

Buyer Power and Price Discrimination: The Case of the UK Care Homes Market

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NON TECHNICAL SUMMARY

Background

In the UK, over 400,000 people aged 65 and over receive long-term care in a care home, and this is projected to more than double over the next 50 years. Local authorities purchase care home places on behalf of a large group of people based on a means test of their income and capital assets. People excluded by the means test are self-funding. The care home market consists of a large number of relatively small private or not-for-profit providers. Local authorities, using their buyer power, may be able to procure assisted places from these providers at a price below the market rate. Low prices paid to care homes could force care homes out of the market resulting in a shortfall in capacity; and care homes might have to charge higher fees to self-funders to subsidise publicly-funded residents. There may therefore be induced 'price discrimination' in the care home market.

Methods

We provide a theoretical model of the care home market to establish the key effects of potential buyer power. A microsimulation model is used to quantify key findings from the theoretical model.

Key findings

Under the theoretical model, if the local authority negotiates a price which is below average costs then, to break even and cover costs, the care home must charge all other users a higher price. The larger is the discount negotiated by the local authority, or the larger is the number of places procured by the local authority, the higher must be the private sector price for the care home to stay in business. People who are no longer willing or able to afford a care home place in the private market but remain ineligible for local authority support lose as a result. This is the 'squeezed middle' who can afford the market (no price discrimination) price but not the higher private rate that care homes charge when local authorities exercise their buyer power. Other self-funders have to pay an inflated price to keep the care homes financially viable so also lose.

Simulation reveals that the first round (unchanged demand) effects of modest price discrimination, local authority costs are about 8% lower than in the absence of price discrimination, costs met by local authority-funded residents are lower by 3% but self-funders pay 11% more. At these fee rates, the squeezed middle amounts to just 0.01% of the total. If local authorities depress the price considerably more, the consequences are greater. The size of the squeezed middle is larger but still a small proportion of the total. The scale of potential demand responses depends on the size of the squeezed middle since this is the group who would withdraw their demand.

Policy issues

One way to address concerns over those priced out of the market is for the local authority to include the squeezed middle by procuring places at the lower fee rate for all those who cannot afford the private fee rate. Including the squeezed middle does not cost local authorities anything directly because under the means test the squeezed middle are liable for the full fee rate negotiated by the local authority. But if the squeezed middle is large, such action may destabilise the market.

Various reforms to the means test remain under debate. Our analysis suggests that the size of the squeezed middle is kept small by the current means test. Reforms to the means test may therefore reduce local authorities' ability to exercise their buyer power meaning that they would have to pay fees which are closer to market rates, so increasing the cost of the reforms. Exercising their buyer power enables local authorities to spend less on care home places (and so more on other things) or buy more care home places. The cost of this is borne by self-funding care home residents. Whether they are the right group to pay for this is debateable.

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Abstract

UK Local Authorities purchase care home places on behalf of a large group of people following a means test of their income and wealth. All other buyers of care home services are atomistic. The care home market is characterised by a large number of small providers. Local authorities may thus have buyer power. We show that any abuse of this buyer power may lead to some people, “the squeezed middle” not being served. We quantify the size of the squeezed middle and assess the implications of the form of the means test for local authorities' ability to exercise buyer power.

Keywords: buyer power, care home market, long term care financing

JEL classification: I11, I19, I31

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1 Introduction

The Office of Fair Trading investigation of the market for Care Homes¹ following a super-complaint by *Which?* in 2003, revealed, almost as an afterthought since this was not part of the original complaint, that there appeared to be an element of induced price discrimination in this market.

“Throughout this study and the OFT's initial response to the super-complaint, the issue of the level of government funding has been raised by a range of stakeholders. In particular, people have told us that the fees paid by Authorities to care homes for older people do not cover the full costs to the care home of providing care, plus a reasonable profit margin. Their concerns about the consequences for older people of low levels of funding are that:

- *excessively low prices paid to care homes may force care homes out of the market and lead to a shortfall in capacity in some areas, and*
- *care homes may be charging higher fees to self funders in order to cross subsidise publicly funded residents.”*

OFT780, paragraph 1.50.

The discrimination allegedly arose because the public sector, using its buyer power, could procure assisted places at a price below the private rates. The primary aim of the paper is to provide a simple model of this market to establish the key effects of such potential buyer power. By stripping the problem down to its bare bones it is possible to identify more clearly the mechanisms which give rise to the problems indicated in the OFT report and summarised in the two bullet points in the quote above.

We model the care home sector as essentially competitive² with relatively free entry, where each firm is characterised by important capacity constraints. Given that the care homes are constrained in the number of places they can offer by the physical space available, this seems a sensible starting point. Modelling the demand side is more challenging, especially the

¹ OFT780: Care homes for older people in the UK: a market study, May 2005

² While some characterise the market as fitting the model of imperfect competition (comprising a large number of small businesses and a smaller number of larger providers), see Gage et al. (2009), the observation and results in Machin and Wilson (2004) suggest otherwise. They argue that the sector “consists of large numbers of small firms ... doing a very homogeneous activity in geographically concentrated markets”. Moreover they find that the main result on the increase in the minimum wage is an exit of firms rather than an ability of firms to internalise the cost increase through reduced profits or by scaling down their activities. It is hard to reconcile the results in Machin and Wilson with an oligopolistic model. Netten et al. (2005) also report on closures and identifies a combination of cost increases and inability of local authority pricing to cover costs as the key causes of care home closure, again suggesting that there is no supra-normal profit arising from oligopoly power from which such cost increases or relative price decreases can be funded.

derived demand of the local authority (LA). One possible minimum aim of the LA is to ensure that all who need a care home place but are not financially able to procure such a place should be given public assistance. That would imply a largely needs based test for eligibility and hence for the number of places the local authority needs to procure. The needs base could be a mixture of health needs and financial needs or simply based on financial need. One of the key effects of an LA using its buyer power is that there will be a section of the population who is not eligible for LA support but who, once care homes respond to the reduced income from LA places, can no longer afford a care home place. The representation which makes this argument in the starkest term is one in which the over-riding criteria for public assistance is income. We use this in our analysis on the basis that people who do not have a health need for a care home place are unlikely to want one even at zero price so will not seek state assistance with the cost.

The analysis identifies the problem with a section of individuals with care home needs being squeezed out of the market solely as a result of the LA's use of buyer power. An equilibrium is characterised which involve closure of some care homes relative to the no-price differential case. In such an equilibrium, the increase in price to the private section of demand may be smaller or larger than the reduction in the price negotiated by LA, depending on the share of the market covered by the LA and in some cases the extent of the discount negotiated. We also demonstrate that if a LA contracts also for the places of those who are priced out through price differentiation, an equilibrium exists in which the LA uses its buyer power but its power to depress the public fee rate is reduced. This result is quite intuitive since the LA will demand more places at the lower price and the care home has fewer places from which to recover its loss.

A key assumption of the analysis is that the LA can prevent entry by a care home who does not have to sell to the LA. If this assumption was violated, new entrants would be able to undercut those supplying places to the LA in the private market since such entrants do not have to recover any losses made on their sales to the LA. This highlights the importance of identifying the source of LA buyer power. The fact that differential pricing was identified by the OFT tells us that this buyer power exists but gives us little guidance as to its source.

Despite their empirical relevance, the issues addressed in this paper have not been raised elsewhere in the UK literature. Looking at the effect of the introduction of and revision to the minimum wage, Machin and Wilson (2004) provide important insight to the effect of cost increases on care homes as well as the response of LAs to such exogenous cost shocks. The

willingness of the LAs to let care homes go to the wall, also noticed in Netten et al. (2005), suggests that the LAs are unlikely to consider providing compensation for those who lose out as a result of their use of buyer power and also casts doubt on being able to buy more places as an objective. Gage et al. (2009) looks at quality differences, an issue not addressed in this paper, but an issue which has implication for the results we derive.

While the state's role in supporting UK care home residents is a little different from in the US, the subject of this paper has some parallels with the US nursing home market. The state-administered Medicaid programme offers means-tested assistance with nursing homes fees to people aged 65 and over, but with considerable variation in how states reimburse nursing homes (Millers et al. 2009). The rate at which the Medicaid programme reimburses nursing homes is typically below that provided by the Medicare insurance programme and that paid by private payers (Grabowski 2007). This has led to concerns that nursing homes may discriminate against Medicaid applicants (Ettner 1993; Harrington Meyer 2001) and that flat-rate as opposed to cost-based reimbursement rates may lead to reduced nursing staff levels (Cohen and Spector 1996). Troyer (2002) addresses the cross-subsidy between private and state-assisted residents. Using data for Florida nursing homes she concludes that the cross-sectional price differential between Medicaid and other residents may be explained as an intertemporal difference; the same individual pays a higher rate before becoming eligible for Medicaid and a lower rate once his/her assets have been depleted such that Medicaid becomes payable. Grabowski (2007) is concerned with the difference between Medicaid and Medicare rules for reimbursing nursing home costs which, he argues, provides neither programme with an incentive to take responsibility for the quality of care while encouraging cost shifting between the two programmes.

The remainder of the paper is organised as follows. Section two provides a simple description of the UK care home market. Section three presents a stylised model of the care home market, aimed at illustrating the effects which may arise from local authorities using their buyer power. Section four uses a simulation model to provide a sense of magnitude of these effects. Section five considers the possible sources of LA buyer power. Section six concludes and proofs of results are provided in the appendix.

2 The market for care homes in England

In the UK over 400,000 people aged 65 and over receive long-term care in a care home and this is projected to more than double over the next 50 years (Hancock et al. 2007). Much of

this care falls to LA social services departments to arrange and hence is ‘social’ care but it includes also on-going nursing care. The majority of care home providers are in the independent sector, that is private (for profit) or voluntary (not for profit) organisations. In April 2010, 74% of all care homes (and 80% of care home places) in England were in the private sector and 19% (13% of places) were from the voluntary sector (Care Quality Commission, 2010). The market is characterised by a large number of small providers. In April 2010 there were around 9,300 providers of care homes in the UK, of which 78% owned just one home. Most care homes for elderly people are small scale. Of the 11,200 care homes for elderly people in the UK, around 70% have fewer than 25 places, 60% have fewer than 20 places and 30% have fewer than 15 places³. Providers of care services are regulated and must satisfy standards prescribed by government. These standards cover matters such as physical aspects of the home and training/qualifications of staff.

