ISER Working Paper Series

Endogenous Job Contact Networks

Andrea Galeotti
Department of Economics
University of Essex

Luca Paolo Merlino
ECARES
Université Libre de Bruxelles

No. 2010-14
April 2010

INSTITUTE FOR SOCIAL
& ECONOMIC RESEARCH

E·S·R·C
ECONOMIC
& SOCIAL
RESEARCH
COUNCIL

www.iser.essex.ac.uk

Non-Technical Summary

We develop a model where workers, anticipating the risk of becoming unemployed, invest in networking with the view of obtaining information about available jobs that other workers may have. The individuals' incentives to network depend on the labor market conditions and this generates a set of novel empirical predictions. First, we show that networking is more pronounced when the probability of losing a job (job separation rate) is moderate; during a boom or a recession there is not much networking. Second, the probability that a worker finds a new job through his network of contacts is positively associated with the separation rate of the labor market, when the separation rate is low; this relationship becomes negative when the separation rate is high.

We use the UK Quarterly Labor Force Survey (QLFS) over the period from 1994 to 2005 to explore how the proportion of workers who found a job via their network of contacts changes over time and how it depends on the UK job separation rate. In line with our theoretical prediction, we show that social networks are more effective in spreading information about jobs in periods of high separation rate.

Overall, this paper shows that the use of social networks and their effectiveness in alleviating labor market frictions depend on labor market conditions.

Endogenous Job Contact Networks

Andrea Galeotti*

Luca Paolo Merlino†

April 2010.

Abstract

We develop a model where workers, anticipating the possibility of unemployment, invest in connections to access information about available jobs. The investment in connections is high when the job separation rate is moderate, otherwise the investment in connections is low. The response of network investment to labor market conditions generates novel predictions. In particular, the probability that a worker finds a new job via his connections increases in the separation rate, when the separation rate is low, and it decreases otherwise. These predictions are supported by the empirical patterns which we document for the UK labor market.

JEL classifications: A14; J64; J63; D85; E24

Keywords: Unemployment; Job search; Endogenous Job Contact Networks.

*Department of Economics, University of Essex, Colchester, UK. E-mail: agaleo@essex.ac.uk

†ECARES, Université Libre de Brusxelles, 50 Avenue Jeanne CP 114, 1050 Bruxelles, Belgium, email: lmerlino@ulb.ac.be. Merlino acknowledges financial support from the Universitat Autònoma de Barcelona and the Spanish Ministry of Education and Science, through grant SEJ2005-01481/ECON and FEDER.

1 Introduction

It is well established that job contact networks play a prominent role in matching workers with vacant jobs. Empirical work, starting with Rees (1966) and Granovetter (1973), shows that between 30% and 50% of jobs are filled through the use of social networks.[1] This evidence has led to a number of theoretical studies in economics which explore the importance of social networks in labor markets, see, e.g., Arrow and Borzekowski (2004), Boorman (1975), Calvó-Armengol (2004), Calvó-Armengol and Jackson (2004, 2007), Calvó-Armengol and Zenou (2005), Fontaine (2007), Cahuc and Fontaine (2009), Montgomery (1991) and Mortensen and Vishwanath (1994). Most of this work takes the network as given and focuses on its implications for aggregate and individual labor market outcomes.

We develop a model where workers, anticipating the risk of becoming unemployed, invest in connections with the view of accessing information that other workers may have. The individuals' incentives to invest in the network depend on the labor market conditions and this generates a set of novel empirical predictions. First, the investment in the network is high when there is a moderate separation rate in the labor market, whereas it is low when the job separation rate is either high or low. Second, the probability that a worker finds a new job through his network of contacts is positively correlated with the separation rate, when it is low; this correlation becomes negative when the separation rate is high. Using the data from the UK Quarterly Labor Force Survey (QLFS) over the period from 1994 to 2005, we show that the probability that a worker finds a job through his network is positively associated with the job separation rate. Overall, this paper illustrates the importance of considering the possible endogeneity of job contact networks to labor market conditions, and that this endogeneity should be taken seriously in future empirical analysis.[2]

[1]The findings of Granovetter (1973) and Rees (1966) have been generalized across countries, industrial sectors and demographic characteristics, see e.g., Bayer et al. (2008), Blau and Robins (1990), Cingano and Rosolia (2006), Loury (2006), Munshi (2003) and Topa (2001). See Ioannides and Loury (2004) for an exhaustive survey on social networks and labor market.

[2]Most of the empirical work on labor market and social networks does not analyze the impact of labor market conditions on the use and productivity of social networks. Some exceptions are Battu et al. (2008), Topa (2001), Bayer et al. (2008) and Elliott (1999). In particular, Elliott (1999) studied the use of information job search channels by less educated urban workers and found that job seekers are more likely to use their social contacts when they live in high poverty neighborhoods.

In our basic model, workers lose their job with some probability (*separation rate*), while with some other probability they access direct information of new available jobs (*vacancy rate*). The protocol of job contact network formation follows Cabrales et al. (2007): workers choose a level of network investment, which in turn determines the meeting possibilities across pairs of workers. When unemployed Paul meets Helen, he receives information about a vacant job only if Helen is employed and has this information, (i.e., Helen has a *needless offer*), and she has chosen Paul among all her unemployed friends. The incentives to form connections therefore depend on the probability of becoming unemployed and the effectiveness of connections in transmitting information, which are both related to the conditions of the labor market.

We first consider the case where the network is exogenously given. We formally derive the likelihood of a match between a job seeker and an available job resulting from social interaction, which we term *network matching rate*. The network matching rate is described by an urn-ball matching function of the kind traditionally used in search models of the labor market.[3] It depends positively on the average connectivity of the network and it is decreasing in the separation rate.

We then turn to the equilibrium analysis. We show that investment in the network is high when the separation rate is at intermediate levels, whereas it is low when the separation rate is either low or high. This non-monotonic relation between network investment and separation rate leads to a similar relation between network matching rate and separation rate. In particular, for low levels of the separation rate, the network matching rate is increasing in the separation rate. This is in sharp contrast with the predictions obtained in a model where the network of contacts is taken as exogenous to the state of the labor market.

We use the UK Quarterly Labor Force Survey (QLFS) over the period from 1994 to 2005 to explore the empirical relation between the probability of finding a job via the network and the separation rate. In the survey, the respondents who found a job in the last three months were asked

[3]Urn-ball models have been extensively studied in probability theory. In labor market contexts, vacancies play the role of urns, which have to be filled by balls, which are the workers. Each ball reaches an urn, chosen at random. From each urn a ball is extracted and a match between a vacancy and a worker is formed. The randomness in the placement of balls determines coordination frictions. We refer to Mortensen and Pissarides (1999) and Petrongolo and Pissarides (2001) for surveys on the matching processes used in the labor market literature, and to Albrecth et al. (2004) for an extension to the case of multiple applications.

how they found their current position. We then construct a binary variable which takes value one if the respondent reported to have found his new job by "hearing from someone who worked there" and zero otherwise. We regress this variable against the job separation rate that we construct at the regional level. Overall, we find support for a positive and concave effect of separation rate on the probability that a recently employed worker found his current job via his network of contacts.[4]

The basic model assumes that a worker accesses a direct offer with some exogenous probability. In actual practice, the probability that a worker accesses directly information of a vacant job depends also on his investment in searching for job offers. In the second part of the paper we extend the model to allow for these considerations. Marginal returns from personal search depend on the network matching rate which, in turn, depends on the extent to which individuals collect information personally. This interplay leads to multiple equilibria. In one equilibrium, personal search is pervasive and job contact networks are not very connected; the reverse holds in the other equilibrium. Despite the multiplicity of equilibria, our analysis leads to three insights.

First, the network matching rate is constant across equilibria, and groups with lower costs of acquiring information personally experience a higher network matching rate. This prediction is consistent with the findings of Battu et al. (2008) that immigrant groups with a higher level of country assimilation (a proxy for higher hosting country language proficiency) experience a higher network matching rate. Second, the equilibrium with high personal search Pareto dominates the equilibrium with low personal search, and it displays a lower unemployment rate. So, two distinctive groups, facing identical labor market conditions, may experience very different unemployment rates depending on the equilibrium on which they coordinate. Finally we show that, regardless of the equilibrium selected, an increase in the job separation rate increases the network matching rate, thereby confirming our main predictions derived in the benchmark model.

Most of the literature on labor market and social networks assumes that the use of job contact networks is exogenous to labor market conditions. Notable exceptions are Boorman (1975) and Calvó-Armengol (2004).[5] Boorman (1975) is the first to provide a model that integrates social

[4]Subsection 3.3 provides a detailed account of the data, of the derivation of variables and different robustness checks.

[5]The theory of network formation is a recent but very active field of study. For a survey of this research, see Goyal (2007) and Jackson (2008). Complex random networks have also received large attention in economics, e.g., Cabrales,

networks with labor markets and his focus is on workers' incentives to form weak *versus* strong ties. Calvó-Armengol (2004) provides a characterization of stable job contact networks in a two-sided link formation model of the type introduced by Jackson and Wolinsky (1996). Our model is complementary to this earlier work. We do not focus on the specific details of the architecture of equilibrium job contact networks because our aim is to explore how labor market conditions affect the use of social networks and their effectiveness in matching job seekers with vacancies. To the best of our knowledge, our paper is the first to analyze these questions systematically.

Our work is also related to the labor market literature with endogenous search effort pioneered by Diamond (1982). For recent surveys see Mortensen and Pissarides (1999) and Pissarides (2000). We borrow from this literature the idea that search units are chosen optimally to maximize the net returns from search and we apply it to the formation of job contact networks. Our extension on Section 5 combines informal search in networks with individual search, which has been the focus of this literature. In our conclusion we elaborate on how to embed our framework in a dynamic labor search model.

The rest of the paper is organized as follows: Section 2 develops the basic model; Section 3 contains the main results; Section 4 extends the basic model to allow for individual search of job offers; Section 5 concludes. All proofs are relegated in Appendix A. Appendix B and Appendix C show that the main results of the paper are robust to indirect information flow in the network and to endogenous vacancy rate.

2 Model

The model has three building blocks: labor market turnover, information diffusion within the network and the formation of job contact networks. There is a large set of risk neutral workers $\mathcal{N} = \{1, ..., n\}$. Initially all workers are employed and earn an exogenous wage that, without loss of generality, is normalized to 1.

Labor Market Turnover. Two exogenous parameters govern the labor market turnover.

Separation rate. A randomly selected sample of size $B = bn > 1$ of workers become unemployed,

Calvó-Armengol and Zenou (2007) and Galeotti et al. (2009); for a survey see Vega-Redondo (2007).

where $b \in (0,1) \cap \mathbb{Q}$. We denote by $\mathcal{B} \subset \mathcal{N}$ the set of workers who lose their job and we call them *job seekers*.

