (k_0, A_0) at the point of hire with subsequent wage paid $w_j(.) = A_0 k_0 e^{(\rho+\gamma_A)\tau}\theta_j(\tau|A_0, k_0)$ at tenures $\tau \geq 0$.

Now given any such piece rate contract $\theta = \theta_j(\tau|A_0, k_0)$, (2) determines its piece rate value $U(.|\theta)$ in a timeless equilibrium. If its starting piece rate value $U(0|\theta) < U^U$, any worker who accepts this offer immediately quits into unemployment and so this contract offer makes zero profit. Suppose instead it yields starting piece rate value $U(0|\theta) \geq U^U$. Given a contact with an outside worker, Bayes rule implies

$$\alpha = \frac{\lambda_0 u + \lambda_q(1-u)}{\lambda_0 u + \lambda_1(1-u) + \lambda_q(1-u)}$$

is the probability the contact is either unemployed on piece rate value U^U or is an exogenous quitter. In either case, $U(0|\theta) \geq U^U$ implies the contact accepts the job offer. Instead with complementary probability $1-\alpha$ this worker is employed where random contacts implies $G(.)$ describes the distribution of current piece rate value earned by this worker. Hence $\alpha + (1 - \alpha)G(U_0)$ with $U_0 = U(0|\theta)$ is the probability this contract offer is accepted.

Suppose the worker accepts the job offer and $U(\tau|\theta))$ is the piece rate value of this contract at tenure τ. As $F(.)$ describes the distribution of starting piece rate values offered by all other firms, the probability this new hire remains employed by tenure τ is

$$\psi(\tau|\theta) = e^{-\int_0^\tau \{\delta+\lambda_q+\lambda_1[1-F(U(s|\theta))]\}ds}. \tag{4}$$

Note, conditional on piece rate contract $\theta(.)$, this survival probability is otherwise independent of (A_0, k_0).

To determine the contract that maximizes expected profit, we first identify the optimal piece rate contract which maximizes expected discounted profit conditional on the worker accepting the job with starting value $U_0 \geq U^U$; i.e. we solve

$$\max_{\theta(.)} A_0 k_0 \int_0^\infty \psi(\tau|\theta)e^{(\rho+\gamma_A-r)\tau}[1-\theta(\tau|.)]d\tau,$$

subject to $U(0|\theta) = U_0$. As the profit maximising contract is independent of (A_0, k_0), we can denote it as $\theta = \theta^*(\tau|U_0)$ and so define

$$\Pi^*(U_0) = \int_0^\infty \psi(\tau|\theta^*)e^{(\rho+\gamma_A-r)\tau}[1-\theta^*(\tau|.)]d\tau$$

as the firm's maximised expected profit (per hire and per unit of worker productivity A_0k_0). As $\alpha + (1-\alpha)G(U_0)$ is the probability this contract offer is accepted, an optimal contract offer yields expected profit:

$$\Omega(U_0|A_0, k_0) = A_0 k_0 \left[\alpha + (1-\alpha)G(U_0)\right]\Pi^*(U_0).$$

The firm then chooses starting piece rate value U_0 to maximise $\Omega(.)$; i.e. the firm solves the program:

$$\max_{U_0} [\alpha + (1-\alpha)G(U_0)]\,\Pi^*(U_0).$$

As this optimal choice is also independent of (A_0, k_0) the following Claim is immediate.

Claim 2: *In a timeless equilibrium, offering a piece rate contract $\theta(\tau|A_0, k_0)$ which is independent of A_0, k_0 is privately optimal.*

Claim 2 establishes that when all other firms offer piece rate contracts which do not discriminate between worker types, then offering such a contract is always a best response for each firm. We now construct such timeless equilibria.

2.1 Optimal Piece Rate Tenure Contracts in Timeless Equilibria.

Here we consider the optimal piece rate contract of a representative firm in a timeless equilibrium conditional on offering a starting piece rate value $U_0 \geq U^U$. Equilibria in which the corner constraint $\theta \geq 0$ is non-binding implies the optimal contract $\theta^*(.|U_0)$ solves:

$$\Pi^*(U_0) = \max_{\theta(.)} \int_0^\infty \psi(\tau|\theta)e^{(\rho+\gamma_A-r)\tau}[1-\theta(\tau)]d\tau, \tag{5}$$

subject to starting value $U(0|\theta) = U_0$, where $\psi(.)$ is given by (4).

Theorem 1: *In a timeless equilibrium and for any $U_0 \geq U_U$, an optimal contract $\theta^*(.|U_0)$ and corresponding worker and firm payoffs U^* and Π^* are solutions to the following dynamical system $\{\theta, U, \Pi\}$ where, at any tenure $\tau \geq 0$,*
(a) $\theta(\tau)$ is given by the implicit function

$$\begin{aligned} & \frac{\theta^{1-\sigma}}{1-\sigma} + \theta^{-\sigma}\left[(1-\theta) - [r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U)]]\,\Pi\right. \\ = \;& [r+\delta+\lambda_q-[\rho+\gamma_A](1-\sigma)]\,U - \delta U^U - \lambda_1\int_U^{\overline{U}}[1-F(U_0)]dU_0 - \lambda_q\int_{\underline{U}}^{\overline{U}} U_0 dF(U_0) \end{aligned} \tag{6}$$

(b) piece rate profit

$$\Pi(\tau) = \int_\tau^\infty e^{-\int_\tau^s[r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U(t))]dt}[1-\theta(s)]ds, \tag{7}$$

(c) piece rate value U evolves according to the differential equation

$$\frac{dU}{d\tau} = -\theta^{-\sigma}\frac{d\Pi}{d\tau} \tag{8}$$

with initial value $U(0|.) = U_0$.
Proof: see Appendix.

Equation (6) is very general: it describes the optimal piece rate paid without assuming $F(.)$ is differentiable. To reveal the structure of the optimal contract, however, we differentiate (6) and (7) with respect to τ and so identify the following system of differential equations for $\{\theta, \Pi, U\}$:

$$\dot{\theta} = \frac{\lambda_1 \left[\theta^{1-\sigma}\right]}{\sigma} F'(U)\Pi - (\rho + \gamma_A)\theta; \tag{9}$$

$$\dot{\Pi} = [r + \delta + \lambda_q - \rho - \gamma_A + \lambda_1[1 - F(U)]]\Pi - (1 - \theta); \tag{10}$$

with $\dot{U}$ given by (8).

Equation (9) describes how the optimal piece rate paid changes with tenure. As the actual wage paid $w(\tau) = A_0 k_0 e^{(\rho+\gamma_A)\tau}\theta(\tau)$ then, along the optimal contract, the wage paid changes according to:

$$\left[\frac{-u''}{u'^2}\right]\frac{dw}{d\tau} = \left[A_0 k_0 e^{(\rho+\gamma_A)\tau}\right]^\sigma \lambda_1 F'(U)\Pi. \tag{11}$$

A quit is costly to the firm as it loses the piece rate profit of the employment relationship. While $\Pi > 0$, (11) implies the wage paid increases with tenure, where $F'(U)$ measures the number of firms whose outside offer will marginally attract this worker. If $F'(U) = 0$ then marginally raising the wage paid at tenure τ has no impact on the worker's quit rate. Optimal consumption smoothing then implies the firm pays a (locally) constant wage. If $F'(U) > 0$, however, a slightly higher wage results in a slightly lower marginal quit rate and wages thus increase with tenure. The scaling term $\left[A_0 k_0 e^{(\rho+\gamma_A)\tau}\right]$ arises as the worker's value of employment at tenure τ is $V(\tau, .) = [A_0 k_0 e^{(\rho+\gamma_A)\tau}]^{1-\sigma} U(\tau|\theta)$ while the firm's continuation profit is $\left[A_0 k_0 e^{(\rho+\gamma_A)\tau}\right]\Pi(\tau)$. (11) thus describes the optimal trade-off in wage space. (10) is more useful, however, for the quit rate depends on piece rate values U rather than V. Most importantly, Theorem 1 describes the optimal piece rate contract which is independent of the worker's type (A_0, k_0) when hired.

Note that a constant wage (perfect consumption smoothing) implies the piece rate paid $\theta(\tau)$ declines at rate $\rho + \gamma_A$. Although an optimal contract implies wages must always increase within an employment spell, it is no longer the case that tenure effects are necessarily positive.

To provide some intuition for the construction that follows, and assuming for the moment F is differentiable, the optimal piece rate contract is the saddle path solution to the differential equation system (θ, U, Π) given by (8)-(10). Let $(\theta^\infty, \Pi^\infty, U^\infty)$ denote the stationary point of this dynamical system; i.e. $(\theta^\infty, \Pi^\infty, U^\infty)$ solve:

$$[\theta^\infty]^\sigma = \frac{\lambda_1}{(\rho + \gamma_A)\sigma} F'(U^\infty)\Pi^\infty \tag{12}$$

$$\Pi^\infty = \frac{1 - \theta^\infty}{r + \delta + \lambda_q - \gamma_A - \rho + \lambda_1(1 - F(U^\infty))}. \tag{13}$$

$$[r + \delta + \lambda_q - [\rho + \gamma_A](1 - \sigma)]\, U^\infty = \frac{[\theta^\infty]^{1-\sigma}}{1 - \sigma} + \delta U^U + \lambda_1 \int_{U^\infty}^{\overline{U}} [1 - F(U_0)]dU_0 + \lambda_q \int_{\underline{U}}^{\overline{U}} U_0 dF(U_0). \tag{14}$$

Unlike Burdett and Coles (2003), there are two possible types of optimal tenure contracts: those whose piece rates converge to θ^∞ from above and those which converge to θ^∞ from below.

Figure 1 illustrates the possible set of optimal contracts $\theta^*(.)$.