Most care is purchased by public bodies (local authorities) on behalf of users. This is the norm where the user qualifies for any state help with the cost of the care. In such cases the LA contracts with the care provider and then collects a contribution to the cost from the user. About 70% of care home residents are currently entitled to some means-tested state contribution to their care home fees. According to Laing (2008) care home places procured by LAs are generally ‘spot’ purchases rather than block contracting. He attributes this to the requirement that LAs offer individuals a choice of care homes and the need to have purchasing arrangements in place with the bulk of care homes in their locality in order to give themselves access to sufficient capacity.

The opening quote from the OFT suggests that LAs may be able to exert monopsony power to keep prices low, possibly below the average cost of provision. This can then result in ‘self-funders’ (those not entitled to any state help with the cost) paying a higher price than ‘LA-supported’ users for identical rooms and other services provided by the care home.

The care home funding system involves a state contribution towards the care home fees of older people who are assessed by a LA as needing care in a care home. If they are assessed as needing nursing care (that is care from a registered nurse, as distinct from personal care), the state pays a flat rate subsidy towards the care home fee. This subsidy is deemed to be the part of the care home fee attributable to nursing care. Any other contribution from the state comes

³ Derived from data available at <http://www.carehome.co.uk>

from LA budgets and is means tested against the person's income and capital assets. If he/she has capital assets in excess of an upper capital threshold, the state pays nothing. Otherwise the person is required to pay the minimum of their income and the care home fee (other than that part attributable to nursing care) less a small 'personal expenses allowance'. One relevant implication of the means test is that an individual's contribution is not very sensitive to the fee level. Typically he/she will either contribute all their income apart from the personal expenses, or have capital above the upper threshold and so be liable for the whole fee, whatever its level.

In 2008-09, LAs in England spent £4.7 billion on care home places for people aged 65+, recouping about £1.4 billion of that from charges to care home residents (NHS Information Centre, 2010). LA funding comes mainly from central government based on an assessment of the each LA's population needs for the services they provide. The funding formula distinguishes the needs of different population groups so the formula for older people's social care takes account of local factors such as costs, levels of deprivation and the numbers of older people, distinguishing the numbers aged 80+ (House of Commons Health Committee, 2010). Funding from central government is mainly not 'ring-fenced' so LAs are free to spend these funds broadly as they wish. They supplement central government funds by levying Council Tax – a local property tax – the rate of which is within LAs' control subject to central government's power to cap annual Council Tax increases.

3 Simple equilibrium model

In this section we build a simple equilibrium model where we can identify the qualitative effects arising from a LA using its buyer power to obtain lower prices on the units they procure.

3.1 Supply side issues

To understand how the buyer-power exercised by the LA operates, we need to model the cost structure of a typical care home. In the short and possible middle-run, a typical care home will have a fixed capacity, K , determined by the physical space in the facility. While it may be possible to up- or down-grade rooms, the capacity level is fundamentally determined by the number of rooms in the care home and fixed by planning regulation and physical space constraints and hence largely exogenously given. We will in the following assume that K is exogenously fixed and that for simplicity it is identical for all.

Average variable costs [and hence marginal costs] are likely to be fairly constant⁴ up to capacity, at which point they become infinite. Given the fixed costs of providing the capacity [mainly buildings] the average total cost curve of the typical care home, ATC, is falling up to capacity. If all care homes are identical, then the price $P^c = ATC(K)$ would ensure that all care homes would just break even.

One implication of any price discrimination induced by a powerful buyer follows immediately from the cost structure. If the LA negotiated a lower price, $P^{LA} < P^c$, for a fixed number of units, k , then to break even and cover its fixed costs, the care home would have to charge all other users a higher price as illustrated in figure 1 below.

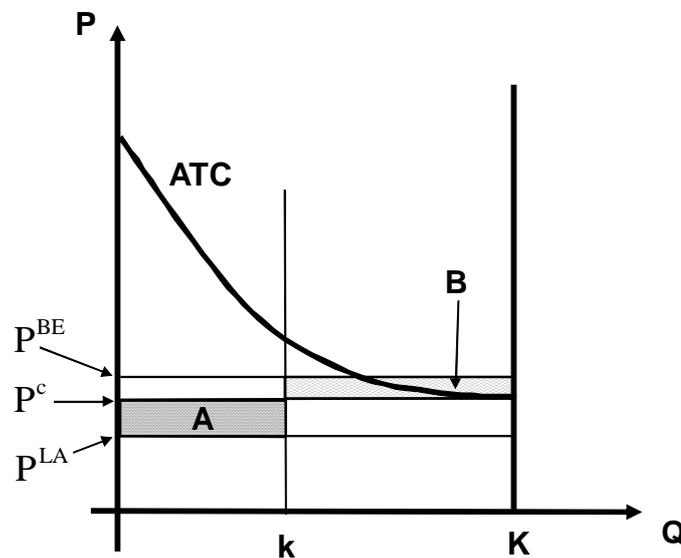


Figure 1: pricing with LA buyer power

By accepting the demand for a lower price from the LA, the care home will be short of an amount A of money to finance its fixed costs. This can only be recovered from the $K - k$ beds sold to private individuals through a higher price. Let P^{BE} be the price paid by a private buyer, then the break-even condition for a care home who had sold k units to the LA at the price $P^{LA} < P^c$ require area A to equal area B, and would be:

$$ATC(K) \cdot K = k \cdot P^{LA} + (K - k) \cdot P^{BE}$$

or

⁴ According to Machin and Wilson (2004) and Laing (2008), the main variable costs is wage costs and the majority of workers in the sector are on minimum wages. This would limit substitution possibilities among types of labour.

$$P^{BE} = ATC(K) + \frac{k}{K-k} \cdot (ATC(K) - P^{LA}) \quad (1)$$

so that the bigger is the discount negotiated with the care home or the larger is the number of units procured by the LA, the higher must the private sector price be in order for the care home to remain in business. This is essentially a “waterbed” effect⁵, where a firm compensates for losses from some sales by increasing the price to others. Note the key caveat that the firm must be able to raise the price to private customers without losing demand.⁶ Thus for such a waterbed effect to be an equilibrium outcome, there must be positive private demand at those higher prices.

Note also from (1) as well as figure 1 that if there is a free entry equilibrium with care homes selling both to the private and the LA market, then any exercise of buyer power by the LA imply a cross subsidy from the private buyers to the LA assisted buyers. In competitive markets, especially with free entry, such cross-subsidies tend to be undermined. It is hence not obvious than an equilibrium in which buyer power is exercised is feasible. To see that this might not be the case, consider an example where all the LA demand was met by one care home at a price $P^{LA} < P^c$. Unless total capacity is so small that it cannot meet total demand at P^{BE} , competition between non-LA providers will drive the price on this market below P^{BE} . This would leave the LA-supplying care home unable to fund the LA discount and as a consequence unwilling to supply the LA at the reduced price.

An immediate implication of the previous discussion is that an entrant who can enter with the same cost structure [and hence break-even price P^c], but without having to supply the LA would be able to undercut the incumbents profitably.⁷ Any price just below P^{BE} would attract the private buyers of all the incumbents without these being able to afford to respond by lowering their prices. Such entry would erode the cross subsidy to the LA covered demand from all other buyers. If the market was remotely contestable, such cross subsidies could not survive in equilibrium. Hence for price differentials to persist, it must be the case that not all forms of entry are free. In particular entry without selling to the LA must not be free. An

⁵ See e.g. Majumdar (2005) and Inderst and Valletti (forthcoming).

⁶ We assume free entry. Thus the care homes would not have been able to raise the price to the private customers in the no price discrimination case.

⁷ An other form of potentially profitable “entry” would be the expansion of the size of a currently active care home. This could similarly undermine the ability of the LA to use its buyer power.

interesting policy question is then how the LA is able to undermine the potential contestability of the market without violating competition law.

3.2 Adding the demand side

One issue which needs to be addressed immediately when it comes to the demand for care homes is whether anyone would want a place if they did not have care needs. It would seem to be reasonable to assume that this is not the case. Thus not all members of society would have a positive demand for such a place even at zero prices. Moreover, unless they are buying on the behalf of a group such as an LA, no buyer would want more than one unit. Thus apart from any LA derived demand, this is a typical case where the individual consumer wants either one unit or none. How much they are willing [or able] to spend for one unit would depend on a number of factors to be discussed further below. Importantly, this willingness to pay is likely to differ across individuals. With the unit of analysis being a care home place, we represent demand in figure 2 below, where each bar in the figure is one unit wide and the height represents a particular consumer's willingness to pay for one unit of care home.

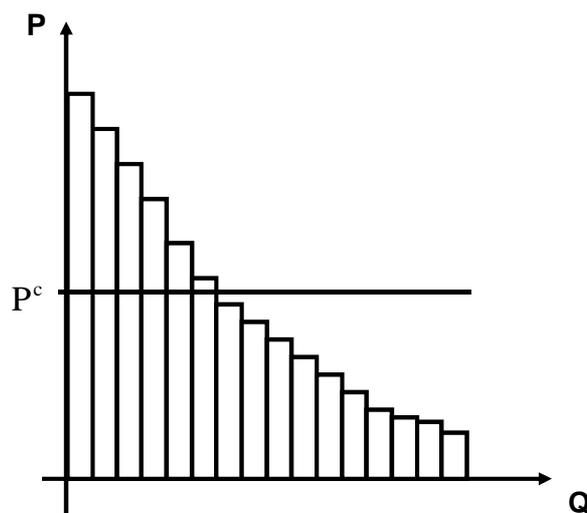


Figure 2: Demand for care homes.

Key to the shape of the demand curve is the willingness to pay which is determined both by income and the “price” of the best available alternative. Conceptually what we mean by income in the present context is not straightforward. Firstly, it tends to be a mixture of pension payments and income generated by accumulated assets and individuals may differ in their desire to leave assets to their descendants. Secondly, the individual needing a care home place may have access to the income or assets of others, such as immediate relatives. Thus depending on the preferences of the individual and possibly their family, the person needing a

care home may have a willingness to pay derived from levels of income and/or assets which are greater or smaller than those used by LAs in assessing their entitlement to state support. Similar difficulties arise when we consider alternatives. This may for a rich person needing simple care involve being cared for at home with privately hired staff or it might be care at home from friends and family. The feasibility of alternatives may differ depending on the extent of care needs and in particular whether this arises from physical or mental fragility. While as a practical matter, determining the willingness to pay is complex, for our modelling purpose, all we need is demand as described in figure 2, from which we get a standard downwards sloping demand curve.

