INSTITUTE FOR SOCIAL & ECONOMIC RESEARCH

ADDRESSING THE INTERPRETATION AND THE AGGREGATION PROBLEMS IN TOTALLY FUZZY AND RELATIVE POVERTY MEASURES

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ABSTRACT

This work is related to the studying of poverty dynamics using the *Totally Fuzzy and Relative* (TFR) approach who allows us to analyse poverty in a multidimensional perspective avoiding the use of arbitrary threshold values as in the traditional approach. This paper focuses on two problems concerning i) the interpretability of TFR measures and ii) their weighting system. For these reason, we propose an alternative specification of the membership function that expands the interpretability of TFR and we compare the weight function originally proposed with the TFR method with other specifications that may be considered equally valid.

NON-TECHNICAL SUMMARY

The Totally Fuzzy and Relative (TFR) method proposed by Cheli and Lemmi (1995) represents a very effective tool for analysing poverty in a multidimensional perspective, at the same time avoiding the use of arbitrary threshold values. TFR indices are ordinal measures that are effective for making cross section and intertemporal poverty comparisons. However their values have no intrinsic meaning and this fact limits both their interpretability and the possibility of comparing the indices that. account for different aspects of poverty to one another as well as the possibility of aggregating them in order to produce an index of global poverty. Here we propose an alternative specification of the membership function that expands the interpretability of TFR indices and makes the aggregation of measures relative to different aspects of poverty less controversial. Another problem that characterises the TFR approach as well as other methods for multidimensional poverty analysis concerns a certain arbitrariness affecting the choice of the weights by means of which we aggregate the information provided by different poverty indicators. Here we do not propose any methodological solution and we just compare some different weight specifications to one another. In the empirical analysis, carried out on BHPS data from 1991 to 1997, we compare the results obtained by applying i) the original versus the new m.f. specification and ii) three different weight functions. The fact that all these sets of results derived according to different methodological variations of the TFR method substantially coincide suggests that both the arbitrary choice of the weight function and the preference for the original or the alternative m.f. are not crucial problems, since they do not seem to condition the results of the analysis. This constitutes empirical evidence of the robustness of the method itself to the mentioned changes. Of course, the empirical nature of this study makes this conclusion dependent on the particular data set that we considered. For this reason, a sensitivity analysis by simulation data would be useful in order to draw more general conclusions.

ADDRESSING THE INTERPRETATION AND THE AGGREGATION PROBLEMS IN TOTALLY FUZZY AND RELATIVE POVERTY MEASURES*

1 Introduction

Most of the methods designed for the analysis of poverty share two main limitations: i) they are unidimensional, i.e. they refer to only one proxy of poverty, namely equivalent income or consumption; ii) they need to dichotomise the population into the *poor* and the *non poor* by means of the so called *poverty line*. This reductionism simplifies the analysis, but also wipes out all the complexity of this phenomenon which, on the contrary, should also be object of study. The *Totally Fuzzy and Relative* (TFR) approach proposed by Cheli and Lemmi (1995) who modified and developed a contribution by Cerioli and Zani (1990), allows us to overcome these limitations and to analyse poverty in a multidimensional perspective avoiding the use of arbitrary threshold values.

Let us explain the three features of the TFR poverty measures: multidimensionality, fuzziness and relativity. Multidimensionality lies in the possibility of referring the analysis to a variety of living condition indicators (that may also include income and/or consumption) and of calculating indices accounting for different aspects of poverty. Making use of *fuzzy sets* theory (Zadeh, 1965; Dubois and Prade, 1980), the degree of deprivation of any household is seen as its degree of membership in the fuzzy subset of the poor. When this membership is associated to a poverty indicator of the continuous type, it takes distinct values in the whole range of this one. In other words the membership function (m.f.) is bijective, that justifies the term "totally fuzzy"[1]. Lastly, relative poverty measures usually depend on a given parameter of the income distribution such as the mean or the median. By contrast TFR measures refer to the entire distribution of the considered poverty indicators and for this reason they are named "totally relative". Thanks to these features TFR measures utilise all the information provided by continuous variables such as income, without the losses caused by the traditional dichotomisation.