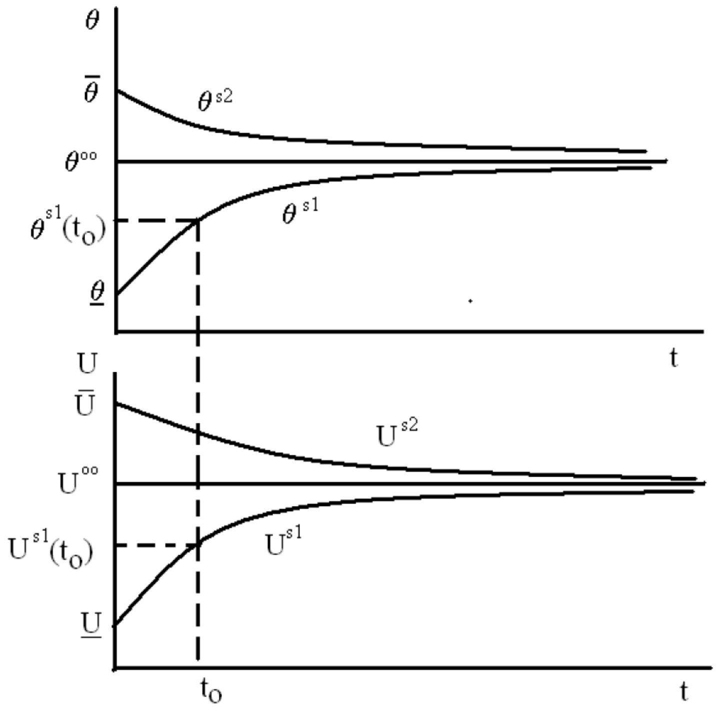


Figure 1: Possible Baseline Salary Scales

Consider first the optimal contract for the firm offering the least generous contract, i.e. one which yields starting value $U_0 = \underline{U} \geq U^U$ and suppose $\underline{U} < U^\infty$. As piece rates increase with tenure along the optimal contract, the piece rate value of employment increases so that $U(\tau|.)$ converges to U^∞ from below. Let $\theta_1(\tau)$ denote this optimal piece rate contract which we define as the lower baseline piece rate scale. Consider instead the optimal contract offered by firms offering the most generous contract $U_0 = \overline{U}$ and suppose $\overline{U} > U^\infty$. Although the wage paid increases with tenure, the piece rate $\theta(.)$ decreases with tenure. The piece rate value of employment thus falls with tenure and $U(\tau|.)$ converges to U^∞ from above. Let $\theta_2(\tau)$ denote this optimal piece rate contract which we refer to as the upper baseline piece rate scale.

Consider now a firm that offers a contract which yields an initial piece rate value U_0 such that $\underline{U} < U_0 < U^\infty$. As depicted in Figure 1, define t_0 as the point on the lower baseline contract where $U_1(t_0) = U_0$. Optimality of the lower baseline piece rate scale yields an important simplification: the optimal contract yielding U_0 is the continuation contract starting at point t_0 on the lower baseline piece rate scale; i.e., the optimal contract $\theta^*(\tau|U_0)$ is $\theta_1(t_0 + \tau)$ where the piece rate paid at tenure τ corresponds to point $(t_0 + \tau)$ on the lower baseline piece rate scale. We denote its corresponding profit as $\Pi_1(t_0)$. Suppose instead $U^\infty < U_0 < \overline{U}$. This time the optimal contract yielding U_0 is the continuation contract starting at point t_0 on the upper baseline piece rate scale where $U_2(t_0) = U_0$ and so yields profit $\Pi_2(t_0)$. It is this baseline property of the optimal contract structure which makes tractable the complete characterisation of timeless equilibria.

2.2 Characterisation and Existence of a Timeless Equilibrium.

In any timeless equilibrium, an optimal wage contract corresponds to a piece rate tenure contract with starting point t_0 on either the upper or lower baseline piece rate scale $i = 1, 2$. Corresponding to any such starting point is a starting piece rate value $U_i(t_0)$ which, if accepted by worker type (A_0, k_0), generates expected profit $A_0 k_0 \Pi_i(t_0)$ for the firm. Thus any such optimal contract offer generates expected profit

$$\Omega_i(t_0|A_0, k_0) = A_0 k_0 \left[\alpha + (1-\alpha)G(U_i(t_0))\right] \Pi_i(t_0)$$

per worker contact. As expected profit is simply proportional to $k_0 A_0$, identifying a timeless equilibrium reduces to solving the constant profit condition:

$$\begin{aligned}
\left[\frac{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G(U_i(t_0))}{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)}\right] \Pi_i(t_0) \quad &= \quad \overline{\Omega} \ > 0 \text{ if } dF(U_i(t_0)) > 0 \qquad (15)\\
\left[\frac{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G(U_i(t_0))}{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)}\right] \Pi_i(t_0) \quad &\leq \quad \overline{\Omega} \text{ otherwise.}
\end{aligned}$$

The proof of Claim 3 uses this constant profit condition to identify a simple equilibrium relationship between piece rate profit Π and piece rate θ.

Claim 3: *Suppose $dF(U_i(t_0))) > 0$ and $\Pi = \Pi_i(t_0)$ and $\theta = \theta_i(t_0)$ at that point. The constant profit condition implies equilibrium relationship:*

$$\Pi = \frac{\sqrt{[r-\rho-\gamma_A]^2\overline{\Omega}^2 + 4[\delta+\lambda_q]\overline{\Omega}(1-\theta)} - [r-\rho-\gamma_A]\overline{\Omega}}{2[\delta+\lambda_q]} \qquad (16)$$

Proof: see Appendix.

Claim 3 is most important: it identifies the equilibrium relationship between firm values Π and piece rates θ paid. Note that piece rate profit Π is strictly decreasing in θ.

We now simplify the equilibrium conditions for a timeless equilibrium by "substituting out" t_0 in Theorem 1. The standard recursive contracting approach with pre-commitment supposes the contract "promises" continuation value U to an employer (e.g. Spear and Srinivastan, 1987). That approach would identify $\theta = \theta(U_0)$ as the piece rate paid when the worker is promised piece rate value U_0, with corresponding firm profit $\Pi = \Pi(U_0)$. Claim 3, however, reveals a more convenient approach. Instead we consider the inverse scenario, that $U = \widehat{U}(\theta)$ describes the piece rate value enjoyed by a worker when the optimal contract pays piece rate θ in a timeless equilibrium. $\widehat{\Pi}(\theta)$ describes the firm's corresponding piece rate profit. We adopt this approach as Claim 3 in fact determines $\Pi = \widehat{\Pi}(\theta)$. Furthermore (8) in Theorem 1 implies worker piece rate values evolve according to:

$$\frac{d\widehat{U}}{d\theta} = -\theta^{-\sigma}\frac{d\widehat{\Pi}}{d\theta}. \qquad (17)$$

Thus given the above relationship (16), worker piece rate values $\widehat{U}$ are strictly increasing in piece rates paid.

Let $F_\theta(\theta)$ denote the distribution of starting piece rates paid by firms and let $[\underline{\theta}, \overline{\theta}]$ denote its support. As (16) and (17) imply $\widehat{U}(.)$ is strictly increasing, we have:

$$F_\theta(\theta) = F(\widehat{U}(\theta)) \text{ for } \theta \in [\underline{\theta}, \overline{\theta}].$$

Let $G_\theta(\theta)$ denote the distribution of piece rates paid across employed workers and so

$$G_\theta(\theta) = G(\widehat{U}(\theta)).$$

The constant profit condition is now more conveniently written as

$$\begin{aligned} \left[\frac{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G_\theta(\theta)}{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)}\right]\widehat{\Pi}(\theta) &= \overline{\Omega} > 0 \text{ if } dF_\theta(\theta) > 0 \\ \left[\frac{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G_\theta(\theta)}{\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)}\right]\widehat{\Pi}(\theta) &\leq \overline{\Omega} \text{ otherwise.} \end{aligned} \tag{18}$$

Let $\overline{U} = \widehat{U}(\overline{\theta})$ denote the highest value enjoyed by workers. A simple contradiction argument establishes $G(\overline{\theta}) = 1$. Putting $\theta = \overline{\theta}$ in (18) now finds $\widehat{\Pi}(\overline{\theta}) = \overline{\Omega}$. Putting $\theta = \overline{\theta}$ and substituting out $\widehat{\Pi}(\overline{\theta}) = \overline{\Omega}$ in (16) now determines

$$\overline{\Omega} = \frac{1-\overline{\theta}}{\delta + \lambda_q + r - \rho - \gamma_A}. \tag{19}$$

Thus (16) and (19) imply (20) described in Theorem 2 below. The following Claim now rules out the existence of the upper baseline piece rate scale (see Figure 1).

Claim 4: *A timeless equilibrium implies* $\theta^\infty = \overline{\theta}$.
Proof: As $\widehat{\Pi}(\overline{\theta}) = \overline{\Omega} = \frac{1-\overline{\theta}}{\delta+\lambda_q+r-\rho-\gamma_A}$, (10) implies $\dot{\Pi} = 0$ at $\theta = \overline{\theta}$. Hence $\overline{\theta}$ is a stationary point of the differential equation system implied by Theorem 1.

By using backward iteration from $\theta = \overline{\theta}$, it is now straightforward to characterise a timeless equilibrium.

Theorem 2: *For* $\theta \leq \overline{\theta}$ *and while* $F'_\theta > 0$ *then:*

$$\widehat{\Pi}(\theta) = \frac{1-\overline{\theta}}{2[\delta+\lambda_q]}\left[\sqrt{\left[\frac{r-\rho-\gamma_A}{\delta+\lambda_q+r-\rho-\gamma_A}\right]^2 + \frac{4[\delta+\lambda_q]}{\delta+\lambda_q+r-\rho-\gamma_A}\frac{1-\theta}{1-\overline{\theta}}} - \frac{r-\rho-\gamma_A}{\delta+\lambda_q+r-\rho-\gamma_A}\right] \tag{20}$$

$$\widehat{U}(\theta) = \overline{U} - \int_\theta^{\overline{\theta}} \frac{[\theta']^{-\sigma}}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\,[\delta+\lambda_q+r-\rho-\gamma_A]\,\frac{1-\theta'}{1-\overline{\theta}}\right]^{1/2}} d\theta' \tag{21}$$

$$\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G_\theta(\theta) = \frac{\left[1-\overline{\theta}\right]\left[\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)\right]}{\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\widehat{\Pi}(\theta)} \tag{22}$$

$$1 - F_\theta = \theta^\sigma \int_\theta^{\overline{\theta}} \sigma[\frac{1}{\theta'}]^{\sigma+1}\Psi(\theta')d\theta', \tag{23}$$

where

$$\Psi(\theta) = \frac{\lambda_q + \delta}{\lambda_1}\left[\frac{\widehat{\Pi}(\theta) - \overline{\Omega}}{\overline{\Omega}}\right] - \left[\frac{(\rho+\gamma_A)}{\lambda_1}\frac{\theta d\widehat{\Pi}/d\theta}{\widehat{\Pi}(\theta)}\right] > 0.$$

Proof: see Appendix.

All that remains is to determine equilibrium $\overline{\theta}$, the highest piece rate paid in the market. Claim 5 establishes the relevant boundary condition.

Claim 5: *A timeless equilibrium implies* $\underline{U} = \widehat{U}(\underline{\theta}) = U_U$ *where:*

$$1 - \underline{\theta} = \frac{(\delta+\lambda_q+\lambda_1)\left[\delta+\lambda_q+\lambda_1+r-\rho-\gamma_A\right]}{(\delta+\lambda_q)[\delta+\lambda_q+r-\rho-\gamma_A]}\left[1-\overline{\theta}\right]. \tag{24}$$

Proof: Standard contradiction argument establishes $\underline{U} = U_U$. To identify $\underline{\theta}$, a simple contradiction argument establishes $G_\theta(\underline{\theta}) = 0$. Putting $\theta = \underline{\theta}$ in the constant profit condition (18), with $\widehat{\Pi}(\underline{\theta})$ given by (20), $G_\theta(\underline{\theta}) = 0$, $\overline{\Omega}$ given by (19) and $u = \delta/(\delta+\lambda_0)$ yields the result.