3.3 Local authority behaviour

Assume first that the market is entirely private, i.e. there is no LA demand, and consider the price P^c shown in figure 2 above. At that price some consumer would be priced out of the market. The first policy question is to decide how to deal with those who are excluded in this way, and who are at least candidates for public supply/subsidy. We will proceed through a number of special cases.

As we saw above, price discrimination between LA funded and privately funded places raises a particular issue, namely how care homes are going to fund the discount offered to the LA. In a market which is largely contestable, the fixed cost short-fall has to be obtained from the private market and moreover, the ability to raise this short-fall may undermine the existence of an equilibrium, because the more the price is increased for the privately funded places, the more buyers will drop out of the private market and the fewer people will be available to pay the higher price. Note from figure 2 that the candidates for paying the missing contribution to fixed costs are those with high willingness to pay.

One would expect there to be a very high correlation between willingness to pay and income (ability to pay). In particular one would expect to find those with very low income [savings] to have low willingness to pay. While the set of those individuals willing to pay the market price would then contain few, if any, with very low income, the set of individuals who are not willing to pay the market price may contain a mixture of those with low income and those with good alternatives. Any criterion based on income is hence likely to leave the set of people who would buy a privately funded place intact. By the same token, any criterion which is based on factors other than income will take away people who would have been

willing to pay the higher private rate. The following combination of assumptions then serve to leave as many as possible in the group who purchases from the private market:

Assumption 1: The test for eligibility for an LA-funded place is based solely on a financial needs test.

Assumption 2: There is a perfect correlation between income and willingness to pay.

The implication is that looking at figure 2, the local authority starts serving the individuals from the right in the diagram and the financial test then determines at what point they stop.

To understand why this set of assumptions are the most helpful for the existence of an equilibrium with price discrimination, consider the following two alternatives. First, maintain assumption 1 and assume an imperfect correlation. In this case, some of the individuals with high willingness to pay will meet the eligibility test and hence be withdrawn from the set of individuals who are willing to purchase a private place. Second, maintain assumption 2, but assume that in addition to a financial based test there is also a health needs test. Now some of the people with high willingness to pay who would fail the financial test may pass the needs based test and again be withdrawn from the set of individuals who are willing to purchase a private place⁸. If we relax both, we are still likely to see individuals withdrawn from the set who are willing to purchase a private place. The importance of the two assumptions will be discussed further in the conclusion.

We have not locked down the exact criteria for the financial needs test. We will assume the following:

Assumption 3: An LA is obliged to fund the places of those with income such that their willingness to pay is at or below P^c .

The motivation for this is that in either a perfectly competitive market or a market in which a social planner had to set a single price for all, this price would be P^c and hence the LA is obliged to ensure that those who will for sure be priced out of the market will be able to obtain an LA-funded place.

⁸ Such a situation exists in the UK for a relatively small number of people with the severe on-going health needs for whom the National Health Service meets 100% of nursing home fees, without a means test.

Given assumption 3, price discrimination raises a new issue when it comes to the behaviour of the LA. From figure 1 we know that with any price discrimination, $P^{BE} > P^c$ and hence, as illustrated in figure 3, there will be some unmet demand

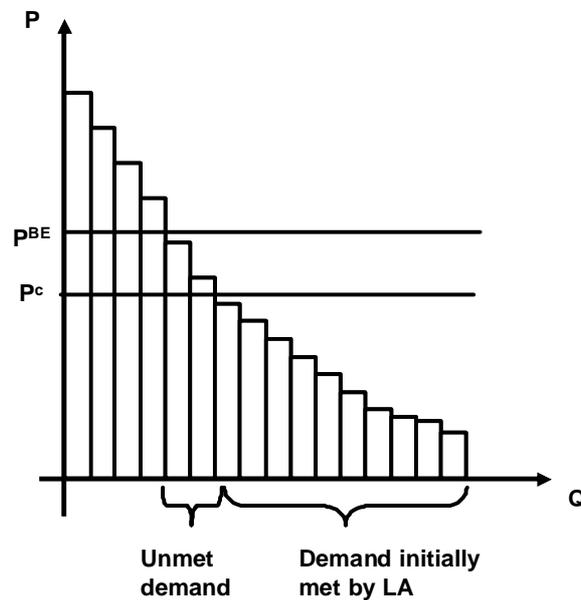


Figure 3: losers from discrimination

A key question is what the LA would do with these consumers. The LA could decide either to leave well alone and accept that some may not be served or they could extend the availability of an LA funded place to those in figure 3 with unmet demand. We will refer to the two cases as “LA non-inclusion” and “LA inclusion” respectively. Recall that those who obtain an LA funded place still have to pay an amount related to their ability to pay. For simplicity, think of this as having to pay up to their willingness to pay as illustrated in the figures above. Note that those with unmet demand, who are not able or willing to pay P^{BE} , are all able to pay the price of an LA funded place. Hence it does not cost the LA anything directly to extend eligibility to cover those with unmet demand. As we shall see later, matters are not so simple when considering the indirect effects of extending eligibility.

3.4 Equilibria

Despite the simplicity of the modelling framework, we are able to extract a number of qualitative insights by considering the equilibrium conditions while varying the assumptions about LA behaviour. Throughout this section we will assume that all care homes are identical and that, while entry is free, all active care homes are obliged (or willing) to sell to the LA if the latter wants to buy. There is also free exit, so a care home can always refuse to sell to the LA by leaving the market.

To simplify the computations, we represent demand by a linear demand curve given by $Q = \alpha - \beta \cdot P$. The interpretation of demand given in the text surrounding figure 3 is still valid and hence the area under the curve represents an overall willingness to pay. Total costs of an individual care home producing q is given by $TC = F + c \cdot q$ up to a maximum of K . The break even price for a care home selling all units at the same price is hence given by

$$P^c = \frac{F}{K} + c \quad (2)$$

To ensure that all care homes are treated equally, the LA will purchase the same number of units from each home. With this we are able to derive the equilibria for the various cases.

3.4.1 Equilibrium in the no-discrimination case

Clearly there is a non-discrimination equilibrium with the price given by (2). To find the demand for the LA, assume that anyone with a positive willingness to pay has a “care home need” so that total demand for care home places is found from the demand curve at zero price, i.e. $q^{\max} = \alpha$. With total capacity of a representative care home set at K , we need:

$$N^* = \frac{\alpha}{K} \quad (3)$$

care homes in equilibrium if there is not discrimination. To get total LA demand we subtract the demand at $P = P^c$ to get:

$$D^{LA} = \alpha - \left(\alpha - \beta \cdot \left(\frac{F}{K} + c \right) \right) = \beta \cdot P^c \quad (4)$$

In equilibrium, the care home sector with N^* care homes supply $\beta \cdot P^c$ places to the LA and $\alpha - \beta \cdot P^c$ to the private sector.

3.4.2 Discrimination with exogenous number of care homes, N^*

Consider the case where the LA uses its buyer power to demand a price $P^{LA} < P^c$. Assume that the number of LA assisted places are determined by those who would not purchase at the price P^c . Thus as in the previous subsection, the LA demand is given by (4). From figure 5, we know that with price discrimination and no further LA intervention, there will be a section of demand which is not served, determined by the number of consumers who have a reservation price in the interval $[P^c, P^{BE}]$.

Assume that N^* is still the number of care homes and that the LA procure the same amount from each of these care homes. We can show that no simple equilibrium price in the private market exist.

Lemma 1: *No pure strategy equilibrium exist in which there is a difference between the private market price and the LA supported price and there are enough care home places to supply the entire market, $N = N^*$.*

The intuition is quite simple. From figure 5 we know that some people will be priced out of the market while unable to get access to an LA supported place. With industry capacity designed to meet all demand, clearly some care homes will carry excess capacity. As the break-even price reflects this possible excess capacity, care homes with empty beds have a strong incentive to cut their prices relative to the break-even price as this will increase sales and hence profits. This downwards pressure on price will imply that no care home set the break even price so that all make a loss. This implies that no simple equilibrium exist.

To obtain an equilibrium, one of two things must happen. Either the LA must intervene further in the market, or else there must be fewer care homes in the market.

3.4.3 Equilibrium with fewer care homes

Consider a proposed equilibrium in which there is zero profits and where the number of suppliers is given by a requirement that there is no spare capacity in equilibrium. The benchmark price is still given by (1). We maintain the assumption that the LA does not intervene further in the market and hence that LA demand is still given by (4).

From the previous subsection we know that the number of care homes must be reduced relative to N^* . Thus the LA demand must be spread on fewer care homes. The implication is that each care home must sell more places at the discounted price and have fewer beds to use for cross subsidy. Hence the break-even price will have to be increased, reducing demand and hence the number of care homes further. But this will increase the number of subsidised units in each care home, necessitating further price increases. It hence seems possible that an equilibrium might not exist. In the appendix we prove the following lemma:

Lemma 2: *For any $P^{LA} \geq \max\left\{P^c - \frac{(\alpha - \beta P^c)^2}{4\beta^2 P^c}, 0\right\}$ there exist an equilibrium with $P^{BE} \geq P^c$*

such that there is no excess capacity and profits are zero.

Let \bar{P}^{LA} be the lowest price for an LA funded place. It follows from the proof of lemma 2 that

$$\bar{P}^{LA}(P^c) = \begin{cases} 0 & \text{if } P^c \leq \frac{\alpha}{3\beta} \\ P^c - \frac{(\alpha - \beta P^c)^2}{4\beta^2 P^c} & \text{if } P^c > \frac{\alpha}{3\beta} \end{cases} \quad (5)$$

The private market price corresponding to $\bar{P}^{LA}(P^c)$ this is then give by

$$\bar{P}^{BE}(P^c) = \begin{cases} P^c + \frac{\alpha - \beta P^c - \sqrt{(\alpha - \beta P^c)^2 - 4\beta^2(P^c) \cdot P^c}}{2\beta} & \text{if } P^c \leq \frac{\alpha}{3\beta} \\ \frac{\alpha + \beta P^c}{2\beta} & \text{if } P^c > \frac{\alpha}{3\beta} \end{cases}$$

The main implication of the lemma is that while an equilibrium with price discrimination always exist, the extent to which the LA can force price down without leading to a collapse of the care home market is possibly quite limited. The extent of the limit depends on P^c so that the larger is P^c , the closer is P^{LA} to P^c . This is quite intuitive when we recall that a large P^c equates to a large LA demand and hence a small number of units on which any missing fixed costs can be reclaimed.