Job opening. A number $V = an$ of new vacancies opens in the market, where $a \in (0,1] \cap \mathbb{Q}$. These vacant jobs are distributed to workers in the following way: bV vacancies reach a randomly selected sample of job seekers and the remaining vacancies reach a randomly selected sample of workers who did not lose their job. Let $\mathcal{A} \subset \mathcal{N}$ be the set of workers who receive a direct job offer.

Under this protocol, nobody receives more than one direct job offer. The set $\mathcal{U} = \mathcal{B} \cap \{\mathcal{N} \setminus \mathcal{A}\}$ is the set of workers who have lost their job and did not receive a direct offer; note that $|\mathcal{U}| = b(1-a)n$. The set $\mathcal{O} = \mathcal{A} \cap \{\mathcal{N} \setminus \mathcal{B}\}$ contains workers who have not lost their job and received a direct offer. We say that a worker $i \in \mathcal{O}$ has a *needless offer* and we note that there are $|\mathcal{O}| = a(1-b)n$ needless offers.

Ex-ante, a representative worker anticipates that with probability $b(1-a)$ he will be unemployed and without a new offer. In that case, he earns an unemployment benefit which, without loss of generality, is normalized to 0. In order to insure themselves against this risk, workers invest in social connections with the view of accessing needless offers.

Job Contact Network. We specify the network formation game below. For the moment, let us assume that workers are located in an undirected network g. A link between workers i and j is denoted by $g_{ij} = 1$, while $g_{ij} = 0$ means that i and j are not linked. The set of all possible undirected networks is $\mathcal{G}$. With some abuse of notation, we denote the set of i's neighbors belonging to $\mathcal{V} \subset \mathcal{N}$ in network g as $\mathcal{N}_i(\mathcal{V}) = \{j \in \mathcal{V} \setminus \{i\} : g_{ij} = 1\}$; $\eta_i(\mathcal{V}) = |\mathcal{N}_i(\mathcal{V})|$ is the number of links that i has with workers belonging to $\mathcal{V}$.

Job transmission in the network. We assume that information about jobs flows only from workers with a needless offer to job seekers. Formally, each $i \in \mathcal{O}$ passes the information to one and only one $j \in \mathcal{N}_i(\mathcal{B})$, chosen at random. If i is not linked to any job seeker, i.e., $\mathcal{N}_i(\mathcal{B})$ is the empty set, the offer is lost.[6] We are assuming that information only flows one-step in the network. In

[6]We are assuming that a worker passes a needless offer to one of his social contacts, chosen at random. The implication of this assumption is that two job seekers, both connected to a worker with a needless offer, "compete" for such offer. If a worker with a needless offer is allowed to give it to both job seekers, then they will have an identical offer and therefore competition for the job will still be present in the hiring process (given that one job offer corresponds to one vacancy).

actual practice, we may receive information from social contacts, which in turn they have received from their acquaintances, e.g., Granovetter (1973, 1974). Indirect information flow can be easily accommodated in our framework, as we show in Appendix B.

Formation of job contact networks. The protocol of network formation follows Cabrales et al. (2007). We consider the following simultaneous network formation game. Each worker i chooses a costly network investment $s_i \geq 0$; the cost of investment s_i is cs_i.[7] The set of pure strategies available to worker i is $\mathcal{S}_i = \mathbb{R}_+$. A pure strategy profile is $\mathbf{s} = (s_1, ..., s_n) \in \mathcal{S} = \mathbb{R}_+^n$, and $\mathbf{s}_{-i}$ indicates the strategies of all workers other than worker i. We denote by $y(\mathbf{s}) = \sum_{i \in \mathcal{N}} s_i$ the aggregate workers' network investment. For a profile $\mathbf{s}$, we assume that a link between an arbitrary pair of workers i and j forms with probability[8]

$$\Pr(g_{ij} = 1|\mathbf{s}) = \begin{cases} \min\left\{\frac{s_i s_j}{y(\mathbf{s})}, 1\right\} & \text{if } y(\mathbf{s}) > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

A profile $\mathbf{s}$ generates a multinomial random graph. When workers choose the same level of investment, say s, the induced random graph is binomial, the probability that two workers are connected equals the per-capita network investment, $\min\{s/n, 1\}$, and the average connectivity of the random graph (the expected number of neighbors of a node) is $\min\{s/n, 1\}(n-1)$.[9]

There are two complementary interpretations of this process of network formation. The first interpretation is that network investment reflects the time that an individual spends in organizations, clubs, conferences, churches etc. An individual who participates in many organizations has greater chances to meet other people and form connections with them. In turn, these connections may provide valuable information about job opportunities. The second interpretation is that individuals are connected in a pre-existing network. For example, we may think of a group of immigrants living in the same neighborhood and define the pre-existing network by geographical proximity. This is

[7] All the results we present can be derived for arbitrary cost functions $C(s)$ which are increasing and convex in s.

[8] Expression (1) can be derived by requiring three axioms on network formation. These axioms are: *one*, undirected links, i.e., $\Pr(g_{ij} = 1|\mathbf{s}) = \Pr(g_{ji} = 1|\mathbf{s})$; *two*, aggregate constant returns to scale, i.e., for all $i \in \mathcal{N}$, $\sum_{j=1}^{n} \Pr(g_{ij} = 1|\mathbf{s}) = s_i$; and *three*, anonymous link formation, i.e., for all $j, l \in \mathcal{N}$, $\Pr(g_{ji} = 1|\mathbf{s})/s_j = \Pr(g_{li} = 1|\mathbf{s})/s_l$, for all $i \in \mathcal{N} \setminus \{j, l\}$. See Cabrales et al. (2007) for details.

[9] We refer to Erdos and Reny (1959) for a study of binomial random graphs, and to Chung and Lu (2002) for a study of multinomial random graphs. Vega-Redondo (2007) and Jackson (2008) provide a detailed overview of the rapidly growing literature on complex networks.

the approach of Topa (2001), which finds a significantly positive amount of social interactions across neighborhoods. While the existence of such a link (e.g., living in closed proximity) is a necessary condition for information exchange, it is not sufficient: for the information to flow from one individual to another their communication links must be active, which requires investment from both workers. In this case, workers' network investment determines the strength of each pre-existing link in the community.

Utilities and Equilibrium. For a strategy profile $\mathbf{s} = (s_i, \mathbf{s}_{-i})$, let $\Psi_i(\mathbf{s})$ be the probability that worker $i \in \mathcal{B}$ accesses at least one offer from the network, which we shall refer as to i's network matching rate and that we will derive in the next section. The expected utility to a worker $i \in \mathcal{N}$ is:

$$EU_i(s_i, \mathbf{s}_{-i}) = 1 - b(1-a)[1 - \Psi_i(\mathbf{s})] - cs_i.$$

The last term represents the cost of investment in the network and the first part is the probability that worker i will be employed and therefore earning a wage 1. This is the complement of the probability that worker i is a job seeker and neither he accesses a direct offer nor he accesses information from the network. A pure strategy equilibrium is $\mathbf{s}$ such that, for all $i \in \mathcal{N}$,

$$EU_i(s_i, \mathbf{s}_{-i}) \geq EU_i(s_i', \mathbf{s}_{-i}), \forall s_i' \in \mathcal{S}_i.$$

We focus on pure strategy symmetric equilibrium in large labor markets, hereafter equilibrium. A large labor market is a labor market in which $n \to \infty$. Note that, by definition, $B/n = b$ and $V/n = a$.[10]

3 Analysis

We first consider the case where the job contact network is given and we explicitly derive the network matching rate. We then turn to our equilibrium analysis. We provide new predictions on how network investment and network matching rate depend on separation rate. We finally use the UK Quarterly

[10]For sake of clarity, we present the analyses for large labor markets. However, our results can be easily extended to an environment with finite and sufficiently large n.

Labor Survey (QLFS) to explore the empirical relation between the probability of a worker to find a job via his network of contacts and the separation rate.

3.1 Exogenous Job Contact Networks

We focus on networks generated by a symmetric profile of network investment, i.e., $s_i = s$ for all $i \in \mathcal{N}$. We recall that a symmetric profile of investments generates a binomial random graph, which in a large labor market coincides with a Poisson random graph.

Since all offers are identical, the probability that an arbitrary job seeker $i \in \mathcal{B}$ finds a job–the *matching rate*– is given by $m(s) = a + (1-a)\Psi(s)$, where we recall that $\Psi(s)$ is the network matching rate, which we now derive.

The probability that $i \in \mathcal{B}$ has η links with workers who have a needless offer is $\Pr(\eta_i(\mathcal{O}) = \eta) = B(\eta|p, |\mathcal{O}|)$, where $B(\cdot|p, |\mathcal{O}|)$ is the binomial distribution, $p = s/n$ is the probability of a success and $|\mathcal{O}|$ is the number of trials. Let j be an arbitrary i's neighbor with a needless offer, i.e., $j \in \mathcal{N}_i(\mathcal{O})$. The probability that j is connected to ω job seekers (given that he is already linked to i) is simply:

$$\Pr(\eta_j(\mathcal{B}) = \omega | g_{ij} = 1) = B(\eta - 1|p, |\mathcal{B}| - 1),$$

and the probability that j passes the job information to i is $1/\omega$. So, the expected probability that the connection to j results in a job information to i is:

$$\sum_{\omega=1}^{|\mathcal{B}|} \Pr(\eta_j(\mathcal{B})) = \omega | g_{ij} = 1)\frac{1}{\omega}.$$

We now observe that the probability that each i's neighbor with a needless offer passes the information to i is independent. Hence, if worker i has η links with workers like j, the probability that i does not hear about a new job is:

$$\left[1 - \sum_{\omega=1}^{|\mathcal{B}|} \Pr(\eta_j(\mathcal{B}) = \omega | g_{ij} = 1)\frac{1}{\omega}\right]^{\eta},$$

and the expected probability that $i \in \mathcal{B}$ does not get an offer via the network is:

$$\phi(s) = \sum_{\eta=0}^{|\mathcal{O}|} \left\{ \Pr(\eta_i(\mathcal{O}) = \eta) \left[1 - \sum_{\omega=1}^{|\mathcal{B}|} \Pr(\eta_j(|\mathcal{B}|) = \omega | g_{ij} = 1) \frac{1}{\omega} \right]^{\eta} \right\}. \quad (2)$$

Using $|\mathcal{B}| = bn$ and $|\mathcal{O}| = na(1-b)$ and the binomial identity, we can rewrite (2) as follows:

$$\phi(s) = \left[1 - \frac{1-(1-p)^{nb}}{bn} \right]^{na(1-b)}. \quad (3)$$

Since $p = s/n$, in a large labor market we obtain that the network matching rate is

$$\Psi(s) = 1 - \lim_{n \to \infty} \phi(s) = 1 - e^{-\frac{a(1-b)}{b}(1-e^{-sb})}. \quad (4)$$

Let's denote unemployment rate by $u(s) = b(1-a)[1-\Psi(s)]$. We note that the network matching rate coincides with the proportional decrease in unemployment rate relative to a situation in which there are not connections among workers, i.e., $\Psi(s) = [u(0) - u(s)]/u(0)$. Hence, the network matching rate describes the extent to which network information transmission alleviates labor market frictions.