This paper focuses on two problems concerning i) the interpretability of TFR measures and ii) their weighting system. TFR indices are ordinal measures[2] that are effective to make cross section and intertemporal poverty comparisons. However their values have no intrinsic meaning[3]. This fact limits

[1] This term was introduced so as to mark the difference between the Cerioli and Zani (1990) contribution, where the membership function in the fuzzy sub-set of the poor according to income was bijective only between two arbitrary threshold values.

[2] See Lemmi *et al*. (1997).

[3] For an extended discussion see Lemmi *et al*. (1997).

both their interpretability and the possibility of comparing the indices that refer to different items (i.e. accounting for different aspects of poverty) as well as the possibility of aggregating them in order to produce an index of global poverty. This question is discussed in section 2: in sub-section 2.1 we point out the limitations of the original shape of the m.f.; in sub-section 2.2 we propose an alternative specification of the m.f. that expands the interpretability of TFR indices and makes the aggregation of measures relative to different aspects of poverty less controversial; finally, sub-section 2.3 is devoted to the shape assumed by this new m.f. in the particular case of binary indicators.

The second problem that we deal with here concerns the weighting system needed for aggregating the measures of deprivation relative to any single item so as to obtain multidimensional poverty indices. Consistently with the relative concept of deprivation, the importance of an item for the measurement of poverty should directly depend on how representative it is of the community's life style. Although the particular specification of the weights usually adopted in TFR poverty analyses satisfies this criterion, its choice might be seen as arbitrary, because many alternative forms can be suggested that agree with the same principle. This question is discussed in section 3, where we compare the weight function originally proposed with the TFR method with other specifications that may be considered equally valid.

Section 4 is devoted to the empirical part of this research, conducted on a data base drawn from the British Household Panel Survey (BHPS), we aim at comparing the results obtained by applying different versions of the TFR method; in particular: i) the original and the new specification of the m.f. that we propose here and ii) different specifications of the weights. Such an effort will let us draw an empirical evaluation of the robustness of the TFR method. Final remarks to this research are reported in section 5.

2 The Membership Function

2.1 The original specification

The TFR method adopts the following specification of the deprivation measure according to a generic poverty indicator *X*:

$$g(x_i) = \begin{cases} H(x_i) & \text{if the degree of poverty grows as } X \text{ increases} \\ 1 - H(x_i) & \text{otherwise} \end{cases} \tag{2.1.1}$$

where *H*( ) represents the observed distribution function of *X* and subscript *i* refers to the *i*-th household. According to the fuzzy sets theory $g(x_i)$ can be interpreted as *membership function* (m.f.) in the fuzzy subset of the poor calculated for the *i*-th household.

However, when the *X* variable is ordinal and the frequency associated to one of the extreme categories is rather high, one should adopt a normalised version of the previous specification given by[4]:

$$g(x_i) = \begin{cases} 0 & \text{if } x_i = x_{(1)} \\ g(x_{(k-1)}) + \dfrac{H(x_{(k)}) - H(x_{(k-1)})}{1 - H(x_{(1)})} & \text{if } x_i = x_{(k)} \quad (k > 1) \end{cases} \qquad (2.1.1\text{bis})$$

where $x_{(1)},\ldots, x_{(m)}$ represent the categories of *X* sorted in increasing order with respect to the risk of poverty. After simple manipulations, the preceding formula can also be written as follows:

$$g(x_i) = \frac{H(x_{(k)}) - h(x_{(1)})}{1 - h(x_{(1)})} \ , \qquad \text{for } x_i = x_{(k)} \ , \quad k = 1,\ldots,m \qquad (2.1.2)$$

where function *h*( ) associates any category of *X* to the corresponding relative frequency. In this way, *g*( ) always takes value 0 in correspondence to the lowest category of *X* (i.e. lowest risk of poverty) and 1 in correspondence to the highest one. In correspondence to the intermediate categories, *g*( ) takes values between 0 and 1 that are not influenced by the extreme categories and depend on the empirical distribution of *X*.