Thus to identify a timeless equilibrium, fix a candidate equilibrium value for $\overline{\theta}$ in the range

$$\overline{\theta} \in (1 - \frac{(\delta+\lambda_q)[\delta+\lambda_q+r-\rho-\gamma_A]}{(\delta+\lambda_q+\lambda_1)\left[\delta+\lambda_q+\lambda_1+r-\rho-\gamma_A\right]}, 1). \tag{25}$$

Such a candidate value implies strictly positive profit ($\overline{\Omega} > 0$) and $\underline{\theta} > 0$ (strictly positive piece rates paid). Given this candidate choice of $\overline{\theta}$, let $\widetilde{F}_\theta(.|\overline{\theta})$ denote the unique candidate distribution function F_θ implied by (23) in Theorem 2. Given the implied distribution of contract offers, Claim 6 now identifies the implied values of $\underline{U}$ and U^U at $\underline{\theta}$, which we denote $\underline{\widetilde{U}}(\overline{\theta})$, $\widetilde{U}^U(\overline{\theta})$ respectively.

Claim 6: *Given* $\overline{\theta}$ *and the implied candidate distribution function* $\widetilde{F}_\theta$ *then:*

$$[r - [\rho+\gamma_A](1-\sigma)]\,\underline{\widetilde{U}} = \frac{\overline{\theta}^{1-\sigma}}{1-\sigma} - \int_{\underline{\theta}}^{\overline{\theta}} \frac{\left[r+\delta-[\rho+\gamma_A](1-\sigma)+\lambda_q\widetilde{F}_\theta(\theta|.)\right]\theta^{-\sigma}d\theta}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}. \tag{26}$$

$$[r+\phi(1-\sigma)-\gamma_A(1-\sigma)]\widetilde{U^U} = \frac{b^{1-\sigma}}{1-\sigma} + \lambda_0\int_{\underline{\theta}}^{\overline{\theta}} \frac{[1-\widetilde{F}_\theta(\theta|.)]\theta^{-\sigma}d\theta}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}. \tag{27}$$

Proof: see the Appendix.

Identifying a timeless equilibrium reduces to finding a $\overline{\theta}$ such that $\underline{\widetilde{U}}(\overline{\theta}) = \widetilde{U}^U(\overline{\theta})$.

Theorem 3: *For* $\sigma > 1$, *a* $\overline{\theta}$ *satisfying (25) exists such that* $\widehat{U}(\underline{\theta}) = U^U$.

Proof: see the Appendix.

If F_θ identified by (23) is a positive increasing function (i.e. has the properties of a distribution function), then Theorems 2 and 3 fully characterise timeless equilibria. By construction, all optimal contracts which offer $\theta \in [\underline{\theta}, \overline{\theta}]$ yield the same expected profit $\overline{\Omega} > 0$. Consider then any deviating contract. Clearly, a suboptimal contract which offers $U_0 \in [\widehat{U}(\underline{\theta}), \widehat{U}(\overline{\theta})]$ yields lower profit. Further as $U^U = \widehat{U}(\underline{\theta})$, any contract which offers value $U_0 < \widehat{U}(\underline{\theta})$ yields zero profit as all workers reject such an offer. Finally any contract which offers $U_0 > \widehat{U}(\overline{\theta})$ attracts no more workers than the optimal contract which offers $U_0 = \widehat{U}(\overline{\theta})$ while the latter contract earns strictly greater profit per hire. As no deviating contracts exist which yield greater profit, Theorems 2 and 3 identify timeless equilibria.

Theorem 2 describes all equilibrium objects apart from the piece rate scale. Equations (9) and (17) imply the piece rate scale is identified by the initial value problem:

$$\begin{aligned} \dot{\theta} &= \frac{\lambda_1 \left[\theta^{1-\sigma}\right]}{\sigma} \frac{F_\theta' \widehat{\Pi}}{d\widehat{U}/d\theta} - (\rho + \gamma_A)\theta \\ &= \left[-\frac{\theta \widehat{\Pi}}{\frac{d\widehat{\Pi}}{d\theta}} \right] \frac{\lambda_1 F_\theta'}{\sigma} - (\rho + \gamma_A)\theta. \end{aligned}$$

with $\theta(0) = \underline{\theta}$, where proof of Theorem 2 establishes:

$$\frac{\theta F_\theta'}{\sigma} = \Psi - (1 - F_\theta).$$

In Appendix B we discuss an algorithm to compute equilibrium. We use this algorithm in our quantitative analysis. Note that the $\theta \geq 0$ constraint may bind on the optimal contract if b is sufficiently small. For example suppose $\lambda_0 = \lambda_1$ and $b = 0$. Foot in the door and learning by doing implies workers would, in theory, accept a lower starting wage $w < b$. Hence $w \geq 0$ binds in this case. Whenever this occurs, the (lower) baseline piece rate scale pays a zero wage for tenures $\tau \leq \overline{\tau}$ and a positive (increasing) wage thereafter. As we show below this does not occur in our calibrations.

3 Quantitative Analysis

We now analyse the quantitative implications of our model in relation to the cost of job loss. We investigate the importance of human capital, job search and worker heterogeneity in determining the long-lasting wage effects of non-employment. To do so, we first calibrate the main parameters of our model to match salient features of the UK labour market, using the British Household Panel Survey (BHPS) and the Labour Force Survey (LFS).

3.1 Data

The BHPS is an annual survey of individuals, age 16 years or more, in a nationally representative sample of about 5,500 households. Approximately 10,000 individuals are interviewed each year.

It started in 1991 and was subsumed by the new and bigger survey "Understanding Society" in 2010. The BHPS contains socioeconomic information, including information about household organization, the labour market, income and wealth, housing, health and socioeconomic values. Using this information one is able to reconstruct the labour market histories of individuals since leaving full-time education. Using Maré (2006) we derive consistent histories that summarize individual's labour market histories, including actual and potential work experience; wages and hours worked; and several socio economic characteristics that are standard in household survey data.

This data set allows us to estimate the long-term wage losses of workers who lost their jobs. We also use this data set as part of our calibration procedure, where we estimate workers' average wage-experience profiles and measures of wage dispersion. For these exercises, we consider real hourly (gross) wages and trim the wage distribution by 5 percent on each side to reduce measurement error and to consider all jobs that pay above the national minimum wage, introduced in the UK in 1999.[9]

The LFS is a quarterly survey of individuals, aged 16 years or more. It has a rotating panel structure, in which individuals that live on the sampled address are followed for a maximum of 5 quarters, also referred to as waves. Each quarter, one-fifth of the sample of addresses is replaced by an incoming rotation group. In each wave, the respondents provide information about, among other things, their labour market status. We use the two-quarter longitudinal sample of the LFS to derive the transitions rates between different employment status and use them as part of our calibration procedure. Relative to the BHPS, the LFS gives more accurate estimates of these empirical transitions rates as it offers higher frequency information and consists of about 60,000 individuals each quarter.

For both data sets we use information on white male workers for the period 1991-2004. We stratify the sample into two educational or skill groups. We consider workers to be low skilled if they reported having no qualification, other qualifications, apprenticeship, CSE, commercial qualifications or no O-levels. We consider workers to be high skilled if they reported having achieved A-levels, nursing qualifications, teaching qualifications, university degree or higher and other higher qualifications. This stratification is motivated by Burdett et al. (2014), who show that in the UK low and high skilled workers reallocate across jobs in very different ways. As shown below differences in turnover patterns across skill groups have important implications for the cost of job loss. Finally, we focus on young workers as it is precisely at this stage of a worker's labour market history that job mobility is most common and our model is more suitable.[10]

3.2 Calibration

To simulate the model we set the time period to a month and fix $r = 0.005$, which gives an annual discount factor of around 6%. We also set $\gamma_A = 0.00183$ to match the estimated slope

[9]Following Dustmann and Pereira (2008), we construct real hourly wages by dividing monthly (gross) earnings by 4.33 weeks and then by the average number of hours worked in a week in full-time jobs. We also take into account overtime hours and use the CPI to deflate nominal wages.

[10]To account for difference in age of graduation, we consider those low skilled workers that have 16 and 30 years of age, and those high skilled who have between 22 and 36 years of age.

of a linear trend on output per worker in the UK for the time period we consider.[11] This parametrisation leaves us with a vector $\Omega = \{\delta, \lambda_0, \lambda_1, \lambda_q, \rho, \phi, \sigma, b\}$, consisting of 8 parameters that we jointly recover by minimizing the sum of squared (percentage) difference between a set of simulated moments from the model and their counterparts in the data, using the identity matrix as weighting matrix. We provide details of the simulation procedure in Appendix B.

Targeted Moments We target 14 moments based on the main characteristics of the labour market to which the model is directly related. Table 1 shows these moments for each skill group. To motivate their choice, first consider the duration moments. Note that the average non-employment spell is the model's counterpart of $1/\lambda_0$. Further, the average employment duration is the model's counterpart of $1/\delta$. Average job duration and the ratio between the average involuntary to voluntary employer-to-employer (EE) transition rates give information about λ_1 and λ_q.[12]

Table 1: Targeted Moments

Moments	Low Skilled		High Skilled	
	Data	Model	Data	Model
Average transitions / duration				
Non-employment spell (months)	12.67	11.91	8.74	8.52
Employment spell (years)	3.78	3.78	12.60	12.63
Job spell (years)	3.10	2.89	5.03	5.43
Invol/vol EE transitions	0.57	0.58	0.50	0.54
OLS returns to experience (%)				
2 years	4.47	3.86	4.58	4.21
4 years	8.62	7.82	8.75	8.40
6 years	12.47	11.87	12.49	12.57
8 years	16.00	16.02	15.81	16.73
10 years	19.23	20.26	18.71	20.86
OLS returns to tenure (%)				
2 years	4.19	4.62	2.27	2.46
4 years	7.29	8.64	3.84	4.65
6 years	9.31	12.03	4.70	6.58
Wage dispersion				
Mm ratio	1.62	1.63	1.62	1.63
re-emp wage / mean wage	0.91	0.92	0.84	0.85

We also target returns to labour market experience and firm tenure. Our theory implies log wages are the sum of worker k fixed effect, current TFP, firm j fixed effect and firm specific tenure effects, such that

$$\log w_{ij}(\tau, k, A) = \log k + \log A + \log \theta_j(0) + \log \frac{\theta_j(\tau)}{\theta_j(0)}. \tag{28}$$

[11] We estimate the slope of the linear trend through OLS, by regressing the log of quarterly output per worker on quarterly dummies and a linear trend. Output per worker is obtained by dividing total output (GDP) over total employment.

[12] In Appendix B we detail how we classified voluntary and involuntary employer-to-employer transitions.

The firm fixed effect, $\log\theta_j(0)$, is central to our explanation of quit turnover: an endogenous quit occurs whenever an employee at firm j on piece rate $\theta_j(\tau)$ receives an outside offer from a different firm j' which offers starting piece rate $\theta_{j'}(0) > \theta_j(\tau)$. The difficulty in estimating (28) is that with endogenous quit turnover, the firm fixed effect is correlated with experience and tenure: the former as workers with long experience have had more time to find a good match, the latter as matches with high value are likely to survive longer and wage changes within a firm depend $\theta_j(0)$.