Proposition 1 *Consider a large labor market and suppose that $s_i = s$ for all $i \in \mathcal{N}$. The matching rate and the network matching rate are decreasing in separation rate, whereas unemployment rate is increasing in separation rate.*[11]

To understand how network information transmission alleviates labor market frictions and determines the level of unemployment rate, it is useful to start considering the extreme case in which the social network is complete, i.e., $s \to \infty$. In a complete network the network matching rate is described by a standard urn-ball matching function of the kind traditionally employed in search models of the labor market, i.e., $1 - e^{a(1-b)/b}$. In our context, each job seeker is an urn and every employed worker with a needless offer places a ball (the job offer) in one randomly selected urn. Not all needless offers result in a match because some job seekers will receive multiple offers, while others none. These frictions depend negatively on the balls-urns ratio, which is the ratio between the proportion of needless offers–*the job network supply*, i.e, $a(1-b)$–and the proportion of job seekers–*the*

[11]It is easy to show that a decrease in vacancy rate has the same effects of an increase in separation rate

job network demand, i.e., b.

The network matching rate is lower when the network is not complete because, with positive probability, an employed worker with a needless offer only connects with other employed workers, a fact which prevents information to flow.[12] In particular, the probability that a worker with a needless offer has only employed friends is $(1 - s/n)^{nb}$. When n goes to infinity this converges to e^{-sb}, where sb is the average number of links that a worker has with job seekers. Thus, the higher is workers' average connectivity with job seekers, the higher is the network matching rate.

Overall, Proposition 1 shows that network matching rate is negatively correlated with separation rate. This prediction is obtained in all existing models with exogenous social networks. Indeed, for a given network of contacts, changes in labor market conditions which decrease job network supply relatively to job network demand will unambiguously decrease the effectiveness of the network in matching jobs with workers.

3.2 Endogenous Job Contact Networks

We now study the implication of allowing individuals to choose strategically how much to invest in the network. The following proposition characterizes interior equilibria.[13]

Proposition 2 *Consider a large labor market. An interior equilibrium s^* exists if and only if $c < ab(1-a)(1-b)$, and s^* is the unique solution to:*

$$b(1-a)\left[\frac{a(1-b)}{bs^*}\left(1-e^{-s^*b}\right)e^{-\frac{a(1-b)}{b}\left(1-e^{-s^*b}\right)}\right] = c \qquad (5)$$

In the unique interior equilibrium, the level of network investment balances worker's marginal returns with marginal costs. The marginal returns are the marginal increase in network matching

[12] Note that this inefficiency is not an artifact of our restriction on one-step information flow. Suppose, for example, that information flow two steps. Then a needless offer is lost whenever the owner of this offer is isolated and, otherwise, when he passes the information to an employed worker, who in turn is only connected with other employed workers. We develop the model for the case of indirect information flow in the Appendix B.

[13] We note that there always exists an equilibrium where workers do not invest in the network, i.e., $s_i = 0$ for all $i \in \mathcal{N}$. However, analyzing the best response dynamics after a perturbation around the equilibrium, it is possible to show that this equilibrium is unstable whenever it coexists with an interior equilibrium. Moreover, when $c \geq ab(1-a)(1-b)$ the equilibrium in which workers do not invest in the network is the only symmetric equilibrium. A formal proof of this statement is available upon request to the authors.

rate (the term in square bracket), weighted by the likelihood that the worker needs the network to find a job, $b(1-a)$.[14]

We now investigate how the separation rate shapes the investment in job contact networks and the network matching rate.[15]

Proposition 3 *Consider a large labor market and suppose that $c < ab(1-a)(1-b)$.*

1. *For every $a \in (0,1)$, there exists $\bar{b}(a) > 0$ such that if $b < \bar{b}(a)$, then the network investment increases in the separation rate, otherwise it decreases in the separation rate.*

2. *For every $a \in (0,1)$, there exists $\hat{b}(a) \geq \tilde{b}(a) > 0$ such that if $b < \hat{b}(a)$, then the network matching rate increases in the separation rate, while if $b > \tilde{b}(a)$ it decreases in the separation rate.*

The first part of Proposition 3 shows that the investment in the network is low when the separation rate is either low or high whereas, when the separation rate is at intermediate levels, workers invest heavily in the network. Intuitively, when the separation rate is low, there is not much value in investing in connections because the risk of unemployment is low. When the separation rate is high, the value of investing in connections is also low because most of the links will be formed with other job seekers and there will be high competition for needless offers. So, the use of the network will be more pronounced when the economy is neither in a boom nor in a recession.

Furthermore, from the theory of random graphs we know that, in a Poisson graph, when the average connectivity is lower than 1, the resulting network comprises different segregated components. Otherwise, the majority of workers are, almost surely, connected directly or indirectly (the network has a giant component).[16] Here, the average connectivity of the network is s^* and Proposition 3 suggests that a giant component will emerge when the level of separation rate is in intermediate

[14] It is possible to show that the equilibrium level of network investment is higher than the level of investment that would be chosen by a social planner who aims at maximizing the expected utility of a randomly selected worker. The formal result is available upon request to the authors.

[15] We focus on separation rate because we can recover it from the data and, therefore, test the predictions of the model. We note, however, that the effects of an increase in the separation rate on investment in the network and network matching rate mimic the effects induced by a decrease in the arrival rate of offers.

[16] We refer to Vega-Redondo (2007) and Jackson (2008) for a formal treatment of random graphs.

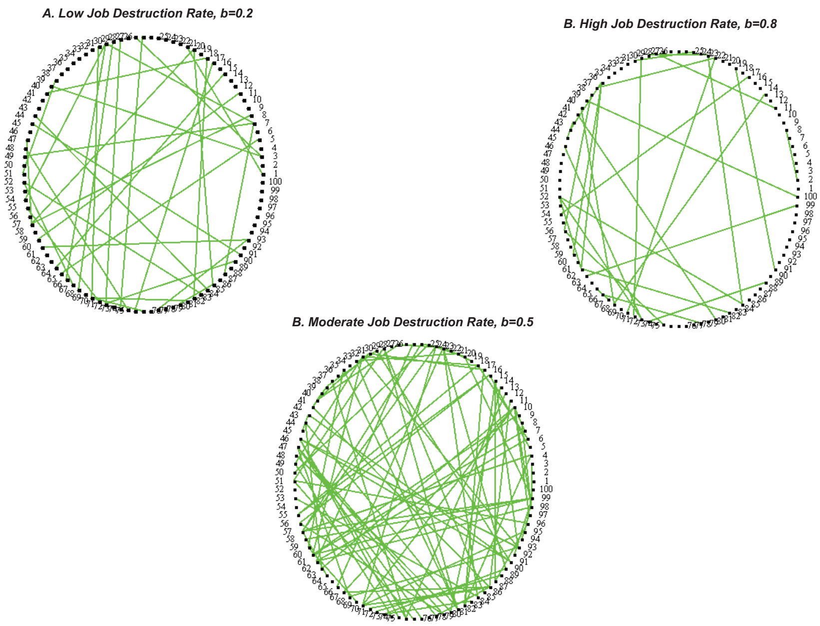

Figure 1: Equilibrium job contact networks and separation rate, when $a = 0.4$, $c = 0.03$, and $n = 100$.

situations. Figure 1 illustrates these findings. Panel A and panel B of figure 1 depict a realization of the equilibrium (random) network for low and high job separation rates, respectively, while panel C of figure 1 depicts a realization of the equilibrium network in a labor market with moderate separation rate.

The non-monotonic relation between network investment and separation rate leads to a similar relation between network matching rate and separation rate. When the separation rate is low, job contact networks are sparse, which implies that the network matching rate is low. In this case, an increase in the separation rate increases the connectivity of the network and, consequently, the probability of a finding a job via the network also increases. However, as the separation rate increases further, the network becomes crowded with job seekers. This creates a congestion effect which eventually reduces the network matching rate. Figure 2 summarizes the comparative statics of the network matching rate with respect to the separation rate. In the figure, we have fixed vacancy rate to $a = 0.2$ and costs of network investment to $c = 0.01$. For different values of separation rate

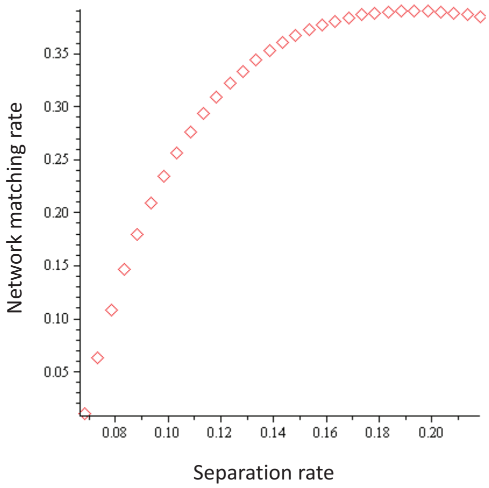

Figure 2: Network Matching Rate and Separation Rate $a = 0.2$, $c = 0.01$, $b \in (0, 0.25]$.

$b \in (0, 0.25]$, we have derived the equilibrium level of network investment and the resulting network matching rate. We remark that this positive relation between network matching rate and separation rate is in sharp contrast from the relation that is obtained in a model where the network is taken as exogenous (see Proposition 1).

3.3 UK Empirical Patterns

We use the UK Quarterly Labor Force Survey (QLFS) to explore the relation between the probability of finding a job via social networks and the job separation rate. We use data from the third quarter of 1994 to the first quarter of 2005. Each wave of the QLFS covers around 60,000 households incorporating from 150,000 to 125,000 individuals, depending on the wave; providing a sample that is representative of the UK population. Only males of working age (aged 16 to 64) resident in Great Britain are considered in our analysis. We exclude Northern Ireland from the sample because of political instability of the region, which we fear could influence the use of social networks. Overall, we are left with about 30-45,000 individuals per wave.