However, as far as monetary variables, such as income or consumption that may be treated as continuous are concerned, later contributions (Cheli, 1995; Lemmi et al., 1997) make use of a modified version of the m.f. (2.1.1), namely:

$$g(x_i) = [1 - H(x_i)]^{\alpha} \qquad (2.1.2\text{bis})$$

where the α exponent determines the relative weight of the poorer with respect to the less poor. The α values can be determined in order to make *E*[*g*(*x*)] equal to the Head Count Ratio of the poor according

[4] For details see the original contribution by Cheli and Lemmi (1995).

toa given poverty line. This also helps to compare the results of the fuzzy analysis to those obtained with the traditional analysis.

In practice, $g(x_i)$ represents an individual index of deprivation specific for item *X*. A collective index specific for *X* is defined as the arithmetic mean of the $g(x_i)$ memberships in the population:

$$P = \frac{1}{n}\sum_{i=1}^{n} g(x_i) = \sum_{k=1}^{m} h(x_{(k)}) \cdot g(x_{(k)}) \tag{2.1.3}$$

At this point we shall try to highlight the problems mentioned in the introduction by means of a numeric example. Let us assume we aim to calculate the TFR deprivation index (2.1.3) specific to a given variable *X* which represents the subjective appraisal of the household's standard of living, liable to assume the three following categories: "satisfactory", "average" and "unsatisfactory". Table 1 illustrates the procedure for calculating the TFR index and allows comparison to be made between two different situations indicated by *A* and *B*.

First of all, intuitively, it would be desirable that, whenever the frequency distribution of *X* is symmetric around a category or value which is in the middle between welfare and hardship, as in situation *A*, the corresponding TFR index were equal to ½ (that is the central value of its maximum range).

This does not happen with the original TFR measures, as shown by the example.

Table 1

	situation *A*			Situation *B*		
X	*h*(*x*)	*H*(*x*)	*g*(*x*)	*h*(*x*)	*H*(*x*)	*g*(*x*)
Satisfactory	0.05	0.05	0.0000	0.05	0.05	0.0000
Average	0.90	0.95	0.9474	0.94	0.99	0.9895
Unsatisfactory	0.05	1.00	1.0000	0.01	1.00	1.0000
P (TFR			**0.9026**			**0.9401**

A second problem seems to arise when we aim at comparing situations *A* and *B*. Although our example reports a clear decrease of subjective deprivation as we move from *A* to *B*, the TFR index paradoxically increases from 0.9026 to 0.9401. However this is not a real problem and it can be easily avoided: when we aim at comparing different situations (either temporal or spatial or characterised by different demographic attributes) we have to anchor the m.f. to a reference situation. In our example let this reference situation be *A*, so that the deprivation index in *B* based on *A* is obtained by multiplying the m.f. calculated in *A* ($g^A(x)$) by the frequency distribution in *B* ($h^B(x)$), as follows:

$$P^{B|A} = \sum g^A(x) \cdot h^B(x) = 0{,}9006$$

which reflects the actual decrease of deprivation. The real problem, however, is that the punctual values that index assumes in *A* and in *B* have no intrinsic meaning. They can only be compared to each other or to other values of the same index, but they do not express a real measure of the phenomenon under study[5].

These remarks, however, take nothing away from the coherency that characterises previous research conducted according to this approach. Besides, the research Authors repeatedly emphasise that TFR indices do not represent cardinal measures[6]. Apart from that, the application of the TFR method produces a system of indices that completely utilise the available statistical information and allow both disaggregated analyses according to relevant socio-demographic characteristics of the population and evaluation of the temporal evolution of the phenomenon. After all this is what we really want from a scientific study. It is worth repeating that counting the poor, by contrast, can only be done by means of an arbitrary choice of the poverty line with consequent loss of information. Such a count has no scientific meaning and its utility is merely political.