3.4.4 LA intervention: Including the squeezed middle

In the previous subsection we showed that when an LA uses its buyer power to reduce the price for LA assisted care home places and where the demand for LA assisted places is exogenously given, some of the demand for care home places would not be met. This creates a “squeezed middle” of consumers who cannot (or will not) afford a private place but who do not qualify for an LA assisted place. A key assumption behind lemma 2 and 3 was that the LA ignored its effect on buyers from their use of market power. In this section we investigate the consequence of them internalising this externality.

Assume that the LA extends the availability of an LA assisted place to all those who are priced out of the market at the equilibrium private market price, P^{BE} . This ensures that all individuals are served and the number of firms is as in section 3.4.1 given by $N^* = \frac{\alpha}{K}$. The

new condition relates to the size of the demand from the LA who now has to meet all demand not met by the private sector:

$$D^{LA}(P^{BE}) = \alpha - (\alpha - \beta \cdot P^{BE}) = \beta \cdot P^{BE}$$

of which each care home gets the same fraction.

Under the ‘no LA inclusion’ case considered in the previous subsection, the consequence of the LA depressing the price for LA supported places too much, is a large fall in the number of care homes so the LA requires an increasing number of beds from each care home, ultimately reducing to zero the number of private places from which to generate the cross-subsidy. In the case of LA inclusion considered in this subsection a similar problem could arise. When the LA extends LA funding to more and more people, even though the number of care homes remain the same, each care home will sell an increasing fraction of its places at the LA funded price, again potentially leading to there being no places from which to generate the cross-subsidy.⁹ However, as above, we can show that so long as P^{LA} is not too small relative to P^c an equilibrium pair of prices (P^{LA}, P^{BE}) for which the care homes remain viable does exist.

Lemma 3: For $\frac{(\alpha + \beta P^{LA})^2}{4\alpha\beta} > P^c$, there exist equilibria in which $P^{BE} > P^c > P^{LA}$.

Let \bar{P}^{LA} be the lowest price for an LA funded place. It follows from the proof of lemma 2 that

$$\bar{P}^{LA}(P^c) = \begin{cases} 0 & \text{if } P^c \leq \frac{\alpha}{4\beta} \\ \sqrt{\frac{4\alpha}{\beta} P^c} - \frac{\alpha}{\beta} & \text{if } P^c > \frac{\alpha}{4\beta} \end{cases} \quad (6)$$

The private market price corresponding to $\bar{P}^{LA}(P^c)$ is given by

⁹ From this we can also see the consequence of relaxing assumptions 1 and 2. Any reduction in the set of individuals with high willingness to pay who have to buy from the private market would undermine the equilibria. At best it would reduce the ability of the LA to press down the price for an LA assisted place. As worse, no equilibrium in which the LA uses its buyer power would exist.

$$\bar{P}^{BE}(P^c) = \begin{cases} \frac{\alpha - \sqrt{\alpha^2 - 4\alpha\beta P^c}}{2\beta} & \text{if } P^c \leq \frac{\alpha}{4\beta} \\ \sqrt{\frac{\alpha}{\beta} P^c} & \text{if } P^c > \frac{\alpha}{4\beta} \end{cases}$$

Recall that a low P^c is equivalent to saying that the LA funded market is quite small. As in the previous section, if P^c is small, the LA could in theory obtain their places for free and still leave enough privately funded spaces for the care homes to break even.

3.5 The effect on the LA

A person eligible for an LA supported place is paying the minimum of the LA price and their willingness (ability) to pay. We illustrate the different effects on welfare gains from the LA using its buyer power in two figures. Where the LA is non-inclusive towards those squeezed out of the market, the effects are summarised in figure 4 below.

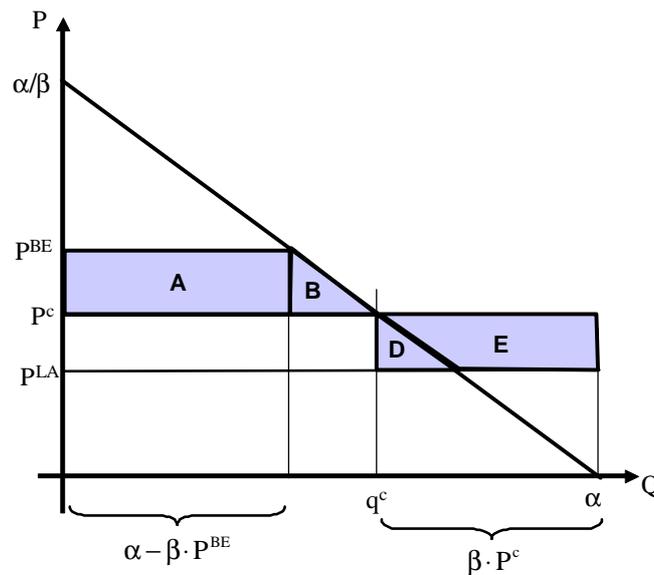


Figure 4: Welfare effects of LA policy with no additional support

The two areas **A** and **B** represent welfare losses to the relatively well off, while **D** represents a welfare gain to a small group of people who would have been willing to pay more than the LA funded price (but not more than the no discrimination price). Finally, **E** represents the LA's saving from obtaining the lower price. We could then crudely measure the net welfare as $\Delta W = \mathbf{D} + \mathbf{E} - \mathbf{A} - \mathbf{B}$. Note that $\mathbf{D} + \mathbf{E}$ represents the total loss to the care home sector from the LA discount and **A** the compensating gain from the self-funders so that $\Delta W = -\mathbf{B} < 0$ and the policy is welfare decreasing.

The welfare effects in the case where the LA is inclusive towards the squeezed middle are illustrated in figure 5 below.

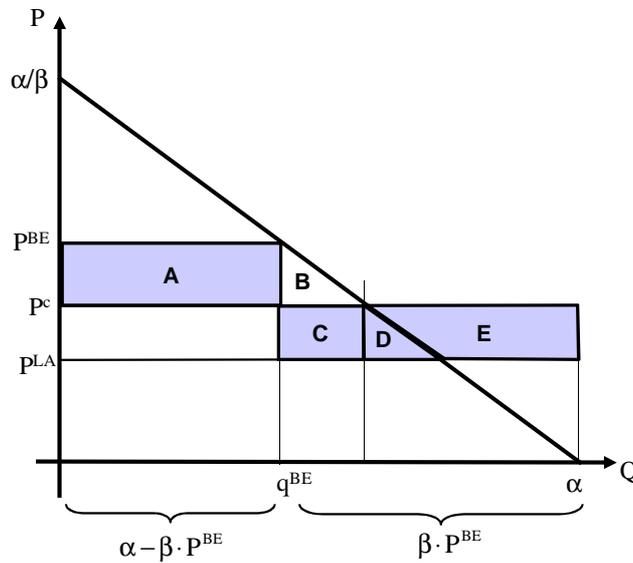


Figure 5: Welfare effects of LA policy with extended access

Note that in figure 5, only **A** represents a welfare loss and that area **C** represents a new gain to mid-valuation individuals. While the areas in the two figures are not directly comparable since the prices do not remain the same when we move from non-inclusion to inclusion, we can still provide a comparison of the overall effect on total welfare. Net welfare is $\Delta W = C + D + E - A$. Now the total loss to the care home sector from the LA discount is given by $C + D + E$, while **A** is the compensating gain from the self-funders so that $\Delta W = 0$ and the policy is neutral with respect to total welfare. On the basis of this, if an LA was to use its buyer power, total consumer welfare would be less adversely affected if the LA was inclusive.

The other relevant measure is the LA saving as given in area **E**. Note that which of the two areas in figures 4 and 5 labelled **E** is largest is simply determined by whether the LA price is lowest with or without inclusion. The lower the LA price, the greater is the saving to the LA. To assess this, we need to compare the equilibrium prices in the two cases. To do so, we first focus on the case where the LA uses its buyer power to the maximum so that LA prices are given by (5) and (6). We can demonstrate the following:

Proposition 1: *The lowest possible LA price, P^{LA} , is weakly the lowest when the LA is non-inclusive, $\bar{P}^{LA} \leq \bar{P}^{=LA}$. For $P^c \geq \alpha/3\beta$, the corresponding private market price is the highest when the LA is not inclusive, $\bar{P}^{BE}(P^c) \geq \bar{P}^{=BE}(P^c)$. For $P^c \leq \alpha/4\beta$, the corresponding private*

market price is the highest when the LA is inclusive, $\bar{P}^{BE}(P^c) \leq \bar{P}^{BE}(P^c)$. As P^c is increased from $\alpha/4\beta$ to $\alpha/3\beta$ the private price moves from being highest with inclusion to highest with non-inclusion.

The result on P^{LA} is far from obvious. How far an LA can depress price depends on how many beds are left in the care home to cross-subsidise the associated losses. With non-inclusion the number of private places is limited by the LA's need to buy more places from each home, arising from the reduction in the number of homes. With inclusion, the number of private places is reduced because the LA's demand for subsidised places has increased. The result tells us that the expansion in LA places in a given care home is greater when the LA is inclusive. Once this result is established, however, the results on P^{BE} are more intuitive. When the LA price is the same with and without inclusion, which occurs when this price is zero, there are fewer private places when the LA is inclusive and hence the private price must be increased relative to the non-inclusive case. If the LA price is positive in both cases, the private market price mirrors the LA price. The implication of this is that the LA obtains greater savings when it is being non-inclusive.

To get a feel for the equilibrium effects, we normalise the variables such that $\alpha = 10$ and $\beta = 1$ and compare the different areas in figures 4 and 5 for a specific example, namely where

$P^c = \alpha/3\beta = \frac{10}{3}$. In this case, we get $\bar{P}^{LA} = 0$, $\bar{P}^{LA} = 10(2\sqrt{\frac{1}{3}} - 1)$, $\bar{P}^{BE}(P^c) = \frac{20}{3}$ and $\bar{P}^{BE}(P^c) = 10\sqrt{\frac{1}{3}}$.

Table 1: numerical example

	Non-inclusive	Inclusive	Effect on
Area A	11.11	10.31	High income users
Area B	5.56		Medium income users
Area C		4.36	Medium income users
Area D	5.56	1.59	Low income users
Area E	5.56	4.36	LA budget
ΔW	- 5.56	0	Total welfare

The numerical example illustrates the points made above. In particular, note that a move from non-inclusion to inclusion, while avoiding welfare losses, leads to a reduction in LA savings of more than 20% and hence may be preferred by the LA.