The main explanatory variable that we use is the job separation rate at a regional level, which has been derived as follows.[17] First, we have constructed unemployment rate using the ILO definition: the number of unemployed workers looking for a job or waiting to start a job in the next two weeks over the whole active population. We obtained an average unemployment over the period of 6.9%, ranging from a minimum of 4.75% to a maximum of 10.86%. Then, we have calculated the job separation rate (i.e., the probability of transition from employment to unemployment) using equation (5) of Shimer (2007). This method takes into account the problem that, while data are available only at discrete date, the underling environment is continuous in time. We obtained an average job separation rate over the period of 2.9%, ranging from a minimum of 2.3% to a maximum of 3.7%, which is consistent with earlier work.[18]

In our estimations we focus on job seekers: at every quarter we consider respondents who, at the previous quarter, reported that they were actively looking for work or a place on a government scheme. Among the job seekers, the ones who found a job (in the last three months) were asked how they found their current position. In particular, the respondents were given a list of available options from which they could choose only one of them. The available options were: replying to a job advertisement; job centre; careers office; job club; private employment agency or business; direct application; hearing from someone who worked there; some other way.[19] We use this question to construct three dependent variables and we regress them against the job separation rate and its squared value, both actual and lagged of one period. For all dependent variables we estimate both a linear panel model and a conditional logit model.[20] In all regressions we include dummies to control for time aggregation, and we include individual fixed effects to control for unobserved heterogeneity

[17]We divided our sample into 19 regions which are defined in the survey design using the information about the respondents usual residence, so that for all the respondents living in the same region, the regional independent variables have the same value. Smaller regions could be used, but this would decrease the precision when calculating the local unemployment.

[18]Another variable that is important, according to our model, is the vacancy rate but a reliable measure of this in UK exists only since 2003. If we restrict attention to the sample from 2003, and we replicate the analysis below introducing (lagged) regional arrival rate of offers, the results are similar but the vacancy rate is not significant, although it has the expected positive sign. Obviously, sample size gets significantly reduced.

[19]There has been some changes in this question throughout the history of the QLFS. From the second quarter of 2005, this question is addressed to everybody who found a new job in the last twelve months or less. For these reasons, we excluded all waves after 2005:Q1 from our sample. Furthermore, the answers of the second quarter of 1992 to some questions we used are clearly miscoded. For this reason, we excluded all waves before 1994:Q3 from our sample.

[20]The standard errors reported for the estimation of the linear models are calculated by bootstrapping.

and regional effects.

Our first dependent variable takes value one if the job seeker reported to have found his new job by "hearing from someone who worked there", and zero otherwise. The results are reported in the first two columns of Table 1. We find that the separation rate has a significant and positive effect on the probability to find a job via a friend and that the coefficient of the square of the lagged separation rate is significant and negative.[21]

The dependent variable used in the first two columns of Table 1 pooled together all job seekers who found a job via a different channels than the network and job seekers who did not find a job. We then investigate how the separation rate influences the probability of finding a job through the social network versus alternative search channels. First, we construct a dependent variable that takes value 0 if the job seeker didn't find a job and 1 if the job was found via the social network. The results for the estimation are reported in the third and fourth column of Table 1 and confirm the results obtained in the first two columns. Finally, we define another dependent variable that takes value 0 if the respondent didn't find a job and 1 if the job was found via a method different from the network. The results are presented in the last two columns of Table 1. In this case, the coefficients of the separation rate are never significant.

Overall, our analysis suggests that the separation rate is positively associated with the probability that a worker finds a job via his network of contacts, but there is no systematic association between job separation rate and other search channels.

4 Extension: job search and job contact networks

The basic model assumes that a worker accesses a direct job offer with some exogenous probability. In actual practice, the probability that a worker accesses direct information of a vacant job depends also on his search activity.[22] In this section we examine how workers' incentives to search for jobs

[21] We repeated the exercise excluding London from our sample. Indeed, London labor market displays different properties with respect to the rest of the UK, as suggested by Petrongolo and Pissarides (2005), who find that, differently than in the rest of the UK, the matching technology in London seems to display increasing returns to scale. Nonetheless, we obtained qualitatively similar results. We repeated the exercise also restricting our attention to each region in isolation, and again we obtained qualitatively similar results for the majority of cases.

[22] Workers generally use different search methods in their attempt to obtain information about jobs, see e.g. Holzer (1987), Osberg (1993), Elliott (1999) and Wabha and Zenou (2005).

Table 1: The probability that a job seeker finds a job through social networks and through other channels regressed against the job separation rate.

	Network Channel 1		Network Channel 2		Other Channels	
	Linear Model[a]	Logit Model[b]	Linear Model[a]	Logit Model[b]	Linear Model[a]	Logit Model[b]
b_t	0.907*	48.77**	0.783	40.13	0.0168	1.270
	(0.521)	(19.89)	(0.612)	(24.46)	(0.707)	(11.12)
b_t^2	-10.94	-572.6*	-9.379	-472.3	-0.474	-41.20
	(7.556)	(312.5)	(8.831)	(361.2)	(8.453)	(138.6)
b_{t-1}	0.880**	49.23***	0.848*	43.66**	-0.246	-1.291
	(0.415)	(17.30)	(0.485)	(17.98)	(0.843)	(14.45)
b_{t-1}^2	-11.97**	-642.0***	-12.51**	-628.7***	4.231	12.83
	(5.437)	(223.6)	(6.351)	(234.6)	(12.44)	(211.8)
$Constant$	-0.199***	-	-0.225***	-	-0.410***	-
	(0.0164)		(0.0192)		(0.0295)	
R^2	0.007	-	0.009	-	0.018	-
$Ps.R^2$	-	0.0854	-	0.105	-	0.0879
Observations	146868	7749	130161	7055	140669	18563
Number of id	83163	2971	74951	2750	80152	7162

Source: UK QLFS, male respondents looking for a job in the previous quarter, aged 16-64, residents in Great Britain, waves from 1994:Q3 to 2005:Q1. Note: Fixed effects and quarterly time dummies included.

[a] Standard errors bootstrapped according to regional clustering in parenthesis.

[b] Robust standard errors adjusted for regional clustering in parenthesis.

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

interact with their incentives to invest in their network of contacts.[23] We shall see that the interaction between search effort and network investment generates multiple equilibria. These equilibria are Pareto ranked, and, regardless of the equilibrium selected, an increase in the separation rate increases the network matching rate.[24]

The formation of the network and the information diffusion process in the network follow the rules described in the basic model. However, each worker i chooses a costly search effort $a_i \in [0,1]$, together with a network investment s_i. If worker i exerts search effort a_i, he accesses a direct offer with probability a_i and incurs a cost αa_i, $\alpha > 0$. A strategy profile is now a profile of network investments $\mathbf{s}$ and a profile of search efforts $\mathbf{a} \in [0,1]^n$.

Given a profile $(\mathbf{s}, \mathbf{a})$, the probability that worker i accesses at least one information via the network is $\Psi_i(\mathbf{s}, \mathbf{a})$, and when $s_j = s$ and $a_j = a$ for all $j \in \mathcal{N}$, then $\Psi(s,a)$ is the same as the network matching rate derived in Section 3, i.e., $\Psi(s,a)$ is given by expression (4). The expected utility to worker i under a profile $(\mathbf{s}, \mathbf{a})$ is

$$EU_i(\mathbf{s}, \mathbf{a}) = 1 - b(1-a_i)[1 - \Psi_i(s_i, s, \mathbf{a}_{-i})] - \alpha a_i - c s_i.$$

The following proposition characterizes interior equilibria. We shall restrict attention to the case in which $\alpha < b$, where we recall that b is the separation rate.[25]

Proposition 4 *Consider a large labor market and suppose $\alpha < b$. For every $b \in [0,1]$, there exists $\alpha(b) > 0$ and $c(b,\alpha) > 0$ such that an interior equilibrium exists if and only if $\alpha \leq \alpha(b)$ and $c \leq c(b,\alpha)$. An interior equilibrium (s^*, a^*) solves*

$$b[1 - \Psi(s^*, s^*, a^*)] = \alpha \tag{6}$$

$$b(1-a)\left[\frac{a(1-b)}{bs^*}\left(1 - e^{-s^* b}\right) e^{-\frac{a(1-b)}{b}\left(1 - e^{-s^* b}\right)}\right] = c \tag{7}$$

[23] In a similar spirit, in Appendix C we extend our basic model by endogenizing the number of vacant jobs that firms are willing to open.

[24] Few papers study the interplay between choice of efforts and the formation of networks under local spillovers, e.g., Cabrales et al (2007), Galeotti and Goyal (2009).

[25] Two remarks are in order. First, if $\alpha > b$ there exists a unique equilibrium: workers do not search and do not invest in the network. Second, when $\alpha < b$ there exists only one corner equilibrium: $s_i = 0$ and $a_i = 1$, for all $i \in \mathcal{N}$.

When $c = c(b, \alpha)$ there exists only one interior equilibrium, otherwise there exist two interior equilibria, (s^H, a^L) and (s^L, a^H), where $s^H > s^L$ and $a^H > a^L$.

Equilibrium condition (6) equates the marginal cost of search effort with its marginal returns. Marginal returns from searching decrease with the network matching rate. Hence, effort to collect information personally and network investment are strategic substitutes. Equilibrium condition (7) is analogous to equilibrium condition (5). The strategic relation between individual search effort and network investment leads to multiple equilibria. Equilibrium (s^H, a^L) describes a labor market where workers search personally with low intensity while the social network is very dense. Low search effort can be justified only if the network matching rate is sufficiently high. Since low search effort induces a low level of job network supply (i.e., few needless offers), this is attainable only if job contact networks are very dense. The reverse holds in the equilibrium (s^L, a^H). Here, workers are highly engaged in collecting information personally, which is consistent with equilibrium only if the likelihood of forming connections is low, i.e., the network investment is low.

Despite the multiplicity of equilibria, the analysis reveals three insights. First, equilibrium condition (6) in Proposition 4 implies that the network matching rate is constant across the two interior equilibria and it decreases with the costs of collecting information personally. So, our model predicts that groups with lower costs of collecting information personally have also higher network matching rate. This is in line with the empirical findings of Battu et al. (2008), which show that ethnic minority groups with a higher level of country assimilation (which could be related to lower language proficiency or lower knowledge of the functioning of local labor markets) are more likely to find jobs via their social networks. Second, the equilibrium with high individual search activity and small network investment has a lower unemployment rate and Pareto dominates the equilibrium with low individual search and dense networks. So, two groups of individuals facing identical labor market conditions may experience very different labor market outcomes depending on the equilibrium on which they coordinate.[26] Finally, an increase in separation rate increases network matching rate. So,

[26]Trejo (1997) found that in the US, taking into account individual characteristics, one can explain only one third of the wage gap for Blacks and three quarters for Hispanics. Blackaby et al. (1998) reports similar findings for ethnic minorities in the UK. As for network usage across groups, Holzer (1987), using US data, finds that, while black and whites do not differ significantly in network usage (21.3% and 23.9% respectively), Hispanics rely much more on social

the main correlation predicted in our basic model is robust to this extension. The following corollary summarizes these considerations.