The aim of this research is to develop the potentialities of the TFR approach, taking advantage of its conceptual validity. In the next section we give an alternative specification of the m.f. that, continuing to be perfectly consistent with the basic frame, allows the inconveniences described above to be avoided.

2.2 The alternative specification

Let us denote by $x_{(1)},\ldots, x_{(m)}$ the sorted categories or values of variable *X* and let us define the following transformation of the sample distribution function:

$$\tilde{H}(x_i) = \begin{cases} \frac{1}{2} h(x_{(1)}) & \text{if } x_i = x_{(1)} \\ H(x_{(k-1)}) + \frac{1}{2} h(x_{(k)}) & \text{if } x_i = x_{(k)} \ (k > 1) \end{cases} \quad (2.2.1)$$

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[5] For an extensive discussion on this topic see Lemmi *et al.* (1997).

If we want to make the preceding formula more compact and, at the same time, extend it to the case that *X* may be either discrete or continuous, we can write:

$$\tilde{H}(x_i) = H(x_{(k-1)}) + \tfrac{1}{2} h(x_i) \qquad \text{for } x_{(k-1)} < x_i \le x_{(k)} \;, \quad i = 1,\dots, n \qquad (2.2.1\text{bis})$$

where, by convention, $H(x_{(0)}) = 0$ and $h(x) = 0$ when $x \notin \{x_{(1)},\dots, x_{(m)}\}$ and where $m \le n$, being $m \equiv n$ only when all the values assumed by *X* in the sample differ from one another.

At this point we can specify the m.f. in the fuzzy subset of the poor as follows:

$$g(x_i) = \begin{cases} \tilde{H}(x_i) & \text{if deprivation grows as } X \text{ increases} \\ 1 - \tilde{H}(x_i) & \text{otherwise} \end{cases} \qquad (2.2.2)$$

whereas the collective index of deprivation is still given by formula (2.1.3).

In practice, the mechanism used for generating the values of the m.f. is conceptually the same as the original one: we want the degree of memberhip of any household in the fuzzy subset of the poor to be equal to the proportion of households that are better off than it is (with respect to the particular item or set of items considered).

The substantial difference is the following: with the old m.f. specification, all the units that shared the same category (or value) of *X* as the *i*-th one were implicitly considered as less deprived. Therefore $g(x_i)$ represented precisely the fuzzy proportion of units that are better or as well off as the *i*-th one. By contrast, with the new specification half of the units (for which $X=x_i$) is considered better off whereas the remaining half is considered worse off than the *i*-th one. With the original specification, therefore, the poverty value associated to any household and the TFR index tends to be higher than with the new one. Such a difference tends to be bigger in the case of binary or ordinal variable with a small number *m* of categories, whereas it tends to disappear as *m* diverges. As a matter of fact, when *X* is continuous the two specifications coincide.

Now let us examine in detail the effects produced by the new specification. First of all let us consider the following result:

Proposition 1. *When m.f. g( ) is specified as in formula* (2.2.2) *it holds that:*

[6] See Lemmi *et al*. (1997).

$$P = \overline{g(x)} = \frac{1}{2} \tag{2.2.3}$$

independently on the distribution of X (the proof is given in Appendix 1).

In the case that X is continuous (it can be for instance income or consumption), we can follow the procedure adopted in previous work and use a theoretic model instead of the sample distribution of X. In this case, after choosing a parametric model[7] $H(x;\theta)$, the TFR index is calculated as follows:

$$P = \overline{g(x)} = 1 - \frac{1}{n}\sum_{i=1}^{n} H(x_i;\hat{\theta}) \tag{2.2.4}$$

Remembering that for any continuous variable X it holds that $H(X)$~U[0,1], it follows that: $P = 1 - E[H(x)] = 0.5$. This is a theoretical result; in practice the value taken by P in the whole sample will tend to be close to 0.5 which represents its expected value.