Estimating returns to experience and tenure directly from (28) while controlling for this endogeneity is problematic.[13] Instead we follow an indirect inference approach and use as an auxiliary model a (misspecified) Mincer wage equation, which we estimate through OLS (see Appendix B for details). Conditional on λ_0 and δ, average returns to experience and tenure give information primarily about ρ, ϕ, λ_1, λ_q and σ. The assumption is that both in the model and in the data, the OLS estimates of the returns to experience depend on the rate of human capital accumulation and the rate of job change; while the returns to firm tenure depend on the rate at which employed workers receive job offer and on their degree of risk aversion.[14]

The final set of moments refer to observed measures of wage dispersion. The ratio between re-employment and average wages and Hornstein et al. (2011) measure of frictional wage dispersion (Mm ratio) give extra information about ϕ, ρ, σ and b. To estimate the Mm ratio we also follow an indirect inference approach, using as an auxiliary equation the one proposed by Hornstein et al. (2007).

Parameters Table 1 shows that the fit of the model is quite good, capturing important features of workers' early careers in the labour market. Table 2 shows the resulting parameters. As mentioned earlier, an important feature of the data is that low and high skilled workers experience very different turnover patterns. For example, low skilled workers experience non-employment spells much more often and take longer to get re-employed than high skilled workers. This explains the large differences between the values of δ, λ_0 and the unemployment rate, u, across skill groups. Further, low skilled workers reallocation across jobs mainly through unemployment, while high skilled workers tend to reallocate across jobs both through unemployment and direct employer to employer transitions.[15] The difference between the values of the job offer arrival rates, λ_1 and λ_q, across skill groups reflect this feature of the data.

Table 2 also shows that both skill groups have similar human capital accumulation rates, to a large extent reflecting that in our data both types of workers exhibit similar returns to labour market experience (see Table 1). Note however that low skilled workers face a higher rate of human capital depreciation when unemployed. Given that low skilled workers also exhibit longer

[13] See for example the classic papers Topel (1991), Altonji and Shakotko (1991) and more recently Dustmann and Meghir (2005).

[14] Of course, in the model all the parameters that determine the shape of the baseline salary scale (BSS), will influence both types of returns. For example, numerical simulations show that, *ceteris paribus*, (i) a higher ρ shifts up and flattens the BSS, (ii) a higher ϕ shifts down the BSS, (iii) a higher λ_1 shifts up and steepens the BSS, (iv) a higher λ_q shifts down and steepens the BSS, and (v) a higher σ shifts up and flattens the BSS.

[15] Table 1 shows that low skilled workers exhibit an average employment duration that is nearly as long as their average job duration. In contrast, high skilled workers exhibit an average employment duration that is more than twice as long as their average job duration.

Table 2: Parameters

	Low Skilled	High Skilled
Parameters		
δ	0.0215	0.0066
λ_0	0.0768	0.1108
λ_1	0.0096	0.0285
λ_q	0.0021	0.0030
ρ	0.0035	0.0036
ϕ	0.0022	0.0016
σ	4.015	2.575
b	0.3252	0.5326
Endog. variables		
$F(\underline{\theta})$	0.071	0.16
u	0.218	0.055
γ_K	0.0023	0.0032
Uemp HC loss	0.026	0.015

average unemployment spells, the values of ϕ and λ_u imply that when unemployed these workers face an expected human capital loss that is nearly twice the size of that faced by high skilled workers, where the expected loss is given by $\phi/(\phi + \lambda_u)$. Further, given the differences between the unemployment rates across skill groups, we also obtain that for high skilled workers the average growth rate of human capital is around 40% larger than that of low skilled workers.

We also find that high skilled workers are less risk averse and face higher values of b than low skilled workers. The values of these parameters seem to allow the model to mainly match the returns to tenure and the extent of frictional wage dispersion.

Figure 2 shows the resulting baseline salary scales. Their relative position implies that high skilled workers receive a higher proportion of their output and, given the human capital and turnover parameters, also higher wages. Further, high skilled workers face a steeper baseline salary scale at short tenures and flatter at longer tenures relative to low skilled workers. This feature reflects that in the calibration high skilled workers are less risk averse and receive job offers at a faster rate than low skilled workers. Notice that Table 1 shows that these differences are consistent with the estimated tenure profiles, where low skilled workers exhibit higher returns to tenure. The reason is that since low skilled workers have a higher δ and a shorter average job duration, they are typically found in the stepper section of their baseline salary scale; while a lower δ and a longer average job duration, imply that high skilled workers are typically found in a flatter section of their baseline salary scale.

4 Long-term Wage Losses

We now turn to evaluate the implications of our model relative to the wage losses faced by displaced workers. We follow the exercise proposed initially by Jacobson et al. (1993). This

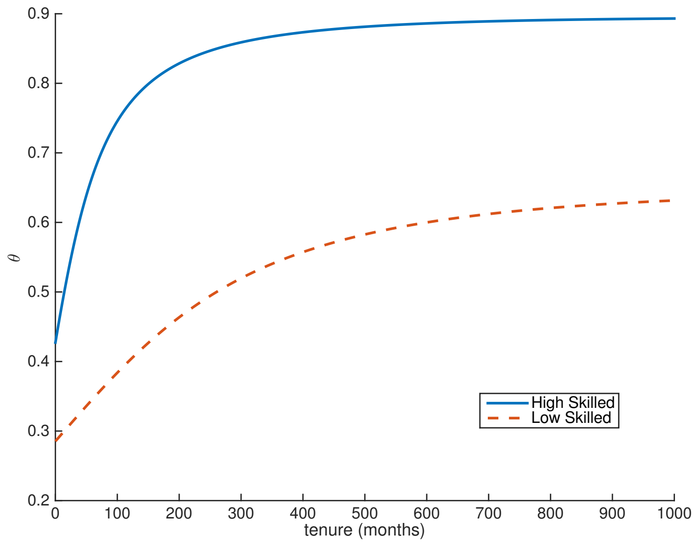


Figure 2: Baseline Salary Scales

basically consists of estimating the parameters of the following distributed-lag equation:

$$w_{it}^{y} = \alpha_i^y + \gamma_t^y + \beta^y X_{it} + \sum_{k=-T_s}^{T_e} \varepsilon_k^y D_{it}^k + \mu_{it}^y, \tag{29}$$

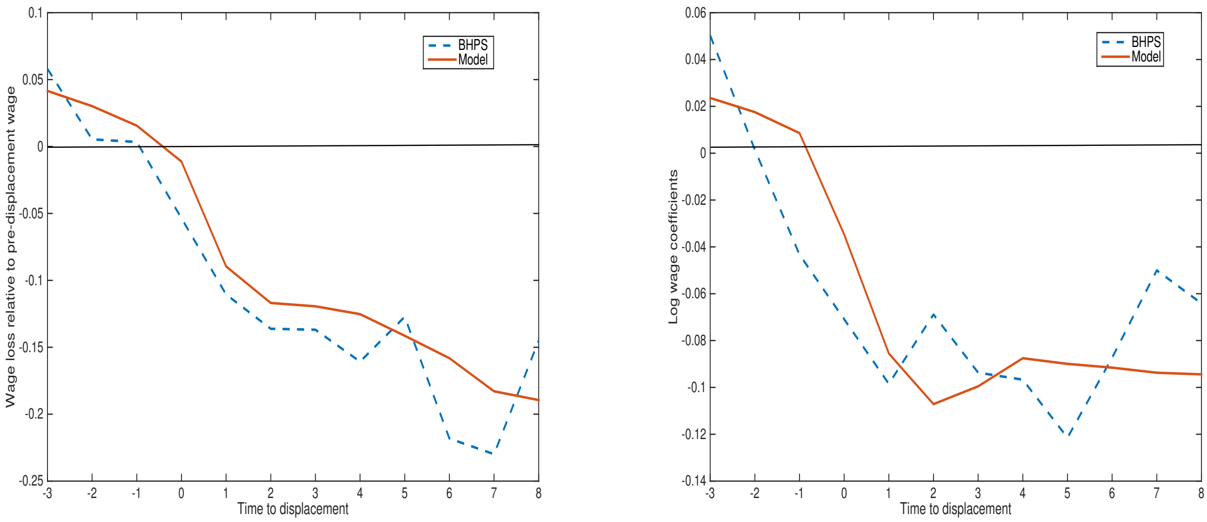
where w_{it}^y denotes real hourly wages, α_i^y individual fixed effect, γ_t^y year dummies, X_{it} quadratic on potential experience, D_{it} a set of dummies that take the value of one if $t - y = k$, where y denotes the separation year and T_s (T_e) the number of years before (after) separation, and μ_{it}^y denotes white noise. The ε_k^y measure the wage difference between those displaced during year y and the control group, where the latter is the set of workers that did not lose their jobs during the separation year. We estimate this equation jointly for those in the displaced and the control groups, using a fixed effects regression in our BHPS data. We consider a displacement window $y = \{1995, 1996, 1997\}$ and set $T_s = 3$ and $T_e = 8$.[16] To keep a sufficiently large sample size, we estimate (29), pooling together high and low skill workers. To be consistent with our theory, we consider as displaced workers those employed workers who transited into non-employment.[17] The observed transitions into non-employment are largely involuntary.[18] In Appendix B we

[16] Note, that for $y = 1997$, we have that $T_e = 7$.

[17] This is the same approached followed by Krolikowski (2014), Jung and Kuhn (2014) and Jarosh (2014) among others.

[18] We consider transitions into non-employment as transitions into unemployment, and into non-participation, excluding transitions due to maternity leave, retirement, further education or training, self-employment. From those who transited to the non-participation category we are left with those that transited due to health reasons or to care for a family member. Further, using the LFS we find that around 90% of those employed workers who transited into unemployment every quarter declared that they did not do so voluntarily, where we define a voluntary transition in the same way as we do in Appendix B. Note that our definition of who is a displaced worker differs from the one used in Jacobson et al. (1993), Couch and Placzek (2010) and others who use administrative data. In such papers a displaced worker is considered to be someone who lost his/her job due to a mass layoff. Jacobson et al. (1993) showed that these workers are the ones that face large and persistent earnings losses, while the earnings of those workers that went to non-employment due to other reasons recover quite quickly.

provide further details of the sample used, estimation results and robustness exercises.