Corollary 1 *Consider the interior equilibria* (s^H, a^L) *and* (s^L, a^H) *described in Proposition 4. Then, (1)* (s^L, a^H) *Pareto dominates* (s^H, a^L)*; (2) the unemployment rate under* (s^L, a^H) *is lower than the unemployment rate under* (s^H, a^L)*; (3) the network matching rate is the same in the two equilibria and it is increasing in the job destruction rate.*

5 Conclusion

This paper documents a positive empirical relation between the job separation rate and the probability that a worker finds a job through his network of contacts. This pattern is not consistent with the prediction of the existing models of social networks and labor market, which assume that the job contact network is exogenous. We explore how labor market conditions affect the formation of job contact networks and how this interplay shapes labor market outcomes. We have shown that taking into account the endogeneity of job contact networks leads to predictions which are in line with the documented empirical patterns.

We have focused on labor markets but the question of how the state of the economy shapes informal institutions is much broader. For example, there is a large amount of empirical work on the effects of social capital on economic growth. Often this work struggles with the fact that social capital is itself an endogenous variable, raising concerns of identification of the models. Referring to this literature, Durlauf (2002, p. F474) noted: "...it seems clear that researchers need to provide explicit models of the codetermination of individual outcomes and social capital, so that the identification problems (...) may be rigorously assessed". Our paper aims to contribute to this line of research by providing a tractable model where the interplay among aggregate variables and individual investment in informal organizations is mapped into equilibrium correlations, among these variables, that cannot be accounted for in a model where informal institutions are taken as given.

Our model is static and this is clearly a limitation of framework. We find the focus on this

networks (32.8%).

simple model to be a useful starting point to illustrate how individuals react to changes in macroeconomic variables and how this affects the informal institutions present in the economy and economic outcomes. However, we also believe that it is important to embed our framework in a full fledged dynamic model. In such a model, at each point in time, labor market conditions would be different and so would workers' incentives to invest in social networks. In addition, the position of a worker in the wage distribution will affect his incentives to keep investing in connections. This model would allow us to provide novel empirical predictions on how, for example, the use of job contact networks changes with job tenure and to what extent social networks can explain durable inequality among groups and individuals.

References

[1] Albrecth J., Gautier T., Tan S. and Vroman S., "Matching with Multiple Applications Revisited", *Economics Letters*, 84, pp. 311-314, 2004.

[2] Arrow K.J. and Borzekowski R., "Limited network connections and the distribution of wages", Finance and Economics Discussion Series 2004-41, Board of Governors of the Federal Reserve System (U.S.), 2004.

[3] Bayer P., Ross S. and Topa G., "Place of work and place of residence: Informal hiring networks and labor market outcomes", *Journal of Political Economy*, 116, pp. 1150-1196, 2008.

[4] Battu H., Seaman P. and Zenou Y., "Job Contact Networks and the Ethnic Minorities", mimeo Stockholm University, 2007.

[5] Blackaby D.H., Leslie D.G., Murphy P.D. and OLeary N.C., "The Ethnic Wage Gap and Employment Differentials in the 1990s: Evidence for Britain", *Economics Letters*, 58, pp. 97-103, 1998.

[6] Blau D. and Robins P., "Job Search Outcomes for the Employed and Unemployed", *Journal of Political Economy*, 98, pp. 637-655, 1992.

[7] Boorman S.A., "A Combinatorial Optimization Model for Transmission of Job Information Through Contact Networks", *Bell Journal of Economics*, 6, pp. 216-249, 1975.

[8] Cabrales A., Calvó-Armengol A. and Zenou Y., "Social Interactions and Spillovers: Incentives, Segregation and Topology", mimeo Universitat Autònoma de Barcelona, 2007.

[9] Cahuc P. and Fontaine F., "On the Efficiency of Job Search with Social Networks", *Journal of Public Economic Theory*, forthcoming, 2009.

[10] Calvó-Armengol A., "Job Contact Networks", *Journal of Economic Theory*, 115, pp. 191-206, 2004.

[11] Calvó-Armengol A. and Jackson M.O., "The Effects of Social Networks on Employment and Inequality", *American Economic Review*, 94, pp. 27-46, 2004.

[12] Calvó-Armengol A. and Jackson M.O., "Networks in Labor Markets: Wage and Employment Dynamics and Inequality", *Journal of Economic Theory*, 132, pp. 426-454, 2007.

[13] Calvó-Armengol A. and Zenou Y., "Job Matching, Social Network and Word-of-mouth Communication", *Journal of Urban Economics*, 57, pp. 500-522, 2005.

[14] Chung F. and Lu L., "The Average Distance in a Random Graph with Given Expected Degree", *Proceedings of the National Academy of Sciences* of the USA, 99, pp. 15879-15882, 2002.

[15] Cingano F. and Rosolia A.,"People I Know: Workplace Networks and Job Search Outcomes", Bank of Italy working paper series, 600, 2006.

[16] Diamond P.A., "Aggregate Demand Management in Search Equilibrium", *Journal of Political Economy*, 90, pp. 881-94, 1982.

[17] Durlauf S., "On the Empirics of Social Capital", *The Economic Journal*, 112, pp. F459-F479, 2002.

[18] Erdos P. and Renyi A., "On Random Graphs". *Publicationes Mathematicae*, 6, pp. 290-297, 1959.

[19] Elliott J.R., "Social Isolation and Labor Market Insulation: Network and Neighborhood Effects on Less-Educated Urban Workers", *The Sociological Quarterly*, 40, pp. 199-216, 1999.

[20] Fontaine F., "A Simple Matching Model with Social Networks", *Economics Letters*, 94, pp. 396-401, 2007.

[21] Galeotti A., Goyal S., Jackson M.O., Vega-Redondo F. and Yariv L., "Network Games", *Review of Economic Studies*, forthcoming, 2009.

[22] Galeotti A. and Goyal S., "The Law of the Few", *American Economic Review*, forthcoming , 2009.

[23] Goyal S., *Connections: an introduction to the economics of networks*, Princeton University Press, 2007.

[24] Granovetter M., "The Strength of Weak Ties", *American Journal of Sociology*, 78, pp. 1360-1380, 1973.

[25] Granovetter M., *Getting a Job: A Study of Contacts and Careers*, First ed., 1974, Harvard U. Press; second ed., 1995, Chicago U. Press, 1974.

[26] Holzer H., "Informal Job Search and Black Youth Unemployment", *American Economic Review*, 77, pp. 446-52, 1987.

[27] Howitt P., and McAfee P.R., "Costly Search and Recruiting", *International Economic Review*, 28, pp. 89-107, 1987.

[28] Ioannides Y.M. and Loury L.D., "Job Information Networks, Neighborhood Effects, and Inequality", *Journal of Economic Literature*, XLII, pp. 1056-1093, 2004.

[29] Jackson M.O., *Social and Economic Networks*, Princeton University Press, 2008.

[30] Jackson M.O. and Wolinsky A., "A Strategic Model of Social and Economic Networks", *Journal of Economic Theory*, 71, pp. 44-74, 1996.

[31] Loury L.D., "Informal Contacts and Job Search among Young Workers", mimeo Tufts University, 2006.

[32] Montgomery J.D., "Social Networks and Labor Market Outcomes: Toward an Economic Analysis", *American Economic Review*, 81, pp. 1408-1418, 1991.

[33] Mortensen D.T. and Pissarides, C.A., "New Developments in Models of Search in the Labor Market", *Handbook of Labor Economics.* Ashenfelter, O.C., and Card, D., eds. Amsterdam: North-Holland, pp. 2567-627, 1999.

[34] Mortensen D. and Vishwanath T., "Personal Contacts and Earnings. It is Who You Know!", *Labour Economics*, 1, pp. 187-201, 1994.

[35] Munshi K., "Networks in the Modern Economy: Mexican Migrants in the US Labor Market", *Quarterly Journal of Economics*, 118, pp. 549-599, 2003.

[36] Osberg L., "Fishing in Different Pools: Job-search Strategies and Job-finding Success in Canada in the Early 1980s", *Journal of Labor Economics*, 11, pp. 348-386, 1993.

[37] Petrongolo B. and Pissarides C.A., "Looking into the Black Box: A Survey of the Matching Function", *Journal of Economic Literature*, 39, pp. 390-431, 2001.

[38] Petrongolo B. and Pissarides C.A., "Scale Effects in Markets with Search", *The Economic Journal*, 116, pp. 21-44, 2005.

[39] Pissarides C.A.,*Equilibrium Unemployment Theory*, 2nd ed. Cambridge: MIT Press (first ed. 1990, Oxford: Blackwell), 2000.

[40] Rees A., "Information Networks in Labor Markets", *American Economic Review, Papers and Proceedings* 56, pp. 559-566, 1996.

[41] Shimer R., "Reassesing the Ins and Outs of Unemployment", mimeo, 2007.

[42] Topa G., "Social Interactions, Local Spillovers and Unemployment", *Review of Economic Studies*, 68, pp. 261-295, 2001.

[43] Trejo S.J., "Why Do Mexican Americans Earn Low Wages?", *Journal of Political Economy*, 105, pp. 1235-68, 1997.

[44] Vega-Redondo F., *Complex Social Networks*, Econometric Society Monograph Series, Cambridge University Press, 2007.

[45] Wabha J. and Zenou Y., "Density, Social Networks and Job Search Methods: Theory and Application to Egypt", *Journal of Development Economics*, 88, pp. 443-473, 2005.