3.5.1 Summary and implications

While the basic model is quite simplistic, it captures some of the salient features of the care home market. In particular, although we have made assumptions which make finding an equilibrium in which the LA exercises its buyer power more likely, establishing the existence of such equilibria is not simple, nor is the ability of the LA to exercise buyer power extensive.

The section has identified two separate ways in which the LA risks sending the market into a tailspin of closures, a concern which was raised in the quote from the OFT in the introduction. Firstly, if the LA does not get the proportions it purchases from each care home right, then some of the care homes may not be able to obtain enough demand for private places to fund the cross-subsidy implied by the exercise of LA buyer power. Secondly, if the LA tries to depress the LA price too much, or is too inclusive, there may again be too few private payers to provide the necessary cross-subsidy. A key concern is the size of the squeezed middle, something we turn to in the next section.

Given the insights of the model, the real surprise turns out to be the willingness of LAs to use their buyer power since this has the potential to disturb the market with potentially catastrophic effects. The dangers are greater the larger is the relative size of the LA market; the more places procured through the LA, the smaller is the discount the LA can demand without undermining the market.

The section also pointed to a group who, depending on the level of inclusiveness of the LA are either the big losers or the big winners, namely the squeezed middle. The location of this group in the income distribution may affect the social value we place on those losses or gains.

4 Quantifying the squeezed middle and the welfare gains/losses from the exercise of buyer power

In the simple model above we assumed that willingness to pay for a care home place, conditional on care needs, was determined largely by income. If willingness to pay is directly proportional to income, then the distribution of income close to P^c will be a primary determinant of the size of the squeezed middle. Recall however, that the means test for state help with care home fees is more complicated than a simple income test. To capture these complications we use a microsimulation model of care home charges, CARESIM, which

applies the means test to the income and assets of a sample of older people from the British Family Resources Survey (see for example, Department of Work and Pensions 2010). The sample is re-weighted by age, gender, marital status and housing tenure to be representative of care home residents in England. Weights are provided by a separate model of aggregate long term care finance developed by the Personal Social Services Research Unit (PSSRU) at LSE. Further details can be found in Hancock et al., 2007 and 2008. CARESIM uses data on the incomes, assets and other relevant characteristics of nearly 17,800 sample members aged 65 and over, from three years of the survey. The model calculates what each sample member would be required to pay towards a given level of care home fees. By varying P^{LA} and P^{BE} for a given P^c , the model is used to quantify the size of the squeezed middle for different price pairs. We consider also the role that the means test plays in determining the size of the squeezed middle and hence how the scope for LAs to use their buyer power might be affected by reforms to that test. Finally we use the simulation model to assess how gains and losses from price discrimination are distributed across care home residents according to their income level.

The analysis relates to independent (private or charitable, but not state-run) residential care homes, i.e. homes that provide personal but not nursing care¹⁰. The means test for assessing entitlement for state help with the fees of such homes operate as follows. Residents with capital assets in excess of an upper capital threshold are liable for the whole of the fee. Capital includes the value of the resident's home (after the first 12 weeks in the care home) unless a qualifying relative continues to live there. Where residents have capital below the upper threshold, the state contribution is a function of their income. The definition of income includes an assumed income from any capital the person has above a lower threshold but excludes actual investment income. Residents are required to pay the minimum of their income and the care home fee, less a small personal expenses allowance.

Residents who have capital above the upper threshold will have to draw on that capital to meet the care home fees unless their income is sufficient to cover their fees. Over time their capital may therefore fall to below the upper threshold, at which point they can become eligible for LA funding. To allow for this, the model randomly assigns a duration of time in

¹⁰ Similar issues arise in the market for care homes which also provide nursing care. Fees are higher in such homes although the state provides a non means-tested subsidy towards the nursing care component of fees. Restricting our analysis to homes that do not provide nursing care simplifies the picture without materially affecting the conclusions

the care home, based on the distribution of such durations reported in a survey of care home residents (Netten et al., 1998). Each sample member's liability to pay towards care home fees is calculated assuming that they have been in the care home for that duration and have drawn on their capital accordingly. In this sense, the model mimics a cross-section of care home residents and calculates liability to pay for the point at which they would be observed in such a cross-section. This highlights a potentially important consequence of the means test.

Residents who are self-funders on entry to a care home may become LA-supported through depleting their capital. The higher the fee paid by self-funders, the faster they will become eligible for LA support and – if there is price discrimination – for the lower LA fee rate.

We work in April 2007 prices when the upper capital threshold was £21,500 and the weekly personal expenses allowance was £20.45. Previous work (see e.g. Hancock et al. 2010), based on the best available data, assumed average weekly care home fees for LA supported and self-funded residents of £419 and £499 respectively in April 2007 prices. The squeezed middle consists of people who would be liable for 100% of the lower fee but less than 100% of the higher fee. An important point is that the means test is such that if a LA purchases care home places for residents in this group, there is no cost to it because the resident meets 100% of the fee. There is however, a reduction in the revenue of care homes.

At these fee rates and under the current means tests, we estimate through CARESIM that about 35% of residents in independent sector residential care are self-funders. This implies an average fee across LA supported and self-funders of £447. We take this to be P^c . The PSSRU long-term care financing model estimates that there were some 175,000 people aged 65+ living in independent residential care homes in England in 2007. At £447 per week, their total annual revenue would be about £4.1 billion. Using the simulation model, we estimate that around 48% of this would be met by Local Authorities, 17% by LA-supported residents and the rest by self-funders.

It is important to emphasise that we do not simulate demand responses to price changes within CARESIM but focus on the first round effects of price changes resulting from LAs exercising their buyer power. We return to this issue in section 4.3.

4.1 Aggregate effects of price discrimination under unchanged demand

Table 1 shows CARESIM estimates of the first round (unchanged demand) effects of price discrimination compared with a uniform price of £447. With $P^{LA} = £419$ and $P^{BE} = £499$, the number of LA supported residents is just over 1% higher than in the absence of price

discrimination, but LAs' costs are about 8% lower. Costs met by LA-funded residents are lower by 3% but self-funders pay 11% more in total. Total care home income is reduced by 0.06%. At these fee rates, the squeezed middle contains just under 200 residents, or 0.1% of the total. If the LA includes them, the effect on care home income is small.

That the percentage of residents who are LA supported is higher under price discrimination even without LA inclusion is because at the higher private fee rate, those who are initially self-funders, deplete their capital faster and hence qualify for LA support sooner. Thus although LAs make savings from the lower price for LA-funded residents, these are partially offset by having to support sooner those who initially self-fund.

If LAs push P^{LA} considerably further below the market rate, e.g. to £319 the consequences would be greater. P^{BE} would need to be at least £685¹¹. At this price, care home revenue would be about 2.5% lower than with no price discrimination and the proportion of self-funders falls to 32%. The size of the squeezed middle is larger (around 360) but still a small proportion of the total.

Table 2: First round effects of price discrimination, current means test, April 2007 prices; Number of residents = 174,500

	No price discrimination $P^c = £447$	Price discrimination compared with no price discrimination			
		$P^{LA}=419, P^{BE}=499$		$P^{LA}=319, P^{BE}=685$	
		LA non inclusion	LA inclusion	LA non inclusion	LA inclusion
Total care home revenue	£4,056m	£-2.6m (-0.06%)	£-3.4m (-0.08%)	£-92m (-2.3%)	£-99m (-2.4%)
Of which:					
LA spend	£1,932m (48%)	£-147m (-8%)	£-147m (-8%)	£-700m (-36%)	£-700m (-36%)
User charges (LA-supported)	£689m (17%)	£-19.5m (-3%)	£-19.8m (-3%)	£-87m (-13%)	£-89m (-13%)
Self-funders	£1,434m (35%)	£+163m (11%)	£+163m (11%)	£+695m (48%)	£+690m (48%)
No. of self-funders	61,702	-1,267 (-1.1%)	-1,459 (-1.3%)	-5,517 (-4.9%)	-5,875 (-5.2%)
No. of LA supported	112,777	+1,267	+1,459	+5,517	+5,875
'squeezed' middle			192		358
% self-funders	35.4%	34.6%	34.5%	32.2%	32.0%

¹¹ This is calculated as $(P^c - s P^{LA}) / (1 - s)$ where s is the proportion of residents who self-fund at P^c . Since that proportion can only increase as a result of price discrimination, this must be the minimum value that P^{BE} can be in equilibrium.

It may seem an anti-climax that the effects are so modest, but a more dramatic result would imply much more instability in the care home industry than any commentators suggest currently exists. However, that is not to say that other means tests would lead to equally modest effects, nor that more significant effects would not be found if LAs became more demanding in terms of price reductions.

The relatively small effects of even a large price differential are partly explained by the operation of the upper capital threshold in the means test because a resident with capital above this threshold is liable for 100% of the fee, whatever the fee level. Under alternative means tests the effects of even small price differentials could be quite large. In Figures 6 and 7 we show care home revenue and LA spending for a range of price pairs (P^{LA} paired with the corresponding minimum P^{BE}), under the current means test and two alternatives to it. The first alternative removes the upper capital threshold. Residents with capital above the lower threshold would be deemed to have an income from all their capital (including housing wealth where relevant) above that threshold. In the second alternative the state would meet the costs of care from the point at which residents have been in a home for two years. This was proposed in the White Paper on social care published immediately before the 2010 UK general election was called. Under such a system there would be an increase in the proportion of care home residents supported by LAs but there would remain self-funders among residents who had been in care homes for under 2 years.