(a) Wage Levels

(b) Log Wages

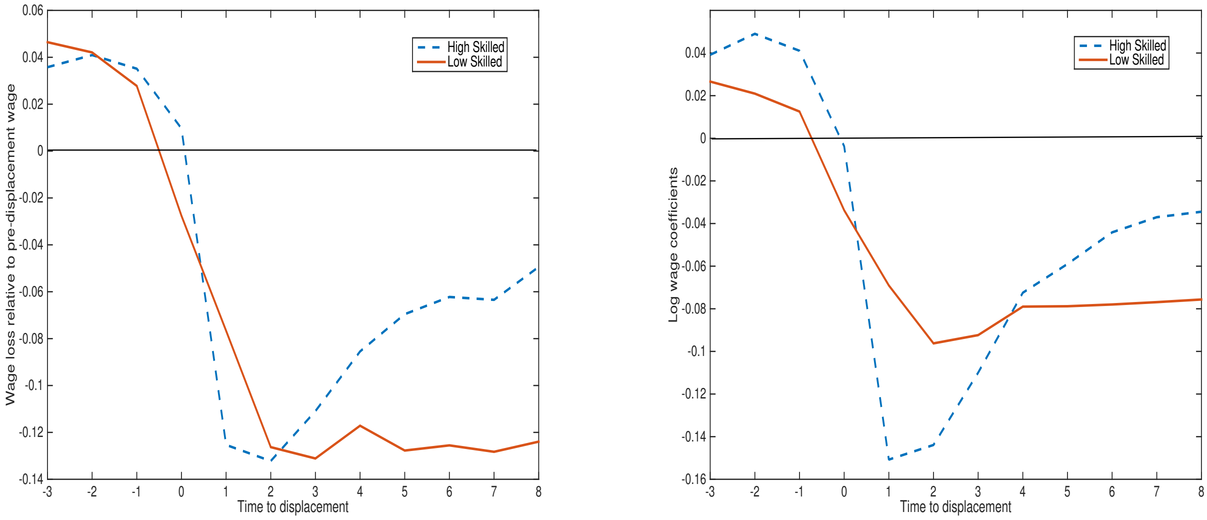
Figure 3: Post Displacement Wage Losses: Data and Model

Figure 3.a shows the estimated sequence of ε_k^y as a proportion of average pre-displacement wages (see also Jacobson et al., 1993). Figure 3.b presents the wage losses in a different way. Here we show the sequences of ε_k^y obtained from estimating (29) using log wages instead of level wages (see also Kletzer and Fairlie, 2003). The ε_k^y now measure the lost wage growth of displaced workers relative to the control group. We highlight three main features: (i) Young workers in the UK have large and persistent displacement wage losses. (ii) When measuring wage losses in relation to pre-displacement wages, the size of these losses increases over time and do not seem to recover. When measuring these losses in terms of growth rates we observe some recovery over time. (iii) Wages losses start occurring before the displacement event takes place.

Figure 3 also shows the corresponding estimated sequence of ε_k^y obtained from doing the same exercises as described above but now on simulated data based on our calibrated model, pooling the data sets for high and low skilled workers together.[19] The fit of the model is remarkable for both wage levels and logs, capturing the three main features of the data described above. The important insight from this exercise is that observed long-term wage displacement losses can be explained through a combination of differences in the turnover rates between different skilled workers, human capital accumulation/depreciation rates and job search effects. We now evaluate the importance of these components in generated the observed wage losses.

Jarosch (2014), however, using administrative data for Germany finds large and persistent wage losses for all those employed workers that transited into non-employment. We find a similar result.

[19]In the simulated data set we maintain the same proportions of high and low skilled workers as found in the pooled sample based on the BHPS.



(a) Wage Levels

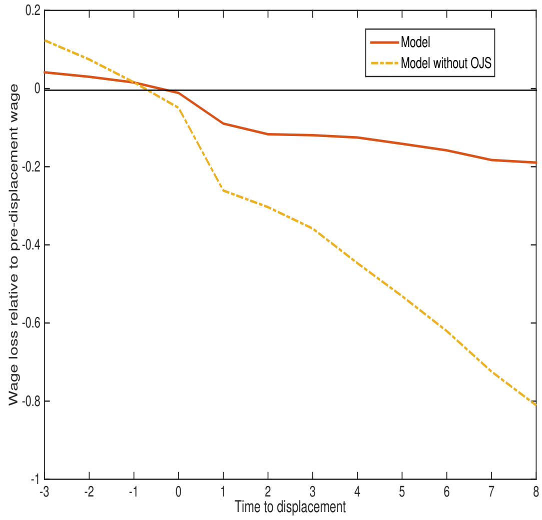
(b) Log Wages

Figure 4: Post Displacement Wage Losses: Skill Groups

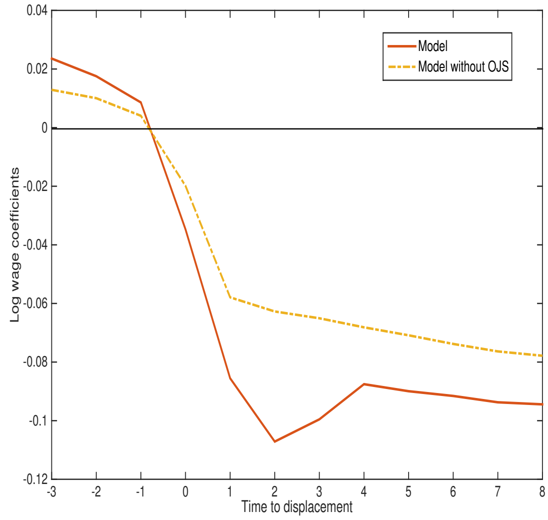
Differences in skills Figure 4 shows the predicted wage losses by skill groups. It shows that there are important differences in the wage loss patterns between high and low skilled workers. Both as a proportion of pre-displacement wages and as growth rates, the model predicts that high skilled workers have an initially large wage loss, but see this loss recover over the years. Low skill workers, however, do not seem to experience a recovery during the 8 year period we consider. This suggest that low skilled workers are the ones that lose the most from job displacement. This decomposition also suggests that the model's predicted wage losses observed in Figure 3 is to a large extent driven by the difference in the turnover patterns between skill groups. When pooling together both types of workers, the difference in the turnover patterns between the skill groups generates a composition effect, where the wage losses of the displaced group (relative to the control group) increases over time as low skilled workers subsequently experience more frequent and longer unemployment spells than high skilled workers.

The role of human capital accumulation In our model, workers' human capital accumulation when employed and depreciation when unemployed generate permanent wage losses for displaced workers. We now want to analyse to what extent the evolution of human capital explains these wage losses. To do so, we perform a counterfactual exercise, where we shut down on-the-job search such that human capital is the only source of wage loss. In particular, we first re-compute the equilibrium outcomes for each skill group under the restriction $\lambda_1 = \lambda_q = 0$. This restriction implies that all firms offer the same constant piece rate contract. We then generate new simulated data sets for each skill group and re-estimate the wage loss equation (29) both on levels and logs.

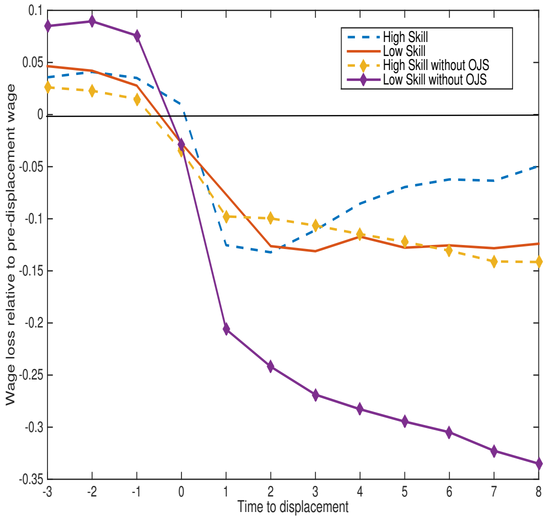
Figure 5 show the estimated parameters when pooling together both skill groups and com-



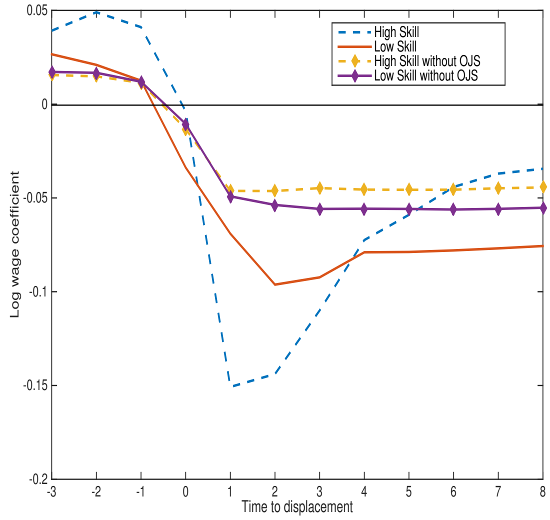
(a) All Workers: Wage Levels



(b) All Workers: Log Wages



(c) Skill Groups: Wage Levels



(d) Skill Groups: Log Wages

Figure 5: Post Displacement Wage Losses without on-the-job search effects

pares it to the results of the benchmark model. Figure 5.a shows that relative to pre-displacement wages, the model with only human capital yields much larger displacement losses. After 8 years, displaced workers would be earning a wage that is on average 80% (relative to 20% in the benchmark model) below their pre-displacement wages. Figure 5.b show that in terms of wage growth, the model with only human capital under-predicts the wage loss of workers. However, in this case the model is much closer to the data than it was when considering wage levels.

Figure 5.c and 5.d depict the results for low and high skilled workers separately. Overall they show a similar picture to the one obtained for the pooled sample of workers. There is, however,

some interesting difference across skill groups. In particular, Figure 5.c shows that the model with only human capital over-predicts the wage losses much more for low skilled than for high skilled workers. Given that low skilled workers do not tend to change jobs through employer to employer transitions, this result suggests that career effects within firms play an important role in reducing the size of the permanent loss induced by human capital. In the case of high skilled workers, although the model with only human capital generates a lower wage loss at the moment of displacement, it generates larger losses over time relative to the benchmark model.

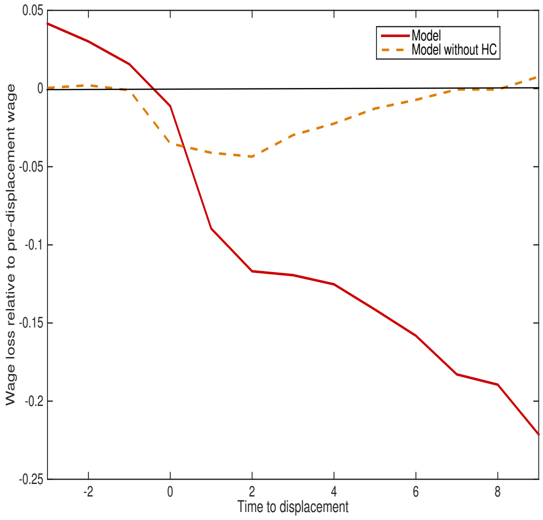
The role of on-the-job search We now turn to analyse the role of on-the-job search on its own. In our model, on-the-job search induces temporary wage losses as displaced workers can catch up over time through employer to employer transitions and within firm career effects. However, it is not clear how large are these temporary wage loss or how long does it take for them to disappear. To answer these questions we perform a similar counterfactual exercise as the one described above, but this time we restrict $\rho = \phi = 0$. This version of the model is essentially the same as Burdett and Coles (2003), but with offer arrival rates that depend on workers' employment status and with reallocation shocks.