Appendix A: Proofs of Lemmas and Propositions

Proof of Proposition 1 First, it is straightforward to verify that $\frac{\partial\Psi}{\partial s}(s) > 0$ and that $\frac{\partial\Psi}{\partial a}(s) > 0$. We now show that $\Psi(s)$ is decreasing in b. The derivative of $\Psi(s)$ with respect to b has the same sign as the derivative of $\frac{(1-b)}{b}\left(1 - e^{-sb}\right)$ with respect to b, which is given by

$$\frac{e^{-sb}[b(1-b)s+1]-1}{b^2}. \tag{8}$$

Expression (8) is negative when $e^{-sb}[b(1-b)s+1] < 1$ which is equivalent to $-sb+\ln(1+b(1-b)s) < 0$. This holds because $-sb + \ln(1 + b(1-b)s)|_{s=0} = 0$ and $-sb + \ln(1 + b(1-b)s)$ is decreasing in s. Hence, $\frac{\partial\Psi}{\partial b}(s) < 0$ for all $s > 0$. Second, the comparative static with respect to the matching rate and the unemployment rate follows by the comparative static of the network matching rate. This concludes the proof of Proposition 1. ■

Proof of Proposition 2 Suppose an interior equilibrium exists. Consider a profile $\mathbf{s}$ where $s_j = s$, $\forall j \neq i$. Note that under profile $\mathbf{s}$ the probability that $i \in \mathcal{B}$ does not receive any offer from the network is

$$\phi_i(s_i, s) = \left[1 - p_i\frac{1-(1-p)^{nb}}{bnp}\right]^{na(1-b)},$$

where $p_i = s_i s/[s_i + (n-1)s]$ and $p = s^2/[s_i + (n-1)s]$. Next note that for every $s > 0$, i's best response, say $\hat{s}_i$, has the property that $\hat{s}_i s \leq [\hat{s}_i + (n-1)s]$. Indeed, if $\hat{s}_i s > [\hat{s}_i + (n-1)s]$, then i could decrease his own networking effort and still be linked to any arbitrary worker with probability 1. Hence, an interior equilibrium s^* solves:

$$\frac{\partial EU_i}{\partial s_i}(s^*, s^*) = -b(1-a)\frac{\partial \phi_i}{\partial s_i}(s^*, s^*) - c = 0.$$

Since

$$\begin{aligned}\frac{\partial \phi_i}{\partial s_i}(s_i, s) &= a(1-b)n\left(1 - \frac{s_i}{bns}\left(1 - \left(1 - \frac{s^2}{s_i + (n-1)s}\right)^{bn}\right)\right)^{a(1-b)n-1} \\ &\quad \left(\frac{s_i s}{(s_i + (n-1)s)^2}\left(1 - \frac{s^2}{s_i + (n-1)s}\right)^{bn-1} - \frac{1}{bns}\left(1 - \left(1 - \frac{s^2}{s_i+(n-1)s}\right)^{bn}\right)\right).\end{aligned}$$

In large labor markets, we have that:

$$\lim_{n\to\infty}\frac{\partial\phi_i}{\partial s_i}(s_i,s) = -\frac{a(1-b)}{sb}\left(1-e^{-sb}\right)e^{-\frac{a(1-b)}{b}\frac{1-e^{-sb}}{s}s_i}.$$

Therefore, s^* must solve

$$\frac{a(1-a)(1-b)}{s^*}\left(1-e^{-s^*b}\right)e^{-\frac{a(1-b)}{b}\left(1-e^{-s^*b}\right)} = c, \tag{9}$$

which is equivalent to condition (5) stated in Proposition 2. It is easy to see that $\frac{\partial\phi_i}{\partial s_i}(s_i,s)$ is continuos in the limit as it converges to (9) when n goes to infinity,[27] which ensures that the solution of the n-game converges to the solution of the limit game.

We now show that there exists a unique solution to this equation and we derive the conditions for existence. We start noticing that the LHS is decreasing in s^* because both $\left(1-e^{-s^*b}\right)/s^*$ and $e^{-\frac{a(1-b)}{b}\left(1-e^{-s^*b}\right)}$ are decreasing in s^*. Furthermore, when s^* goes to 0, the LHS converges to $ab(1-a)(1-b)$, while when s^* goes to infinity the LHS converges to 0. Since marginal returns are continuous in s_i, it follows that an interior symmetric equilibrium exists if and only if $c < ab(1-a)(1-b)$, in which case there is only one symmetric interior equilibrium. This concludes the proof of Proposition 2. ■

Proof of Proposition 3 We first prove part 1. We derive ds^*/db by implicit differentiation of (5) and obtain

$$\frac{ds^*}{db} = \frac{e^{-s^*b}\left[s(1-b)+1\right]-1+a(1-b)(1-e^{-s^*b})\frac{1-e^{-s^*b}-b(1-b)se^{-s^*b}}{b^2}}{(1-b)\left(\frac{1-e^{-s^*b}-s^*be^{-s^*b}}{s^*}+a(1-b)(1-e^{-s^*b})e^{-s^*b}\right)}. \tag{10}$$

Since the denominator is positive, the sign of this derivative depends on the sign of the numerator. Defining $x=\frac{a(1-b)\left(1-e^{-s^*b}\right)}{b}$, we rewrite expression (10) as follows

$$\frac{ds^*}{db} = \frac{\frac{dx}{db}+(1-x)\frac{dx}{db}-b\left(\frac{dx}{db}\right)^2+b(1-x)\frac{d^2x}{d^2b}}{a\left[\frac{1-e^{-s^*b}-bs^*e^{-s^*b}}{s^*}+\left(1-e^{-s^*b}\right)a(1-b)e^{-s^*b}\right]}, \tag{11}$$

[27] In other words, it is possible to show that for each $\epsilon > 0$, there exists an $n(\epsilon)$ such that for each $n > n(\epsilon)$, the distance of the two expressions is at most ϵ.

where

$$\frac{dx}{db} = a\frac{e^{-s^*b}[b(1-b)s^*+1]-1}{b^2} < 0.$$

The derivative of the numerator of expression (11) with respect to b when $ds/db = 0$ is

$$\frac{(1-a)}{s^*}e^{-x}\left[\frac{dx}{db} + (1-x)\frac{dx}{db} - b\left(\frac{dx}{db}\right)^2 + b(1-x)\frac{d^2x}{d^2b}\right], \tag{12}$$

where,

$$\frac{d^2x}{d^2b} = -\frac{2}{b}\frac{dx}{db} - \frac{sae^{-bs^*}(2+s-bs)}{b}.$$

After some algebra, (12) can be written as

$$\frac{(1-a)}{bs^*}e^{-x}\left[\frac{dx}{db}\left(x - b\frac{dx}{db}\right) - (1-x)sae^{-bs^*}(2+s-bs)\right].$$

The first term in the square brackets is always negative because $\frac{dx}{db}$ is always negative, and x is positive. On the other hand, $2+s-bs > 0$ for $b \in [0,1]$ and $s \geq 0$. Note furthermore that when $\frac{dFOC}{db} = 0$, by (11) it follows that $\frac{dx}{db} = -\frac{x}{b(1-x)} < 0$, which implies that $(1-x) > 0$. Hence the last term is also negative. Furthermore, $\frac{dFOC}{db}$ is positive when $b = 0$ and negative when $b = 1$. Since when $\frac{dFOC}{db} = 0$, $\frac{d^2FOC}{d^2b}$ is negative, by continuity, (11) crosses the $b-axis$ only once, precisely at $\bar{b}(a) > 0$. Hence, if $b < \bar{b}(a)$, the derivative is positive, while if $b > \bar{b}(a)$, it is negative. Note that this is the value of b where s^* is maximum. This concludes the first part of the proof of Proposition 3.

We now turn to the second part of Proposition 3. The change in equilibrium network productivity when b changes is described by:

$$\frac{d\Psi(s,a,b)}{db} = a\phi(s^*)\left[-\frac{1-e^{s^*b}}{b^2} + (1-b)e^{-s^*b}\left(\frac{s^*}{b} + \frac{ds^*}{db}\right)\right]. \tag{13}$$

While the first term in the square parenthesis is always negative, the sign of the second one depends on the sign of ds^*/db. Using part 1 of Proposition (3), ds^*/db is negative when b is sufficiently high, and hence $d\Psi(s,a,b)/db$ would be negative as well. When b tends to 0, ds^*/db tends to ∞. Furthermore, the limit of expression (13) as b goes to 0 is also ∞. Since the derivative is continuous, its sign is positive when b is close enough to 0. This concludes the proof of Proposition 3. ■

Proof Proposition 4 It is easy to see that an interior equilibrium (a^*, s^*) solves conditions (6) and (7) stated in Proposition 4. We now show existence. Using condition (6) we have that $\phi(s^*, a^*) = \alpha/b$, which is equivalent to

$$a^* = -\frac{b}{(1-b)(1-e^{-s^*b})} \ln\left(\frac{\alpha}{b}\right).$$

Since $a^* \in (0,1)$ it must be the case that $1 + [b\ln(\alpha/b)]/[(1-b)(1-e^{-s^*b})] > 0$. Let $\alpha(b)$ be such that: $1+[b\ln(\alpha(b)/b)]/[(1-b)] = 0$. Note that if $\alpha \geq \alpha(b)$ then $1+[b\ln(\alpha/b)]/[(1-b)(1-e^{-s^*b})] \leq 0$ for all s^*, while if $\alpha \leq \alpha(b)$ then $1 + [b\ln(\alpha/b)]/[(1-b)(1-e^{-s^*b})] > 0$ for sufficiently high s^*. So, a necessary condition for an interior equilibrium is that $\alpha < \alpha(b)$. Suppose $\alpha < \alpha(b)$. Next, using the above expression for a^* and $\phi(s^*, a^*) = \alpha/b$, we obtain that condition (7) holds if and only if

$$V(b, c, s^*) = \frac{cs^*}{\alpha} + \ln\left(\frac{\alpha}{b}\right)\left(1 + \frac{b}{(1-b)(1-e^{-s^*b})}\ln\left(\frac{\alpha}{b}\right)\right) = 0.$$

Note that $V(b, c, s^*)$ is the sum of two convex function in s^* and therefore it is convex in s^*. Moreover, $\lim_{s^*\to 0} V(a, b, s^*) = \lim_{s^*\to\infty} V(b, c, s^*) = \infty$. Finally, note that the LHS is strictly decreasing in c and, since $\alpha < \alpha(b)$, $V(b, 0, s^*) < 0$. For every $\alpha < \alpha(b)$, there exists a $c(\alpha, b) > 0$ such that if $c = c(\alpha, b)$, $V(b, c, s^*) = 0$ has a unique solution in s^*, while for all $c < c(\alpha, b)$ the equation has two solutions, s^H and s^L, with $s^H > s^L$. Since a^* is decreasing in s^*, it follows that under s^H, the equilibrium vacancy rate is a^L which is lower than the equilibrium vacancy rate a^H under s^L. This completes the proof of Proposition 4. ■

Proof of Corollary 1 The equilibrium unemployment rate under equilibrium (s^*, a^*) is $u(s^*, a^*) = b(1-a)\phi(s^*, a^*) = (1-a)\alpha$, where the last equality follows from equilibrium condition (6). Hence, the equilibrium unemployment rate is decreasing in a^*. So unemployment rate is lower under (s^L, a^H) than under (s^H, a^L). Next, note that in an equilibrium (s^*, a^*), $Eu(s^*, a^*) = 1 - b(1-a)\phi(s^*, a^*) - \alpha a^* - cs^* = 1 - \alpha - cs^*$, where the second equality follows from equilibrium condition (6). Hence, expected equilibrium utility is decreasing in s^* and therefore equilibrium (s^L, a^H) Pareto dominates equilibrium (s^H, a^L). Finally, from equilibrium condition (6), it is immediate to see that network matching rate is constant across the two interior equilibria and that an increase in b implies an increase in the nework matching rate. ■

Appendix B: Indirect information flow

This appendix examines the implications of indirect information flow in the matching process of workers with firms and how it shapes workers' socialization incentives. Information flow in the network now follows the following process. As in the basic model, each worker l with a needless offer gives it to one of his unemployed neighbors, chosen at random. If worker l has only employed friends, then he chooses one of them at random, say j, and gives him the information. For simplicity, we assume that if j had himself a needless offer, then the offer he receives from l is lost. If, on the contrary, worker j did not have a needless offer, then he selects at random one of his unemployed friends, say i, and passes the information to him. We also assume that the information passed from j to i reaches i with probability $\delta \in [0,1]$, where δ is the decay in the information flow. So, information now may flow two-steps away in the network.