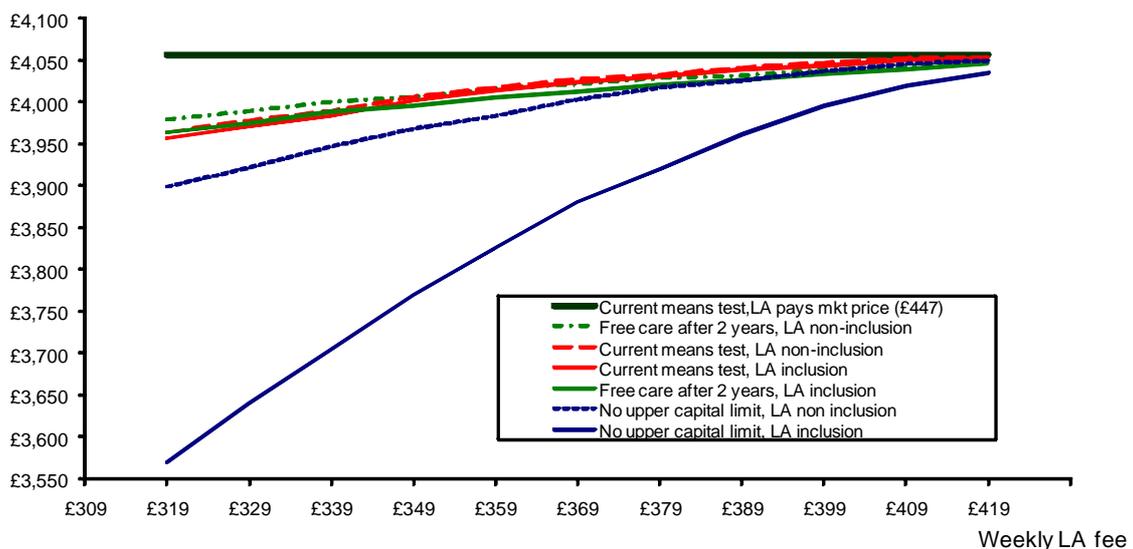


Figure 6: Annual care home revenue in £ millions as P^{LA} varies, without demand response

Figure 6 confirms the limited scope for LAs to depress P^{LA} without reducing annual care home revenue by amounts which are likely to destabilise the market. It also confirms the

importance of the capital test in enabling LAs to pay below P^c . Without the capital test, care home revenue falls much faster as P^{LA} falls.

Figure 7 shows the clear budgetary incentive for LAs to try to negotiate low prices with care homes. Continuing with a capital threshold for those in care homes for under 2 years protects the ability to exercise buyer power. Faced with the additional costs of meeting the care costs of those in care homes for more than two years, it seems likely that LAs would seek to exercise this buyer power and reap the not inconsiderable savings from it were this reform to be implemented.

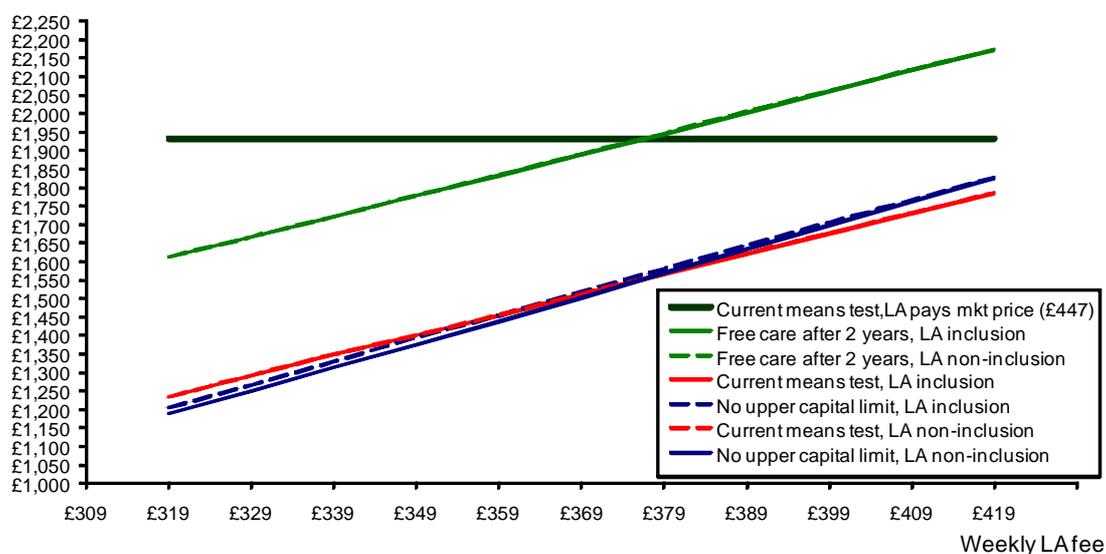


Figure 7: Annual LA spend in £ millions as P^{LA} varies, without demand response

Note also from Figure 7 that if the LA is aiming at a specific annual spend, then while a change in policy towards a system with no upper capital limit would have little or no effect, a change to a system with free care after 2 years would increase the LA spend considerably. One response by the LA to such a policy change might be to demand a larger discount on LA funded places. In evaluating the consequence of a policy change, it is then necessary to allow for this type of re-optimisation by the LA.

Finally, Figure 8 shows that the size of the squeezed middle is contained by the capital test, and would be much larger and grow rapidly as P^{LA} falls, if the upper capital threshold were to be removed.

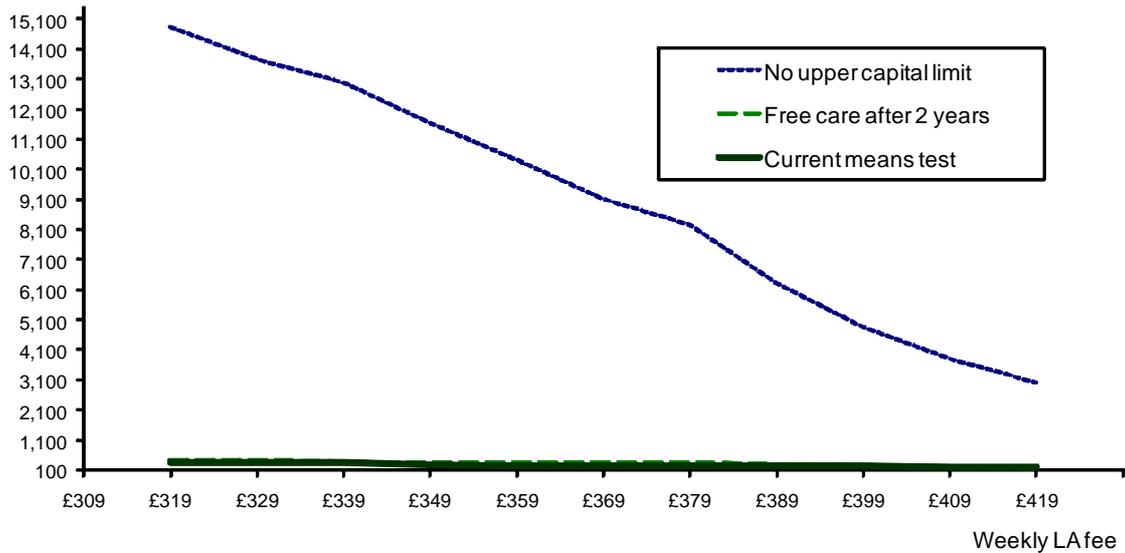


Figure 8: Size of the squeezed middle as P^{LA} varies

In the analysis above we assume that in the non-inclusion case LAs apply the means test to the non discrimination fee. In practice they may use the lower LA-supported rate because the non discrimination price is not observed. The effect on the size of the squeezed middle is small for small differences between P^{BE} and P^{LA} , but could be important where the price differential is larger. For example under the current means test, with $P^{LA} = £319$ and $P^{BE} = £685$, the squeezed middle would be about 1,200 if LAs apply the means test to P^{LA} compared with under 400 if the test uses P^c .

4.2 Distributional effects of price discrimination under unchanged demand

Table 3 shows the difference in residents' contributions towards their fees under price discrimination compared with no price discrimination. The first two columns of the table show mean changes in residents contributions to care resulting from price discrimination distinguishing residents who in the absence of price discrimination would be LA supported from those who would be self-funding. The former group potentially pay less under price discrimination while the latter pay more.

By definition LA supported residents are on lower incomes and/ or have lower assets that self-funders. With one exception they gain very little from price discrimination. Only those who in the absence of price discrimination, contribute between P^{LA} and P^c benefit from the fee falling from P^c to P^{LA} . Self-funders initially experience an increase in their contribution of the difference between P^c and P^{BE} but since this means they deplete their capital faster,

some of them become LA-supported, even without compensation, and so at the point at which we ‘observe’ them, are paying less than without price discrimination. The exception is where care is free after two years. For this version of the means test, the care home fee has to be apportioned between care costs and so-called ‘hotel’ costs – accommodation, meals etc. We assume roughly £250 per week is for care costs irrespective of the total level of fee. The rest is deemed to be hotel costs and the LA contribution to that part of the fee remains subject to the means test even after 2 years. Under no price discrimination this amount is £447-£250 = £197. Where the amount that is subject to the means test is relatively low, rather more people benefit from a reduction in it. To put it another way, more people will be meeting all of the hotel costs and so stand to gain from any reduction in it. Thus if LAs are able to push the price they pay to £419, the means-tested component for people in care homes for more than two years, falls from £197 to £169 and the average gain to LA supported residents, including those in care home for under 2 years, is £6.50. If the LA negotiates a price as low as £319, the average gain to LA supported residents is £30.30.

Table 3: Mean gain/loss from price discrimination under different means tests, by status under no price discrimination, assuming a non discriminatory fee of £447

	Status if there were no price discrimination		
	LA supported	Self-funding	All residents
	Mean <i>reduction</i> in residents' contribution	Mean <i>increase</i> in residents' contribution	Mean <i>increase</i> in residents' contribution
	£ pw	£ pw	£ pw
Current means test			
$P^{LA}=419, P^{BE}=499$			
LA non inclusion	0.05	45.50	16.10
LA inclusion	0.05	45.50	16.00
$P^{LA}=319, P^{BE}=685$			
LA non inclusion	0.20	192.30	67.80
LA inclusion	0.20	190.40	67.20
No upper capital threshold			
$P^{LA}=419, P^{BE}=512$			
LA non inclusion	1.00	62.20	18.80
LA inclusion	0.70	59.00	19.70
$P^{LA}=319, P^{BE}=746$			
LA non inclusion	2.70	241.90	71.40
LA inclusion	0.60	133.10	39.90
Free care after 2 years			
$P^{LA}=419, P^{BE}=575$			
LA non inclusion	6.50	118.60	15.90
LA inclusion	6.50	118.10	15.80
$P^{LA}=319, P^{BE}=1030$			
LA non inclusion	30.30	531.00	70.20
LA inclusion	30.30	522.60	68.70

Table 4 shows mean increases resulting from price discrimination in residents' contributions according to their location in the distribution of income¹² for the 65+ population. It is clear from this, that price discrimination hits those on highest incomes the most and those on the lowest incomes least. The fact that price discrimination produces increases in residents' contributions in every income quintile indicates that there are self-funders in all parts of the distribution. This is a consequence of including capital – or an assumed income from it – in the means test. The benefits to care home residents of LA inclusion are confined to those in the top 20% of the income distribution except where the capital threshold is removed.