Figure 6 shows the estimated parameters for this case. Figures 6.a and 6.b present the results for the pooled sample and compares it to the benchmark model. Figures 6.c and 6.d depict the results for low and high skilled workers separately. In all these cases we observe that the wage losses induced by the model with only on-the-job search are much smaller than those induced by the benchmark model. Note, however, that since high skilled workers rely more on employer to employer transitions to change jobs, their losses are larger than the wage losses of low skilled workers. Furthermore, our exercise shows that although on-the-job search effects on wage losses are temporary, it still takes a considerable amount of time for these effects to disappear. For both skill groups we find that these wage losses disappear 9 years after job displacement.

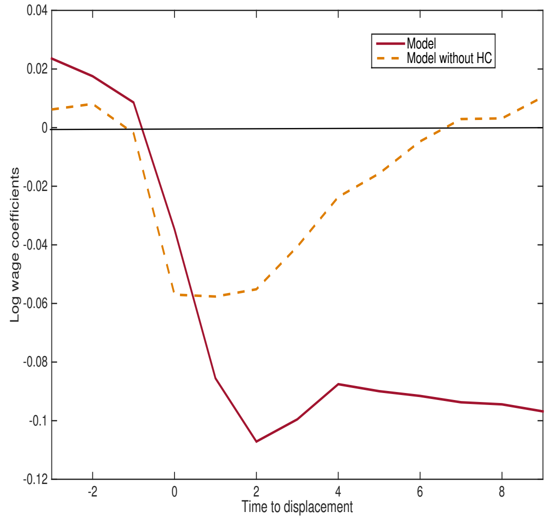
5 Further Discussion

In this paper we have proposed an equilibrium theory of the labour market in which risk averse workers accumulate human capital through learning-by-doing and engage in on-the-job search. Workers lose some of their human capital while searching unemployed. Firms optimally react to workers' search behaviour by posting long-term contracts in which wages increase smoothly with tenure. These contracts provide workers with some degree of consumption smoothing and at the same time reduce their quit probability. An important innovation of the paper is to show that our theory remains very tractable, while providing rich job turnover dynamics that allows us to study the cost of job loss. Further, our theory extents previous theoretical work that studies on-the-job search in a "timeless" equilibrium setting (see, for example, Postel-Vinay and Moscarini, 2014) by incorporating optimal contracts and human capital accumulation.

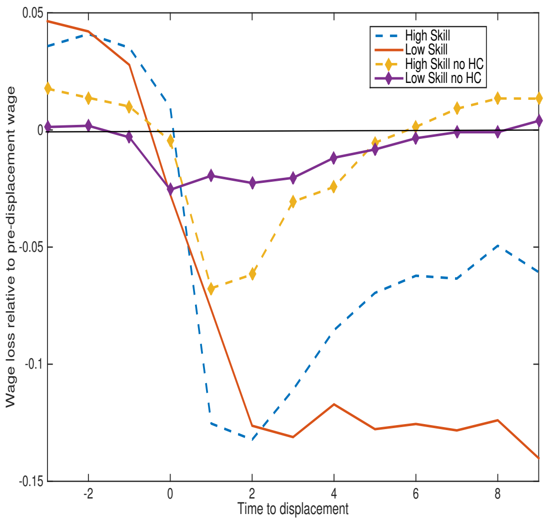
Our quantitative analysis shows that our model is able to reproduce the loss in hourly wages after a job separation observed for young workers in the UK. From the lenses of our model three ingredients are important to explain these wage losses: human capital accumula-



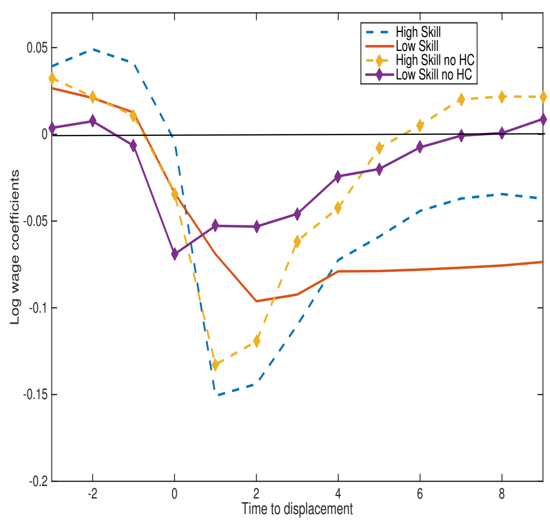
(a) All Workers: Wage Levels



(b) All Workers: Log Wages



(c) Skill Groups: Wage Levels



(d) Skill Groups: Log Wages

Figure 6: Post Displacement Wage Losses without human capital effects

tion/depreciation, within firm and across firm wage growth and differences in turnover patters across skill groups. We find that the strong permanent effects induced by the loss of human capital are tempered by workers' on-the-job and firms' optimal responses to worker turnover. Differences in worker turnover patterns across skill groups suggest that human capital losses play a more important part in explaining the extent and persistence of wage losses among low skilled workers. On-the-job search exerts an strong moderating force that allows the wages of high skilled workers to start recovering sooner.

Our model can be further used to understand the impact of long-run changes in the growth rate of aggregate output on the cost of job loss. In particular, Pessoa and Van Reenen (2014) document that since the onset of the Great Recession, the UK has experienced a sharp drop in the growth rate of aggregate output and of labour productivity. In our model changes in the growth rate of an economy affect returns to job search through the dependence of the baseline salary scale on γ_A. This implies that the temporary effects of on-the-job search on wage losses could be significantly affected. To quantify the latter we set $\rho = \phi = 0$ in the model and perform the same exercise as described in Section 4. This time, however, using a value of γ_A that is 20% lower than the one used in the original exercise.[20] We find that with a lower γ_A, the wages of displaced workers are still below their pre-displacement levels 10 years after the separation episode and this occurs for both high and low skilled workers. That is, our model predicts that decreases in the long-run growth rate of aggregate output will lead to even larger wage losses for displaced workers.

[20]Such a drop in γ_A reflects the observed reduction in the growth rate of output per worker during the 2009-2012 period relative to the 1991-2004 period, as considered in the original exercise.

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Appendix

A Proofs

Proof of Theorem 1: Substituting out ψ in the objective functions gives the dynamic optimisation problem:

$$\max_{\theta(.)} \int_0^{\infty} e^{-\int_0^{\tau}\{r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U(s|\theta))]\}ds}[1-\theta(\tau)]d\tau,$$

subject to starting value $U(0|\theta) = U_0$ where (2) describes how $U(.)$ evolves with tenure. Define transformed variable

$$\psi_0 = e^{-\int_0^{\tau}\{r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U(s|\theta))]\}ds}$$

and note it satisfies the differential equation

$$\dot{\psi}_0 = -\left[r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U)]\right]\psi_0. \tag{30}$$

The dynamic optimisation problem is equivalently rewritten as

$$\max_{\theta(.)} \int_0^{\infty} \psi_0[1-\theta]d\tau, \tag{31}$$

where ψ_0, U are state variables which evolve according to the autonomous, first order differential equations (30) and (2) respectively with initial values $\psi_0 = 1$, $U = U_0$ at $\tau = 0$. We can solve this dynamic optimisation problem using the Hamiltonian approach. Define

$$\begin{aligned} H &= \psi_0[1-\theta] - \xi_{\psi_0}\left[r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U)]\right]\psi_0 \\ &\quad +\xi_U \left[\begin{array}{c} [r+\delta+\lambda_q-[\rho+\gamma_A](1-\sigma)]\,U \\ -\left[\frac{\theta^{1-\sigma}}{1-\sigma}+\delta\phi^{1-\sigma}U^U+\lambda_1\int_U^{\overline{U}}[1-F(U_0)]dU_0+\lambda_q\int_{\underline{U}}^{\overline{U}}U_0dF(U_0)\right] \end{array}\right] \end{aligned}$$

where ξ_{ψ_0}, ξ_U are the respective costate variables. The Maximum principle yields the following necessary conditions for optimality:

$$\theta^{-\sigma} = -\frac{\psi_0}{\xi_U} \tag{32}$$

$$\dot{\xi}_{\psi_0} = -[1-\theta] + \xi_{\psi_0}\left[r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U)]\right] \tag{33}$$

$$\dot{\xi}_U = -\left[\xi_{\psi_0}\lambda_1F'(U)\psi_0 + \xi_U\left[[r+\delta+\lambda_q-[\rho+\gamma_A](1-\sigma)]+\lambda_1[1-F(U)]\right]\right] \tag{34}$$

along with autonomous differential equations (30), (2) for $\dot{\psi}_0$ and $\dot{U}$. As we do not wish to assume F is differentiable, however, we drop condition (34) and instead note that as the objective function in (31) does not depend explicitly on tenure, optimality also implies

$$H = 0 \tag{35}$$

(e.g. p298, Leonard and Long, 1992). Now integrating (33) forward yields:

$$\begin{aligned}
\xi_{\psi_0}(t) &= \int_t^\infty e^{-\int_t^s [r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U(\tau))]d\tau}(1-\theta(s))ds + B_0 e^{\int_0^t [r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1(1-F(U(x)))]dx} \\
&= \Pi(t) + B_0 e^{\int_0^t [r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1(1-F(U(x)))]dx}
\end{aligned}$$

where B_0 is the constant of integration and $\Pi(.)$ is the firm's continuation profit as defined in Theorem 1. (32) implies $\xi_U = -\psi_0\theta^\sigma$. Substituting out ξ_U and ξ_{ψ_0} in the definition of H, the restriction $H = 0$ yields the optimality condition:

$$\begin{aligned}
0 &= [1-\theta] - [r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F(U)]]\left[\Pi(t) + B_0 e^{\int_0^t [r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1(1-F(U(x)))]dx}\right] \qquad (36) \\
&\quad -\theta^\sigma\left[[r+\delta+\lambda_q-[\rho+\gamma_A](1-\sigma)]\,U - \frac{\theta^{1-\sigma}}{1-\sigma} - \delta\phi^{1-\sigma}U^U - \lambda_1\int_U^{\overline{U}}[1-F(U_0)]dU_0 - \lambda_q\int_{\underline{U}}^{\overline{U}} U_0 F(U_0\right]
\end{aligned}$$

Now the restriction $r+\delta-\rho-\gamma_A > 0$ ensures the exponential term becomes arbitrarily large as $\tau \to \infty$. As Π and U must be bounded, then (36) implies $B_0 = 0$. (36) now yields (6) given in the Theorem. Using this to substitute out $\frac{\theta^{1-\sigma}}{1-\sigma}$ in (2) then yields (8). This completes the proof of Theorem 1.