We observe that a job seeker worker does not hear about new jobs from his friends if: 1) he does not access information from his friends who received a needless offer directly and 2) he does not get information from his contacts who are employed, do not have a needless offer directly, but have heard of a job opportunity from at least one of their friends. The probability associated to the event described in 1) is given by (3). We now derive the probability associated to the event described in 2). For concreteness, in what follows, $i \in \mathcal{B}$ and chooses s_i, while all other workers choose effort s. Moreover, worker j is employed and he does not have a needless offer, $j \in \mathcal{N} \setminus \{\mathcal{B} \cup \mathcal{O}\}$, while worker $l \in \mathcal{O}$.

First, suppose j and l are linked, i.e., $g_{jl} = 1$. The probability that j receives information from l is:

$$(1-p_i)(1-p)^{nb-1} \sum_{v=1}^{n(1-b)} \Pr(\eta_l(\mathcal{N} \setminus \{\mathcal{B}\}) = v | g_{lj} = 1) \frac{1}{v}.$$

That is, worker l's friends must be all employed, $(1-p_i)(1-p)^{nb-1}$, and, conditioning on having v links, worker l gives the information to j with probability $1/v$. So, if worker j is linked with ω

workers such as l, the probability that j does not receive information is:

$$\left[1-(1-p_i)(1-p)^{nb-1}\sum_{v=1}^{n(1-b)}\Pr(\eta_l(\mathcal{N}\setminus\{\mathcal{B}\})=v|g_{lj}=1)\frac{1}{v}\right]^{\omega}.$$

Summing across all possible number of j's neighbors who are employed and with a needless offer, we obtain the probability that j accesses at least an indirect offer:

$$\Theta(\mathbf{s})=1-\sum_{\omega=0}^{|\mathcal{O}|}B(\omega|p,|\mathcal{O}|)\left[1-(1-p_i)(1-p)^{nb-1}\sum_{v=1}^{n(1-b)}\Pr(\eta_l(\mathcal{N}\setminus\{\mathcal{B}\})=v|g_{lj}=1)\frac{1}{v}\right]^{\omega},$$

and in a large labor market it is equal to:

$$\Theta(\mathbf{s})=1-e^{-a\left(e^{-sb}-e^{-s}\right)}. \tag{14}$$

Note that in a complete network worker l has always links with unemployed workers and therefore every worker like j will never receive information. When the network is not complete, it is easy to verify that the probability that j gets information is non-monotonic in s–it first increases when s is low to begin with and then it decreases. Therefore, greater connectivity of workers other than unemployed i may have a positive effect on the probability that i gets a job. This illustrates a novel effect which emerges from indirect information flow. In fact, when the network is not very connected to start with, high socialization investments of other workers have a positive effect on the value of worker i's socialization investment because it makes more likely that i's neighbors have job information to pass along.

Second, consider our original job seeker i and suppose he has η links with workers like j above. The probability that i does not receive an offer from each of these η contacts is:

$$\sum_{d=0}^{\eta}B(d|\Theta(\mathbf{s}),\eta)\left[1-\sum_{t=1}^{nb}\Pr(\eta_j(\mathcal{B})=t|g_{ij}=1)\frac{\delta}{t}\right]^{d}.$$

In words, with probability $B(d|\Theta(\mathbf{s}),\eta)$, d out of the η contacts of i have received an offer from one of their employed friends. Suppose j is one of these individuals; then the probability that i receives information from j depends on the level of the decay in the information flow and the number of unemployed workers connected to j. Summing across all possible number of links that worker i can have with workers like j, we obtain the probability that i does not access an indirect offer:

$$\phi_i^{in}(s_i, \mathbf{s}_{-i}) = \sum_{\eta=0}^{n(1-a)(1-b)} B(\eta|p_i, n(1-a)(1-b)) \sum_{d=0}^{\eta} B(d|\Theta(\mathbf{s}), \eta) \left[1 - \sum_{t=1}^{nb} \Pr(\eta_j(\mathcal{N}^U) = t|g_{ij} = 1)\frac{\delta}{t}\right]^d,$$

and clearly the probability that i gets at least an indirect offer is $\Psi_i^{in}(s_i, \mathbf{s}_{-i}) = 1 - \phi_i^{in}(s_i, \mathbf{s}_{-i})$. In a large labor market, this is equal to

$$\Psi^{in}(s_i, \mathbf{s}_{-i}) = 1 - e^{-\frac{\delta(1-b)(1-a)}{b}\frac{s_i}{s}\left(1-e^{-sb}\right)\Theta(\mathbf{s})}. \tag{15}$$

In a symmetric profile where $s_i = s$ for all $i \in \mathcal{N}$, the probability that an unemployed worker hears a job indirectly is non monotonic in socialization effort and the intuition follows from the effect of indirect information flow which we have discussed above.

Finally, the overall probability that an unemployed worker gets at least an offer in a symmetric profile is:

$$\tilde{\Psi}^{in}(s) = 1 - \phi(s)\phi^{in}(s) = 1 - e^{-\frac{1-b}{b}\left(1-e^{-sb}\right)(a+\delta(1-a)\Theta(s))}. \tag{16}$$

The network matching rate is decreasing in the decay of information flow and under full decay we are back to the network matching rate (4) developed in Section 3. Under indirect information flow, the expected utility of a worker i choosing s_i and facing a strategy of others $s_j = s$ for all $j \neq i$ is:

$$EU_i(s_i, \mathbf{s}_{-i}) = 1 - b(1-a)\phi_i(s_i\mathbf{s}_{-i})\phi_i^{in}(s_i, \mathbf{s}_{-i}) - cs_i.$$

Proposition 5 *Consider a large labor market and consider indirect information flow. An interior equilibrium exists if and only if $c < ab(1-a)(1-a)$. In a symmetric interior equilibrium every worker chooses $\hat{s}$ which is the unique solution to:*

$$\frac{(1-b)(1-a)}{\hat{s}}\left(1 - e^{-\hat{s}b}\right)(a + \delta(1-a)\Theta(\hat{s}))\left(1 - \tilde{\Psi}^{in}(\hat{s})\right) = c \tag{17}$$

Proof of Proposition 5. Equilibrium condition (17) is obtained by taking the partial derivatives of $EU_i(s_i, \mathbf{s}_{-i})$ with respect to s_i and imposing symmetry, i.e, $s_j = s$ for all $j \in \mathcal{N}$. We now show that a solution exists and it is unique if and only if $c < ab(1-a)(1-b)$. To see this note that when $s \to 0$ the LHS of (17) equals $ab(1-a)(1-b)$ and when $s \to \infty$ the LHS of (17) equals 0. So, it is sufficient to show that the LHS of (17) is decreasing in s, which we now prove. We first claim that the following expression is decreasing in s:

$$\frac{1}{s}\left(1 - e^{-sb}\right)(a + \delta(1-a)\Theta(s))$$

Taking the derivatives of the above expression with respect to s we have that

$$\frac{1}{s^2}\left[-(a + \delta(1-a)\Theta(s))(1 - e^{-sb}(1-b)) + (1 - e^{-sb})\delta(1-a)\frac{\partial\Theta(s)}{\partial s}\right].$$

Since $\frac{\partial\Theta}{\partial s}(s) = (1 - \Theta(s))a\left(e^{-s} - be^{-sb}\right)$, then the above derivative is negative if and only if

$$\delta(1-a)a(1-\Theta(s))(1-e^{-sb})\left(e^{-s} - be^{-sb}\right) < (a + \delta(1-a)\Theta(s))(1 - e^{-sb}(1-b))$$

Since the RHS of the inequality is always positive, if $\left(e^{-s} - be^{-sb}\right) < 0$ the claim follows. So, suppose that $\left(e^{-s} - be^{-sb}\right) > 0$. Here note that

$$1 - e^{-sb}(1-b) > (1 - e^{-sb})\left(e^{-s} - be^{-sb}\right)$$

if and only if, taking the log, $\ln(1-b) + sb > 0$ which is obviously true. Next, note that $a + \delta(1-a)\Theta(s) > \delta(1-a)a(1-\Theta(s))$ if and only if $a + \delta[\Theta(s) - a(1-\Theta(s))] > 0$, which holds because $a + \delta[\Theta(s) - a(1-\Theta(s))] > a(1-\delta(1-a)) > 0$. These two observations imply our first claim. Using similar arguments, it is easy to show that $(1 - \tilde{\Psi}^{in}(s))$ is also decreasing in s. Hence, the LHS of expression 17 is decreasing in s. Proposition 5 follows. ■

Appendix C: Endogenous arrival of offers

We now study how workers' incentives to invest in socialization interplay with firms' incentives to open new jobs, and how this interplay shapes overall labor market outcomes. Firms' incentives to create new jobs depend on the probability that vacant jobs are eventually filled. This depends on network matching rate and therefore on workers' socialization effort. We show that the decision of firms to create new vacancies and the decision of workers to invest in social connections are strategic complements. This complementarity leads to multiple equilibria with the feature that equilibrium with a high vacancy rate also entails a high socialization effort.[28]

For simplicity we assume that the productivity of each worker equals 2. Since wage is set to 1, the firm's per-worker profit is also 1. To define the expected profits to a firm opening a new job we need to define the probability that the job is filled. This is the vacancy filling rate and it equals $m_f(\mathbf{s}, a) = b + (1-b)\Psi_f(\mathbf{s}, a)$, where $\Psi_f(\mathbf{s}, a)$ is the probability that a vacant job is filled via the network. Using the condition that the expected number of filled vacancies, $Vm_f(\mathbf{s}, a)$, must equal the expected number of workers who find a new job, $Bm(\mathbf{s}, a)$, we obtain a relation between the network matching filling rate $\Psi_f(\mathbf{s}, a)$ and the network matching rate $\Psi(\mathbf{s}, a)$. Formally,[29]

$$\Psi_f(\mathbf{s}, a) = \frac{b(1-a)}{a(1-b)}\Psi(\mathbf{s}, a). \tag{18}$$

The expected profits to a firm opening a vacancy is:

$$\Pi_f(a, \mathbf{s}) = b + (1-b)\Psi_f(a, \mathbf{s}) - k,$$

where $k \in [0, 1]$ is the cost of opening a vacancy.