Table 4: Mean gain/loss from price discrimination under different means tests, by income quintile, assuming a non discriminatory fee of £447

	LA supported and self-funders combined					
	Mean increase in residents' contribution by income quintile (£s pw)					
	Q1	Q2	Q3	Q4	Q5	All income levels
	£ pw	£ pw	£ pw	£ pw	£ pw	£ pw
Current means test						
$P^{LA}=419, P^{BE}=499$						
LA non inclusion	12.40	12.70	14.40	19.60	35.40	16.10
LA inclusion	12.40	12.70	14.40	19.60	35.10	16.00
$P^{LA}=319, P^{BE}=685$						
LA non inclusion	51.40	47.70	64.90	87.40	154.20	67.80
LA inclusion	51.40	47.70	64.90	87.40	147.20	67.20
No upper capital threshold						
$P^{LA}=419, P^{BE}=512$						
LA non inclusion	13.30	12.50	18.00	23.10	42.90	18.80
LA inclusion	12.60	11.80	17.11	22.40	41.90	19.70
$P^{LA}=319, P^{BE}=746$						
LA non inclusion	52.20	47.80	63.80	92.20	182.90	71.40
LA inclusion	24.20	21.20	27.00	54.20	146.30	39.90
Free care after 2 years						
$P^{LA}=419, P^{BE}=575$						
LA non inclusion	13.00	12.90	17.00	18.70	28.00	15.90
LA inclusion	13.00	12.90	17.00	18.70	27.10	15.80
$P^{LA}=319, P^{BE}=1030$						
LA non inclusion	58.00	55.90	73.30	82.00	127.20	70.20
LA inclusion	58.00	55.90	73.30	82.00	111.10	68.70

¹² Income is defined as the after tax income of the care home resident and any spouse, before entering a care home. It includes all sources of income except means-tested benefits and disability benefits. It includes actual income from capital rather than the assumed income used in the means test. It does not include any assumed income from housing wealth. A couple's income is divided by an equivalent income scale of 1.5.

4.3 Demand response

The above analysis makes no allowance for a fall in private demand in response to an increase in the private fee rate. Following the model developed in section 3, a fall in private demand would affect the analysis in two ways. It would further reduce the income of care homes and reduce the proportion of residents who were being charged the higher fee rate. It would also imply that the welfare loss experienced by private residents is less than implied by the average increase in fee paid by them, because they would presumably only substitute away from care home places if this increased their welfare. Thus we must regard the welfare losses in table 3 as maximum losses. The scale of potential demand responses depends on the size of the squeezed middle since this is the group who would withdraw their demand. As we have seen, this group is small when there is a capital threshold but larger in the absence of such a threshold.

5 Source of buyer power

We have so far ignored the question of the source of LA buyer power. There is nothing in the model in section 3 which provides a source. This is problematic for this paper's motivation and because we have implicitly assumed that the LA can prevent entry by a care home who refuses to supply the LA. However, observing price differences in many areas, as was reported in the OFT report, indicates that such buyer power must be present. We consider a number of possible explanations below.

Firstly, dealing with an LA saves on transactions costs, especially marketing costs. Because of the nature of the product, the costs of selling a unit to a self-funder could be significant. How substantial depends to an extent on the involvement of third parties in advising the purchaser, such as social workers. If care homes in the main only need to be known to a small number of social workers these costs may not be able to explain the LA discounts.

Secondly, a number of those who are initially self-funded will, through the associated reduction in their assets, become eligible for LA support. By accepting LA supported individuals, the transition from private to LA-assisted can happen within the same care home. This parallels the reasoning in Troyer (2002) for the US. Note however that in the US housing wealth is not included in the means test for Medicaid. US nursing home residents are thus likely to run down their assessable capital assets faster than their counterparts in the UK. Moreover, the prospect of self-funders becoming LA supported residents who attract the

lower LA rate suggests that homes will need a steady flow of new self-funders in order to be able to cross subsidise.

A third explanation arises from the asymmetry in information between self-funders and care homes regarding the quality of care. One could think of the relationship between the buyer and provider of the care home place as being in a principal-agent relationship. As such there are two problems. One is for the buyer to select a high quality care home when this is not directly observable. In this case the number of LA supported places may serve as a signal of quality, partly because one would think that as a repeat purchaser, the LA has better ability to assess quality *ex ante*. The other problem is a moral hazard issue related to the quality of the ongoing stream of services provided. We know that extensive monitoring can solve the moral hazard problem, but in the case of care homes with some private supply, there are multiple principals and hence the usual problem of free-riding in monitoring may arise. The fact that the LA is responsible for several places not only makes it more likely that the LA will monitor, but also that it will observe shortcomings and will act on such observations. Hence both from an adverse selection and from a moral hazard perspective, being an LA provider may work as a kite mark for a care home and hence make it more attractive to a privately funded individual. If this argument holds, both existing care homes and new entrants would be very keen to transact with the LA even if this meant at a lower price.

Finally, while the model is basically static, this is really a dynamic market where at any point in time there is both new demand and places which become available.¹³ Both the rate of new demand and the rate of death can be forecasted but with an element of error. As an implication, an individual care home may at any given time have a number of empty places which may threaten the financial survival of the care home. In such an environment, selling a block of places to the LA provides a security for the care home as it is sure to sell at least that many units¹⁴. This type of take-or-pay arrangements¹⁴ is well known from other areas in the economy where fixed costs are relatively high.¹⁵ This explanation for the willingness to trade with the LA is then based on a form of risk-sharing between the care home and the LA.

¹³ Note that the model in this paper is neither dynamic nor does it allow for demand uncertainty. While the model could be adapted to allow for this, it is not obvious that it would throw up new or different qualitative results.

¹⁴ Although we noted in section 2 that LAs generally procure care home places as spot purchases.

¹⁵ For example oil pipe lines bringing the oil to shore.

6 Conclusion

From the theoretical model derived above and the simulation results we are able to draw a number of conclusions. We can also identify some hypotheses which could be tested empirically in future work.

Using a relatively simple cost and demand representation, we have confirmed the two assertions by the OFT quoted in our introduction. If a Local Authority uses its buyer power, some care homes will be forced out of the market and prices offered to self-funders will increase. We demonstrated that a local authority is able to use buyer power to reduce the price it pays for assisted care home places and where it chooses to do so, this will lead to two types of losers. One group will no longer be able or willing to afford a care home place on the private market while remaining ineligible for LA support. The other group now have to pay an inflated price to keep the care homes financially viable. By using its buyer power, a dominant LA generates an externality on the “middle income” group who may be priced out of the market. The consequence of excluding middle income groups is likely to depend on their numbers. According to our microsimulation model, the size of this squeezed middle is kept small by the current means test, albeit on the rather strong assumption that willingness to pay for a care home place is approximated by liability to pay under this means test. Under alternative forms of the means where this assumption may be more plausible, the squeezed middle could be considerably larger. This is important given the continuing UK debate on how to finance long term care in which various reforms to the means test are being suggested.

One way to address concerns over those priced out of the market is for the LA to include the squeezed middle by procuring places at the lower fee rate for all those who cannot afford the private fee rate. In this case, the immediate losers from the LA using its buyer power are confined to those who remain self-funders. However if the squeezed middle is large, such action may destabilise the market. An empirically testable hypothesis is that LAs in areas with relatively large numbers in the squeezed middle are less likely than those with smaller numbers to exercise buyer power. We would also expect that any reforms to the means test which increase the size of the squeezed middle would lead to a reduction in extent to which LAs can exercise buyer power.

The model also highlighted the importance of (almost) all care homes selling to the LA when the latter is exercising buyer power. The testable implication of this is that the exercise of

buyer power is positively correlated with the LA procuring places widely among the available care homes.

We have identified the exercise of buyer power as a strategic means of increasing what can be purchased with the budget available to a LA. In effect buyer power is being used to tax self-funding care home residents to enable the LA to contribute to the costs of a greater number of residents who cannot meet the costs themselves, than would otherwise be the case. We might therefore expect to observe a positive association between the exercise of buyer power and the number of care home places a LA buys, controlling for the financial and health needs of its population. However we would observe the same association for LAs for whom the central government allocation process underestimates the local need for assisted care home places. If one wanted to discriminate between these two explanations, one would need to be able to identify those LAs which are likely to have greater than predicted needs. If these are predominantly the ones using their buyer power, then the “tax raising” story is not supported. Rather use of buyer power is compensating for imperfections in the formula for setting LAs’ budgets.

Our analysis also shows that the use of buyer power requires the LA to be vigilant against entry of new places, either through expansion of current care homes or the entry of new ones. These have the potential to undermine the pricing structure in the market and hence the ability of the LA to demand lower prices. The existence of such cost makes one wonder about the size of the benefits perceived by the LAs.

We have also been able to establish the scale and distribution of costs to (mainly) self-funding care home residents. These costs are highest for those who are located within the higher parts of the income distribution but exist also for those on lower incomes, because the means tests for care homes take into account assets as well as income. The exercise of buyer power makes rather little difference to what LA-supported care home residents pay towards their care home fees. It enables LAs either to spend more on other services or to support more care home residents. Given that the means test is set nationally, supporting more residents implies relaxing the health-related eligibility criteria rather than the means test. Whether the LA spends more on other services or on supporting residents with lower health needs, one may ask whether self-funding care home residents are the most appropriate group in the population to be financing this extra activity.

That local authorities have buyer power vis-à-vis care homes is evident from the OFT finding that they can negotiate lower prices than those in the private market. What is less obvious is the source of this buyer power and possibly whether it is desirable. For that we need a better understanding of the aims of the local authorities.

Finally one may wonder whether the use of buyer power by the local authorities amounts to abuse under the UK Competition Act 1998. The LA is carrying out commercial activity and is hence very likely an undertaking. Moreover it is at least in some areas also likely to be deemed dominant. The complication is that the LA is purchasing on behalf of consumers and the aim of competition law is protection of consumer welfare. They are clearly maximising the welfare of some of these consumers (and arguably the most vulnerable), but to the disadvantage of others. It is debatable whether that implies that there is no abuse.