Proof of Claim 3: Equation (16) follows by solving the constant profit condition. To do so, note that standard turnover arguments imply G satisfies

$$\begin{aligned}
&u\lambda_0[1-F(U)] + (1-u)G(U)[\lambda_q+\lambda_1][1-F(U)] + (1-u)G'(U)\dot{U} \\
= \;&(1-u)(1-G(U))[\delta+\lambda_q F(U)],
\end{aligned}$$

where the left hand side describes the flow of workers into employment with piece rate value more than U while the right hand side describes the flow out through job separation. As (8) and (10) together imply

$$\dot{U} = \widehat{\theta}^{-\sigma}\left\{1-\theta-[r+\delta+\lambda_q-\rho-\gamma_A+\lambda_1[1-F]]\Pi\right\},$$

rearranging the previous expression yields

$$\frac{dG}{dU} = \frac{(1-u)\delta(1-G(U)) - u\lambda_0[1-F(U)] - (1-u)G(U)\lambda_1[1-F(U)]}{(1-u)\theta^{-\sigma}[1-\theta-[r+\delta-\rho-\gamma_A+\lambda_1(1-F(U))]\Pi]}, \qquad (37)$$

where $\Pi = \Pi_i(t_0)$ and $\theta = \theta_i(t_0)$.

While $dF(U) > 0$, differentiating the constant profit condition implies:

$$[\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G(U)]\,\widehat{\Pi}'(U) + \lambda_1(1-u)G'(U)\widehat{\Pi}(U) = 0.$$

As (8) implies

$$\frac{d\widehat{U}}{d\theta} = -\theta^{-\sigma}\frac{d\widehat{\Pi}}{d\theta}, \qquad (38)$$

and using (37) to substitute out $\widehat{\Pi}'(U)$ and $G'(U)$ gives

$$\begin{aligned} & [\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G]\left\{1 - \widehat{\theta} - [r + \delta + \lambda_q - \rho - \gamma_A + \lambda_1[1-F]]\widehat{\Pi}\right\} \\ = \; & \lambda_1(1-u)\widehat{\Pi}\left[\delta(1-G) - \frac{u}{1-u}\lambda_0[1-F] - G\lambda_1[1-F] + \lambda_q[F-G]\right]. \end{aligned}$$

Inspection finds the F-terms all cancel out and so:

$$\begin{aligned} & [\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)G]\left\{1 - \widehat{\theta} - [r + \delta + \lambda_q - \rho - \gamma_A + \lambda_1]\widehat{\Pi}\right\} \\ = \; & \lambda_1(1-u)\widehat{\Pi}\left[\delta(1-G) - \frac{u}{1-u}\lambda_0 - G\lambda_1 - G\lambda_q\right]. \end{aligned}$$

But the constant profit condition also implies

$$G(U) = \frac{[\lambda_0 u + \lambda_q(1-u) + \lambda_1(1-u)]\frac{\overline{\Omega}}{\widehat{\Pi}} - [\lambda_0 u + \lambda_q(1-u)]}{\lambda_1(1-u)}.$$

Using this to substitute out G, substituting out $u = \delta/(\delta + \lambda_0)$ finds the previous expression, after a lot of simplification gives the quadratic equation

$$\widehat{\Pi}^2[\delta + \lambda_q] + [r - \rho - \gamma_A]\overline{\Omega}\widehat{\Pi} - (1-\theta)\overline{\Omega} = 0. \tag{39}$$

As $dF(U) > 0$ implies the firm must make positive profit at this offer point, the solution to this quadratic completes the proof of Claim 3.

Proof of Theorem 2: (17) and (20) imply

$$\frac{d\widehat{U}}{d\theta} = \frac{\theta^{-\sigma}}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q][[\delta+\lambda_q+r-\rho-\gamma_A]]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}, \tag{40}$$

whose solution is given by (21). Given $\widehat{\Pi}(\theta)$, the constant profit condition (**??**) implies (22).

To determine the equilibrium distribution of offers F_θ, standard turnover arguments imply G_θ must satisfy

$$u\lambda_0[1-F_\theta(\theta)] + (1-u)G_\theta(\theta)[\lambda_1+\lambda_q][1-F_\theta(\theta)] + (1-u)G'_\theta(\theta)\dot{\theta}(\theta) = (1-u)(1-G_\theta(\theta))\left[\delta + \lambda_q F_\theta(\theta)\right],$$

where the left hand side describes the flow of workers into employment with piece rate more than θ while the right hand side describes the flow out through job separation. Now (9), (17) and $F'_\theta(\theta) = F'(\widehat{U})d\widehat{U}/d\theta$ together imply

$$\dot{\theta} = \frac{\lambda_1 F'_\theta}{\sigma}\left[-\frac{\theta\widehat{\Pi}}{\frac{d\widehat{\Pi}}{d\theta}}\right] - (\rho + \gamma_A)\theta.$$

Using this solution for $\dot{\theta}$ and $G_\theta, \widehat{\Pi}$ described in the Theorem, a lot of algebra finds the turnover equation for G implies the following first order differential equation for F:

$$\frac{\theta F_\theta'}{\sigma} + (1 - F_\theta) = \Psi(\theta),$$

where

$$\Psi(\theta) = \frac{\lambda_q + \delta}{\lambda_1}\left[\frac{\widehat{\Pi}(\theta) - \overline{\Omega}}{\overline{\Omega}}\right] - \left[\frac{(\rho + \gamma_A)}{\lambda_1}\frac{\theta d\widehat{\Pi}/d\theta}{\widehat{\Pi}(\theta)}\right] > 0.$$

Integration now yields the stated solution for F_θ while using (16) it is easy to show that $\Psi(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. This completes the proof of Theorem 2.

Proof of Claim 6: Integration by parts finds

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} \widehat{U}(\theta)dF_\theta(\theta) &= \overline{U} - \int_{\underline{\theta}}^{\overline{\theta}} \frac{d\widehat{U}}{d\theta}F_\theta(\theta)d\theta \\ &= \overline{U} - \int_{\underline{\theta}}^{\overline{\theta}} \frac{[\theta]^{-\sigma}F_\theta(\theta)d\theta}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}. \end{aligned} \tag{41}$$

Putting $\theta = \underline{\theta}$ in (21) implies

$$\overline{U} = \underline{U} + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^{-\sigma}d\theta}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}. \tag{42}$$

Putting $\theta = \overline{\theta}$ in (6), noting $U^U = \underline{U}$ in a timeless equilibrium (Claim 5) implies:

$$\frac{\overline{\theta}^{1-\sigma}}{1-\sigma} = [r + \delta + \lambda_q - [\rho + \gamma_A](1-\sigma)]\,\overline{U} - \delta\underline{U} - \lambda_q\int_{\underline{\theta}}^{\overline{\theta}} \widehat{U}(\theta)dF_\theta(\theta). \tag{43}$$

Using (41) and (42) to substitute out $\overline{U}$ implies:

$$[r - [\rho+\gamma_A](1-\sigma)]\,\underline{U} = \frac{\overline{\theta}^{1-\sigma}}{1-\sigma} - \int_{\underline{\theta}}^{\overline{\theta}} \frac{[r+\delta-[\rho+\gamma_A](1-\sigma)+\lambda_q F_\theta(\theta)]\,\theta^{-\sigma}d\theta}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}. \tag{44}$$

Equation (3) with $U^U = \underline{U}$ [Claim 5], (41) and substituting out $\overline{U}$ using (42) implies

$$(r+\phi(1-\sigma)-\gamma_A(1-\sigma))U^U = \frac{b^{1-\sigma}}{1-\sigma} + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^{-\sigma}\lambda_0[1-F_\theta(\theta)]}{\left[[r-\rho-\gamma_A]^2 + 4[\delta+\lambda_q]\left[\delta+\lambda_q+r-\rho-\gamma_A\right]\frac{1-\theta}{1-\overline{\theta}}\right]^{1/2}}d\theta. \tag{45}$$

As a timeless equilibrium requires $\underline{U} = U^U$, we obtain the equilibrium condition stated with $F = \widetilde{F}$. This completes the proof of Claim 6.

Proof of Theorem 3: Note that as $\overline{\theta} \to 1$, (24) implies $\underline{\theta} \to 1$ and so all piece rates paid lie in a neighbourhood of 1. Frictions $\lambda_0 < \infty$, $b < 1$ and $\phi \geq 0$ ensure the value of being unemployed $\widetilde{U}^U(\overline{\theta}) < \underline{\widetilde{U}}(\overline{\theta})$ in this limit.

Suppose instead $\overline{\theta} \to 1 - \frac{\delta[\delta+r-\rho-\gamma_A]}{(\delta+\lambda_1)[\delta+\lambda_1+r-\rho-\gamma_A]}$ and so $\underline{\theta} \to 0^+$. As

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{[r+\delta-(\rho+\gamma_A)(1-\sigma)][\theta']^{-\sigma}}{\left[[r-\rho-\gamma_A]^2+4\delta\left[\delta+r-\rho-\gamma_A\right]\frac{1-\theta'}{1-\overline{\theta}}\right]^{1/2}} d\theta'$$

$$> \int_{\underline{\theta}}^{\overline{\theta}} \frac{[r+\delta-(\rho+\gamma_A)(1-\sigma)][\theta']^{-\sigma}}{\left[[r-\rho-\gamma_A]^2+4\delta\left[\delta+r-\rho-\gamma_A\right]\frac{1-\underline{\theta}}{1-\overline{\theta}}\right]^{1/2}} d\theta'$$

$$= \frac{[r+\delta-(\rho+\gamma_A)(1-\sigma)]}{[[r-\rho-\gamma_A]^2+4(\delta+\lambda_1)\left[\delta+\lambda_1+r-\rho-\gamma_A\right]]^{1/2}}\left[\frac{\overline{\theta}^{1-\sigma}}{1-\sigma}-\frac{\underline{\theta}^{1-\sigma}}{1-\sigma}\right]$$

then $\underline{\theta} \to 0^+$ and (44) implies $\widetilde{\underline{U}}(\overline{\theta}) \to -\infty$ in this limit. As (45) implies $\widetilde{U}^U \geq \frac{b^{1-\sigma}}{1-\sigma}/(r+\phi(1-\sigma)-\gamma_A(1-\sigma))$ and so is finite, then $\widetilde{U}^U(\overline{\theta}) > \widetilde{\underline{U}}(\overline{\theta})$ in this limit. As these are continuous functions for $\overline{\theta}$ satisfying (25), a $\overline{\theta}$ satisfying (25) exists such that $\widetilde{\underline{U}}(\overline{\theta}) = \widetilde{U}^U(\overline{\theta})$. This completes the proof of Theorem 3.

B Quantitative Analysis

B.1 Simulation

To recover the parameters of the model we solve $min_\Omega \sum_{i=1}^{14}\left[\frac{M_i^S-M_i^D}{M_i^D}\right]^2$, where M_i^S denotes the i^{th} moment obtained from the simulations and M_i^D the corresponding data moment. To obtain the empirical moments, M_i^D, we use data drawn from the LFS and the BHPS as mentioned in the main text. To obtain the simulated moments at each iteration, we first compute the equilibrium of our model, then simulate workers' employment histories, and then compute each M_i^S from this data.