An equilibrium is a symmetric profile $\mathbf{s}^*$ and a vacancy rate a^* such that: 1) $\mathbf{s}^*$ is a Nash equilibrium, given a^*, and 2) $\Pi_f(a^*, \mathbf{s}^*) = 0$.

We assume that $k \in (b, 1]$ and we focus on interior equilibria.[30] The following proposition characterizes interior equilibria.

Proposition 6 *Consider a large labor market and suppose that $k > b$. There exists a number*

[28]The presence of multiple equilibria in the context of endogenous search and strong positive externalities (increasing returns to matching) have been addressed by early papers, e.g., Diamond (1982) and Howitt and McAfee (1987). A novelty of our result is that here externalities are generated by the transmission of information in the network.

[29]We observe that, for a given symmetric profile of socialization s, we can use Lemma 1 to show that the network filling vacancy rate is increasing in s and b, while it is decreasing in a.

[30]Two observations are in order. First, it is easy to verify that if $k < b$ then the only equilibrium is $a^* = 1$ and $s^* = 0$. Two, when $k > b$, there always exists an equilibrium in which $a^* = 0$ and $s^* = 0$. Since firms believe that the job contact network is empty, the vacancy filling rate equals b, which is lower than the costs of the vacancy, so firms will not create new jobs. Given that no new jobs are created, it is optimal for workers to set zero effort.

$\bar{c}(k,b) > 0$ such that an interior equilibrium exists if and only if $c \leq \bar{c}(k,b)$. An interior equilibrium (s^*, a^*) solves:

$$b(1-a^*)\frac{\partial \Psi_i}{\partial s_i}(s^*, s^*, a^*) \quad = \quad c \tag{19}$$

$$\frac{b}{a^*}\left[a^* + (1-a^*)\Psi(s^*, s^*, a^*)\right] \quad = \quad k \tag{20}$$

When $c = \bar{c}(k,b)$, there exists only one interior equilibrium, while if $c < \bar{c}(k,b)$ there are two interior equilibria, $(\hat{s}^H, \hat{a}^H)$ and $(\hat{s}^L, \hat{a}^L)$, where $\hat{s}^H > \hat{s}^L$ and $\hat{a}^H > \hat{a}^L$.

Proof of Proposition 6 It is easy to verify that an interior equilibrium (s^*, a^*) solves condition (19) and condition (20) stated in Proposition 6. We now prove existence. First, condition (20) can be rewritten as follows:

$$e^{-\frac{a^*(1-b)}{b}\left(1-e^{-s^*b}\right)} = \frac{b-ka^*}{b(1-a^*)}. \tag{21}$$

Taking the log of expression (21) we obtain

$$(1-e^{-s^*b}) = -\frac{b}{a^*(1-b)}\ln\left(\frac{b-ka^*}{b(1-a^*)}\right), \tag{22}$$

and taking again the log we have that:

$$s^* = -\frac{1}{b}\ln\left(1 + \frac{b}{a^*(1-b)}\ln\left[\frac{b-ka^*}{b(1-a^*)}\right]\right). \tag{23}$$

Let $\tilde{s}(a^*, b, k)$ be the s^* which solves (23).

Second, using condition (21) and condition (23), we rewrite condition (19) as follows

$$s^* = -\frac{b-ka^*}{c}\ln\left[\frac{b-ka^*}{b(1-a^*)}\right], \tag{24}$$

and we denote by $\bar{s}(a^*, c, b, k)$ the s^* which solves 24.

An interior equilibrium is (a^*, s^*) such that $\bar{s}(a^*, c, b, k) = \tilde{s}(a^*, c, b, k) = s^*$. Now note that $\bar{s}(a^*, c, b, k)$ is a well defined function when (1.) $b - ka^* > 0$ and (2.) $b - ka^* < b(1-a^*)$. We observe that (2.) holds because $k > b$. So, consider all $a^* \in (0, b/k]$ and note that $\bar{s}(a^*, c, b, k) > 0$, $\bar{s}(0, c, b, k) = \bar{s}(b/k, c, b, k) = 0$ and that $\bar{s}(a^*, c, b, k)$ is concave in a^*, since $\partial^2\bar{s}(a^*, c, b, k)/\partial a^2 =$

$-(b-k)^2/[(1-a^*)^2(b-ka^*)] < 0$.

Next, note that $\tilde{s}(a^*,b,k)$ is a well defined function when (1.) above holds, (1a.) $1+\frac{b}{a(1-b)}\ln(\frac{b-ka}{b(1-a)}) > 0$ and (2a.) $\ln(1+\frac{b}{a(1-b)}\ln(\frac{b-ka}{b(1-a)})) < 0$. It is easy to check that condition (2.) above implies condition (2a.). It is also easy to verify that there exists a unique $\bar{a}(b,k) > 0$ such that $1+\frac{b}{\bar{a}(b,k)(1-b)}\ln\left(\frac{b-k\bar{a}(b,k)}{b(1-\bar{a}(b,k))}\right) = 0$, that $\bar{a}(b,k) < b/k$ and that condition (1a.) holds if and only if $a < \bar{a}(b,k)$. Hence, (1a.) implies (1.). So, a necessary condition for an interior equilibrium is that $a^* \in [0,\bar{a}(b,k)]$, which we now assume. Note here that: $\tilde{s}(0,b,k) = -\frac{1}{b}\ln(\frac{b-ka^*}{b(1-a^*)}) > 0$, $\tilde{s}(\bar{a}(b,k),b,k) = +\infty$ and that $\tilde{s}(a^*,b,k)$ is increasing and convex in $a \in [0,\bar{a}(b,k)]$.

Combining the properties of $\bar{s}(a^*,c,b,k)$ and $\tilde{s}(a^*,b,k)$, we obtain that for all $a^* \in [0,\bar{a}(b,k)]$, the function

$$G(a,c,b,k) = \bar{s}(a,c,b,k) - \tilde{s}(a,b,k)$$

is concave in a^*, $G(0,c,b,k) < 0$ and $G(\bar{a}(b,k),c,b,k) < 0$. Let $\tilde{a} = \arg\max_{a^*\in[0,\bar{a}(b,k)]} G(a^*,c,b,k)$. It is then clear that an interior equilibrium exists if and only if $G(\tilde{a},c,b,k) \geq 0$. Furthermore, if $G(\tilde{a},c,b,k) = 0$ then there is only one interior equilibrium, while if $G(\tilde{a},c,b,k) > 0$ there are two interior equilibria, a low $a^* = a^L$ with corresponding high $s^* = s^H$, and a high a^* a with corresponding low $s^* = s^L$. Since for fix $k > b$ and $a^* \in [0,\bar{a}(b,k)]]$, $G(a^*,c,b,k)$ is strictly decreasing in c, $\lim_{c\to 0} G(a^*,c,b,k) = \infty$ and $\lim_{c\to\infty} G(a^*,c,b,k) < 0$, it follows that there exists a $\bar{c}(k,b)$ such that if $c = \bar{c}(k,b)$ there exists one interior equilibrium, while for all $c \leq \bar{c}(k,b)$ there are two interior equilibria. This concludes the proof of Proposition 6. ■

When the cost of socialization is sufficiently low, there are two interior equilibria. In one equilibrium few vacant jobs are created and workers choose a low level of socialization. When the vacancy rate is low, the job network supply is also low and so workers have low incentives to form connections. Overall, the network matching rate is low, which justifies the low equilibrium level of the vacancy rate. In the other equilibrium, quite the opposite occurs. Firms create many new jobs under the expectation of a high network matching rate. Since under a high vacancy rate the job network supply is also high, workers heavily invest in connections and this self fulfils firms' expectation of a high network matching rate. So, the endogenous formation of job contact networks determine *per se* strategic complementarities between the incentive of firms to open new vacancies and the incentives of workers to invest in job contact networks.

The discussion above suggests that the network matching rate is higher under equilibrium $(\hat{s}^H, \hat{a}^H)$ than under equilibrium $(\hat{s}^L, \hat{a}^L)$. Since in the former equilibrium it is also more likely that a worker accesses a direct offer than in the latter equilibrium, this suggests that unemployment rate is lower in the equilibrium with dense networks and high vacancy rate. This intuition is confirmed in the next result, which also shows that the two equilibria can be ranked in the Pareto sense.

Corollary 2 *Consider the interior equilibria* $(\hat{s}^H, \hat{a}^H)$ *and* $(\hat{s}^L, \hat{a}^L)$ *described in Proposition 6. Then,*
1) $(\hat{s}^H, \hat{a}^H)$ *Pareto dominates* $(\hat{s}^L, \hat{a}^L)$*;*
2) *the network marching rate under* $(\hat{s}^H, \hat{a}^H)$ *is higher than the network matching rate under* $(\hat{s}^L, \hat{a}^L)$*;*
3) *the unemployment rate under* $(\hat{s}^H, \hat{a}^H)$ *is lower than the unemployment rate under* $(\hat{s}^L, \hat{a}^L)$*.*

Proof Corollary 2 The vacancy filling rate under equilibrium (s^*, a^*) is $\Psi_f(s^*, a^*) = [k-b]/[1-b]$. Since $\Psi_f(s^*, a^*) = \Psi(s^*, a^*)b(1-a^*)/[a^*(1-b)]$, it follows that $\Psi(s^*, a^*) = a^*(k-b)/[b(1-a^*)]$, which is increasing in a^*. So the network matching rate under $(\hat{s}^L, \hat{a}^H)$ is higher than the network matching rate under $(\hat{s}^H, \hat{a}^L)$. Second, unemployment rate under equilibrium (s^*, a^*) is $u(s^*, a^*) = b(1-a^*)\phi(s^*, a^*) = b-a^*k$, which is decreasing in a^*. So, unemployment rate under under $(\hat{s}^L, \hat{a}^H)$ is lower than unemployment rate under $(\hat{s}^H, \hat{a}^L)$. Finally, note that expected utility under equilibrium (s^*, a^*) is $EU(s^*, a^*) = 1 - b(1-a^*)\phi(s^*, a^*) - cs^* = 1 - b + a^*k - cs^*$, which is increasing in a and decreasing in s. So, $(\hat{s}^L, \hat{a}^H)$ Pareto dominates $(\hat{s}^H, \hat{a}^L)$. ■