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Appendix

Proof of Lemma 1: The demand of the LA from each care home, k , is given by D^{LA}/N^* , which given (3) and (4) we can write as:

$$k = \frac{\beta \cdot P^c \cdot K}{\alpha} = \frac{\beta \cdot P^c}{N^*} \quad (5)$$

For any price agreed for the LA market, P^{LA} , the break even [free entry equilibrium] price for the private market, P^{BE} , is given by the solution to the zero profit condition:

$$(P^{LA} - c) \cdot k + (P^{BE} - c) \cdot \min\left\{\frac{\alpha - \beta \cdot P^{BE}}{N^*}, K - k\right\} - F = 0$$

where we assume that if the demand at P^{BE} is less than the remaining total capacity, $N^* \cdot (K - k)$, the firms share this residual demand equally. Using (5) and the definition of N^* , we can rewrite this condition as

$$(P^{LA} - c) \cdot k + (P^{BE} - c) \cdot \frac{1}{N^*} \cdot \min\{\alpha - \beta \cdot P^{BE}, \alpha - \beta \cdot P^c\} - F = 0$$

Since we are looking for an equilibrium in which the LA uses its market power, we must have $P^{LA} < P^c$ and hence $P^{BE} > P^c$ so that the zero profit condition is:

$$\Pi_i = (P^{LA} - c) \cdot \frac{\beta \cdot P^c}{N^*} + (P^{BE} - c) \cdot \frac{1}{N^*} \cdot (\alpha - \beta \cdot P^{BE}) - F = 0 \quad (6)$$

From (6) it is evident that the total amount of places sold in a proposed equilibrium must fall short of total capacity. The amount sold is given by

$$\beta \cdot P^c + (\alpha - \beta \cdot P^{BE}) = \alpha - \beta \cdot (P^{BE} - P^c) < \alpha$$

An immediate implication is that if an equilibrium with price discrimination and $N = N^*$ exists, it cannot be in pure strategies. At the proposed price each care home has unsold units. If it drops its price slightly below the common price charged by all, it will attract all the private demand and hence be able to expand its sales. Since the drop in price is marginal while the expansion in sales is discrete, the care home must be better off.

Proof of Lemma 2: We require zero excess capacity so that

$$\frac{\alpha - \beta \cdot P^{BE}}{N} = K - \frac{\beta \cdot P^c}{N}$$

which we can rewrite as

$$N = \frac{\alpha - \beta \cdot (P^{BE} - P^c)}{K} \quad (7)$$

Profit of care home i is given by:

$$\Pi_i(P^{BE}) = (P^{LA} - c) \cdot \frac{\beta \cdot P^c}{N} + (P^{BE} - c) \cdot \frac{1}{N} \cdot (\alpha - \beta \cdot P^{BE}) - F$$

Recall from (2) that $F = K(P^c - c)$, using (7) and noticing that both N and c cancel out, profits can be rewritten as

$$N \cdot \Pi_i(P^{BE}) = (P^{BE} - P^c) \cdot (\alpha - \beta \cdot P^{BE}) - (P^c - P^{LA}) \cdot \beta \cdot P^c \quad (8)$$

where the first term is the gain from the private market and the second term is the loss in the LA market. The zero profit condition in (8) can be written as

$$-\beta(P^{BE})^2 + (\alpha + \beta P^c) \cdot P^{BE} - (\alpha + \beta P^c - \beta P^{LA}) \cdot P^c = 0$$

from which we can find P^{BE} as

$$P^{BE} = \frac{\alpha + \beta P^c \pm \sqrt{(\alpha + \beta P^c)^2 - 4\beta(\alpha + \beta P^c - \beta P^{LA}) \cdot P^c}}{2\beta}$$

which we can rewrite as

$$P^{BE} = P^c + \frac{\alpha - \beta P^c \pm \sqrt{(\alpha - \beta P^c)^2 - 4\beta^2(P^c - P^{LA}) \cdot P^c}}{2\beta}$$

Consider the limit as $P^{LA} \rightarrow P^c$. For the “+” solution, $P^{BE} = \frac{\alpha}{\beta}$ while for the “-“ solution

$P^{BE} = P^c$. Hence the latter is the relevant solution and we find:

$$P^{BE} = P^c + \frac{\alpha - \beta P^c - \sqrt{(\alpha - \beta P^c)^2 - 4\beta^2(P^c - P^{LA}) \cdot P^c}}{2\beta}$$

Note that the highest value Π_i can attain is for $P^{BE} = P^c + \frac{\alpha - \beta P^c}{2\beta}$. Using this in the profit

function gives us

$$\Pi_i = \frac{(\alpha + \beta P^c)^2}{4\beta} - (\alpha + \beta P^c - \beta P^{LA}) \cdot P^c$$

This provides a limit on how low P^{LA} can be since profits have to be non-negative. As profits are decreasing in P^{LA} , the lowest admissible value of P^{LA} solves

$$\frac{(\alpha + \beta P^c)^2}{4\beta} - (\alpha + \beta P^c - \beta P^{LA}) \cdot P^c = 0$$

which we can write as

$$\hat{P}^{LA} = P^c - \frac{(\alpha - \beta P^c)^2}{4\beta^2 P^c}$$

For $P^c < \frac{\alpha}{3\beta}$, $\hat{P}^{LA} < 0$, so the requirement used in the lemma becomes:

$$P^{LA} \geq \max \left\{ P^c - \frac{(\alpha - \beta P^c)^2}{4\beta^2 P^c}, 0 \right\}$$

This proves the lemma.

Proof of lemma 3: The break even condition is now given by:

$$N \cdot \Pi_i = (P^{LA} - c) \cdot \frac{\beta \cdot P^{BE}}{N^*} + (P^{BE} - c) \cdot \frac{1}{N^*} \cdot (\alpha - \beta \cdot P^{BE}) - F = 0$$

Recall from (2) that $F = K(P^c - c)$, using that $N^* \cdot K = \alpha$ and noticing that both N^* and c cancel out, profits can be rewritten as:

$$\Pi_i = (P^{BE} - P^c) \cdot \alpha - (P^{BE} - P^{LA}) \cdot \beta \cdot P^{BE} \quad (10)$$

Solving the zero profit condition yields:

$$P^{BE} = \frac{(\alpha + \beta \cdot P^{LA}) \pm \sqrt{(\alpha + \beta \cdot P^{LA})^2 - 4\alpha\beta P^c}}{2\beta}$$

Again looking at the limit as P^{LA} approaches P^c , only the “-“ solution is relevant, i.e.

$$P^{BE} = P^{LA} + \frac{(\alpha - \beta \cdot P^{LA}) - \sqrt{(\alpha - \beta \cdot P^{LA})^2 - 4\alpha\beta(P^c - P^{LA})}}{2\beta}$$

Note that for profits to be non negative for some value of P^{BE} , the expression under the square root sign must be non-negative, which yield the expression in the lemma. This completes the proof.

Proof of Proposition 1: To establish the first part, recall that the highest value P^c can take on is α/β at which price private demand is zero, and note from (5) and (6) that

$$\bar{P}^{LA} \left(\frac{\alpha}{\beta} \right) = \frac{\alpha}{\beta} = \bar{P}^{LA} \left(\frac{\alpha}{\beta} \right), \text{ that } \frac{d\bar{P}^{LA}(P^c)}{dP^c} > 0, \frac{d\bar{P}^{LA}(P^c)}{dP^c} > 0, \frac{d\bar{P}^{LA}(\frac{\alpha}{\beta})}{dP^c} = 1 = \frac{d\bar{P}^{LA}(\frac{\alpha}{\beta})}{dP^c} \text{ and that}$$

both $\bar{P}^{LA}(P^c)$ and $\overline{\overline{P}}^{LA}(P^c)$ are concave in P^c . In other words both $\bar{P}^{LA}(P^c)$ and $\overline{\overline{P}}^{LA}(P^c)$ are concave functions of P^c with identical value and slope at the maximal value of P^c . At

$P^c = \frac{\alpha}{3\beta}$ we have that $\bar{P}^{LA}\left(\frac{\alpha}{3\beta}\right) = 0 < \overline{\overline{P}}^{LA}\left(\frac{\alpha}{3\beta}\right)$. Given the shape of the functions, once

$\overline{\overline{P}}^{LA}(P^c)$ is bigger than $\bar{P}^{LA}(P^c)$, it remains so until it reaches the limit at $P^c = \frac{\alpha}{\beta}$.

To establish the second part, note that for $P^c \leq \frac{\alpha}{4\beta}$, $\bar{P}^{LA}(P^c) = \overline{\overline{P}}^{LA}(P^c) = 0$. To show that

$\bar{P}^{BE}(P^c) \leq \overline{\overline{P}}^{BE}(P^c)$ we need to show that

$$P^c + \frac{\alpha + \beta P^c - \sqrt{(\alpha - \beta P^c)^2 - 4\beta^2 (P^c)^2}}{2\beta} \leq \frac{\alpha - \sqrt{\alpha^2 - 4\alpha\beta P^c}}{2\beta}$$

This can be rewritten as

$$\beta P^c + \sqrt{(\alpha - \beta P^c)^2 - 4\beta^2 (P^c)^2} \geq \sqrt{(\alpha - 2\beta P^c)^2 - 4\beta^2 (P^c)^2}$$

which clearly holds since both terms under the square root are positive and the first is bigger than the second.

For $P^c \geq \frac{\alpha}{3\beta}$, direct comparison of the prices show that $\bar{P}^{BE}(P^c) \geq \overline{\overline{P}}^{BE}(P^c)$ if

$$\frac{\alpha + \beta P^c}{2\beta} \geq \sqrt{\frac{\alpha}{\beta} P^c}$$

Squaring both sides and rearranging yields: $\alpha^2 + 2\beta\alpha P^c + \beta^2 (P^c)^2 \geq 4\beta\alpha P^c$, which clearly holds.

Finally, for the case where $P^c \in \left[\frac{\alpha}{4\beta}, \frac{\alpha}{3\beta}\right]$, $\bar{P}^{BE}(P^c)$ is increasing and convex while $\overline{\overline{P}}^{BE}(P^c)$

is increasing and concave so that these price functions can only cross once from above and once from below. Given the results for the first two intervals, the prices can only cross one in

the interval $\left[\frac{\alpha}{4\beta}, \frac{\alpha}{3\beta}\right]$ and has to do so, which completes the proof.