For a set of parameter values, computing the equilibrium implies picking a $\overline{\theta}$ satisfying (25), then using Theorem 2 to compute F_θ over $[\underline{\theta}, \overline{\theta}]$ with $\underline{\theta}$ given by (24). Then computing $\widetilde{\underline{U}}(\overline{\theta}), \widetilde{U}^U(\overline{\theta})$ as defined in Claim 6. The equilibrium value of $\overline{\theta}$ is then determined by $\widetilde{\underline{U}}(\overline{\theta}) = \widetilde{U}^U(\overline{\theta})$. Using the corresponding value of $\underline{\theta}$ we then solve the differential describing the evolution of θ to obtain the baseline salary scale.

Given these equilibrium outcomes, we simulate the employment histories of 10,000 workers. We assume that all workers start unemployed and experience different types of shocks during their lifetime depending on the worker's employment status. In particular, every time a worker is unemployed, he receives a job offer at rate λ_u. We obtain his unemployment duration by drawing a random number, $r1 \in [0,1]$ and then exploiting the fact that the inter-arrival time between events in a Poisson process follows an exponential distribution with parameter equal to the rate of the process. That is, the duration until the worker receives a job offer is determined by $tu = -log(1-r1)/\lambda_u$. After deriving tu, we sample a position in the baseline salary scale from the offer distribution F, by choosing a random number between 0 and 1 and interpolating between the sample value of F and the corresponding value of θ.

Now that the worker is employed, he faces three shocks: a reallocation shock, a job offer shock and a displacement shock. All these shocks follow Poisson process with rates, λ_q, λ_e and

δ, respectively. What is important here is the duration of the job and the employment spells, where the latter is defined as the sum of job spells that start with the worker transiting from unemployment to employment and end with the worker becoming unemployed. To obtain these durations we need to compute the durations until the worker receives a job offer tj, receives a displacement shock, tu, or receives a reallocation shock, tr. We do this by drawing three random number between 0 and 1 and using the inverse of the corresponding exponential distribution. The job duration until the worker experiences one of these three events in then $\min\{tj, tu, tr\}$. If the worker becomes unemployed, $tu = min\{tj, tu, tr\}$, we repeat the corresponding procedure. If the worker receives an outside offer, $tj = min\{tj, tu, tr\}$, we draw a new position in the baseline salary scale. If the current position is greater than the one drawn, the worker stays employed in his current job. Otherwise, we move the worker to the new position and compute a new set $\{tj, tu, tr\}$. If the worker receives a reallocation shock, $tr = min\{tj, tu, tr\}$, we obtain a new position in the baseline salary scale, move the worker to the new position and compute a new set $\{tj, tu, tr\}$. When we compute these events, we also compute workers' labour market experience defined as the sum of employment spells. This information, together with the length of the worker's unemployment spells, can then be used to compute wages at each point in which an event has occurred taking into account that human capital accumulation occurs at rate ρ and human capital depreciation occurs at rate ϕ.

We follow workers for 40 years to guarantee that we converge to the ergodic distributions for each M_i^S. To compute the transition moments, we use average durations, except for the average Invol/vol EE transitions, for which we compute the average number of involuntary and voluntary transitions. To compute the returns to experience and tenure, we construct a yearly panel resembling the BHPS structure and regress log wages on a constant, a quadratic on experience and a quadratic on tenure. To compute the Mm ratio we use this panel and follow the procedure described in Burdett et al. (2014). We also use this panel to compute the ratio between the average re-employment wage to average wage.

B.2 Data Moments

To compute the returns to experience and tenure we estimate workers' wage-experience profile using the BHPS. In particular, we obtain these returns from an OLS regression of log wages on a quadratic on experience, a quadratic on tenure, a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies, dummies for cohort effects and a time trend. To compute the Mm ratio we also use the BHPS and follow the procedure described in Hornstein et al. (2009) and described in Burdett et al. (2014). We also use the BHPS to compute the ratio between the average re-employment wage to average wage.

To compute the transition moments we use the 2-quarter sample of the U.K. LFS. The non-employment to employment transition rate is computed as the ratio between the flow of those non-employed workers in a given quarter q gaining employment the following quarter and the stock of non-employed workers in quarter q. The employment to non-employment transition rate is computed as the ratio between the flow of those employed workers in a given quarter q that lost their jobs the following quarter and the stock of employed workers in quarter q. The inverse of these rate gives the average duration of a non-employment and of an employment spell. We also use the LFS to obtain the average duration of a job. In this case, we use directly the question asked to employed workers about the length of their current job spell. To construct the ratio of involuntary to voluntary employer to employer transitions, we use the reported reasons of why workers left their current job and construct three groups. The *voluntary* group consists of those employed workers who changed jobs because they"resigned", went to "education or training" and "gave up for family or personal reasons". The *involuntary* group consists of those employed workers who left their last job because they were "dismissed", "made redundant/took voluntary redundancy", "temporary job finished" and "gave up work for health reasons". The *other* group consists of those employed worker who left their last job because they "took early retirement", "retire" and due to "other reasons. We then use the first two for our statistic.

B.3 Empirical Wage Losses

Our analysis of wage losses focus on a sample of workers that in 1991 were between 16 and 30 years old. From this sample we dropped all non-white workers, workers with employment spells in the government, with spells in training or full-time education and with spells in self-employment. We also drop reported spells of employment and non-employment that were shorter than a month. This selection left us with 1,911 individuals and 21,875 individual/spell observations. Using this sample we estimate (29) using fixed effects, which controls for differences in gender, skills and other fixed observable and unobservable characteristics.

Tables 3 and 4 show the full regression estimates, based on the set of displacement years $y = \{1995, 1996, 1997\}$. This set of years allows us to control for the 3 years previous the displacement event and go forward at least 7 years after displacement. We also considered the set of displacement years $y = \{1994, 1995, 1996, 1997\}$ without much variation in the results. In addition we estimate equation (29) controlling for 2-digit industry dummies, without minor effects on the estimated values of ε_k^y. Following Davis and von Wachter (2011), we also run separate regressions for different value of y. In particular, we considered $y = \{94/95, 95/96, 96/97\}$ and then average out the resulting estimated ε_k^y. This approach gives similar estimates when estimating (29) using wage levels, but does give statistically significant coefficients when using log wages.

Table 3: Wage losses using wage levels, BHPS: 1991-2004

real hourly wage	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
cumexp	0.010522	0.005054	2.08	0.037	0.000615	0.020428
cumexp2	-5.7E-05	6.65E-06	-8.54	0	-7E-05	-4.4E-05
yr15	-8.11205	0.647501	-12.53	0	-9.38124	-6.84286
yr16	-7.77285	0.600108	-12.95	0	-8.94914	-6.59656
yr17	-7.32525	0.55136	-13.29	0	-8.40599	-6.24451
yr18	-6.74079	0.501928	-13.43	0	-7.72464	-5.75695
yr19	-6.19072	0.453639	-13.65	0	-7.07991	-5.30152
yr20	-5.55935	0.404837	-13.73	0	-6.35288	-4.76581
yr21	-4.7106	0.356797	-13.2	0	-5.40997	-4.01123
yr22	-4.0699	0.309639	-13.14	0	-4.67684	-3.46297
yr23	-3.39544	0.265413	-12.79	0	-3.91568	-2.8752
yr24	-2.61977	0.222249	-11.79	0	-3.0554	-2.18413
yr25	-1.93181	0.186702	-10.35	0	-2.29777	-1.56585
yr26	-1.08056	0.164163	-6.58	0	-1.40234	-0.75878
disp_m3	0.434164	0.230987	1.88	0.06	-0.0186	0.886928
disp_m2	0.03999	0.218906	0.18	0.855	-0.38909	0.469074
disp_m1	0.025382	0.219715	0.12	0.908	-0.40529	0.456053
disp_m0	0	(omitted)				
disp_p1	-0.82714	0.198713	-4.16	0	-1.21664	-0.43764
disp_p2	-1.0192	0.222463	-4.58	0	-1.45526	-0.58315
disp_p3	-1.02452	0.233056	-4.4	0	-1.48134	-0.5677
disp_p4	-1.20289	0.24843	-4.84	0	-1.68984	-0.71593
disp_p5	-0.95209	0.273181	-3.49	0	-1.48757	-0.41662
disp_p6	-1.63288	0.28385	-5.75	0	-2.18926	-1.07649
disp_p7	-1.72019	0.340445	-5.05	0	-2.38751	-1.05288
disp_p8	-1.08533	0.486755	-2.23	0.026	-2.03943	-0.13122
_cons	10.8558	0.922816	11.76	0	9.046957	12.66464

Table 4: Wage losses using log wages, BHPS: 1991-2004

log real wage	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
cumexp	0.001991	0.00063	3.16	0.002	0.000757	0.003226
cumexp2	-1.2E-05	8.12E-07	-14.77	0	-1.4E-05	-1E-05
yr15	-1.16925	0.081308	-14.38	0	-1.32863	-1.00988
yr16	-1.10051	0.075199	-14.63	0	-1.24791	-0.9531
yr17	-0.99079	0.068954	-14.37	0	-1.12595	-0.85563
yr18	-0.87122	0.062702	-13.89	0	-0.99412	-0.74831
yr19	-0.76528	0.056519	-13.54	0	-0.87606	-0.65449
yr20	-0.67212	0.050367	-13.34	0	-0.77084	-0.57339
yr21	-0.53291	0.044253	-12.04	0	-0.61965	-0.44616
yr22	-0.4341	0.038282	-11.34	0	-0.50914	-0.35906
yr23	-0.3484	0.032721	-10.65	0	-0.41253	-0.28426
yr24	-0.2444	0.027351	-8.94	0	-0.29801	-0.19079
yr25	-0.18189	0.022925	-7.93	0	-0.22682	-0.13695
yr26	-0.10102	0.020134	-5.02	0	-0.14048	-0.06155
disp_m3	0.050291	0.027268	1.84	0.065	-0.00316	0.10374
disp_m2	0.001471	0.025517	0.06	0.954	-0.04855	0.051488
disp_m1	-0.04331	0.025763	-1.68	0.093	-0.09381	0.007185
disp_m0	0	(omitted)				
disp_p1	-0.09848	0.023674	-4.16	0	-0.14488	-0.05208
disp_p2	-0.06885	0.026327	-2.62	0.009	-0.12045	-0.01724
disp_p3	-0.09368	0.027362	-3.42	0.001	-0.14732	-0.04005
disp_p4	-0.09669	0.02947	-3.28	0.001	-0.15446	-0.03893
disp_p5	-0.12147	0.03272	-3.71	0	-0.1856	-0.05733
disp_p6	-0.08732	0.034499	-2.53	0.011	-0.15494	-0.01969
disp_p7	-0.05	0.040098	-1.25	0.212	-0.1286	0.028597
disp_p8	-0.06412	0.057746	-1.11	0.267	-0.17731	0.04907
_cons	2.125103	0.115312	18.43	0	1.899074	2.351133