As far as the interpretation is concerned, index P expresses the relative social position of the "average" household in the population analysed, according to indicator X. In order to make cross and/or temporal comparisons let A be the reference population and B the population that we want to compare to A. Typically, A is the whole population (or a sample drawn from it) and B may be either a subgroup of A or A itself surveyed at a different time. Index $P^{B|A}$ expresses the position of B's "average" household in A's deprivation scale and is calculated by averaging the positions occupied by the households of B in the deprivation scale of A. If X is a continuous variable, we can say that B's average household has a value of X equal to the quantile of order $P^{B|A}$ in the reference distribution. In this sense, since the position of A's average household in A itself lays exactly in the middle ($P^{A|A}$=0.5), such an average household corresponds to the median in the distribution of X.

Let us illustrate the calculation and the use of the new m.f. specification given by formula (2.2.1) by means of the example reported in Table 1. The new situation is illustrated in Table 2.

Table 2

	situation *A*			situation *B*		
X	*h*(*x*)	$\tilde{H}(x_i)$	*g*(*x*)	*h*(*x*)	$\tilde{H}(x_i)$	*g*(*x*)

[7] In the empirical part of this research we use Dagum's model (Cf. Dagum, Lemmi, 1989).

Satisfactory	0.05	0.025	0.025	0.05	0.025	0.025
Average	0.90	0.50	0.50	0.94	0.52	0.52
Unsatisfactory	0.05	0.975	0.975	0.01	0.995	0.995
P (TFR			**0.5**			**0.5**

First of all let us notice that the *P* index calculated for the two situations, independently of each other, takes always value 0.5 in spite of the fact that the distribution of *X* changes when we move from *A* to *B*. In order to compare the two situations, instead, let *A* be the reference one and let us calculate $P^{B|A}$ as follows:

$$P^{B|A} = \sum g^A(x) \cdot h^B(x) = 0.481$$

that reveals an improvement. This can be interpreted in the sense that *B*'s median household is less deprived than *A*'s. More precisely, if we imagine moving this household from *B* to *A*, the proportion of households that are better off than it is decreases from 0.5 to 0.481.

Finally we emphasise that, when the new specification of the m.f. is adopted, the TFR indices calculated for different poverty indicators become homogeneous to one another and similarly interpretable. Therefore they also become more suitable for aggregation so as to give a synthetic measure of different aspects of poverty. The aggregated indices are liable to the same interpretation as the specific ones that compose them. We deal with the aggregation problem in section 3. In the next subsection we describe the shape of the m.f. in the particular case in which *X* is dichotomous.

2.3 The particular case of binary indicators

A poverty symptom which is just either present or absent can be treated by means of a binary variable. Denoting by $x_{j(1)}$ and $x_{j(2)}$ the categories[8] of indicator X_j that correspond respectively to absence and presence of the poverty symptom to which X_j refers, the m.f. defined in (2.2.1) and (2.2.2) assumes the following shape:

[8] Henceforth we introduce subscript j in the notation of the poverty indicator so as to treat the case in which we consider several indicators in a multidimensional perspective.

$$g(x_{ij}) = \begin{cases} \frac{1}{2} h(x_{j(1)}) \; = \; \frac{1-p_j}{2} & \text{if} \;\; x_{ij} = x_{j(1)} \\ h(x_{j(1)}) + \frac{1}{2} h(x_{j(2)}) \; = \; 1 - \frac{1}{2} p_j & \text{if} \;\; x_{ij} = x_{j(2)} \end{cases} \tag{2.3.1}$$

where p_j stands for the (crisp, i.e. non fuzzy) proportion of households that manifest the *j*-th poverty symptom (i.e. $X_j = x_{j(2)}$) in the reference population.

With this specification of the m.f. and consistently with the relativity of the approach, the values assigned to the two categories depend on the observed proportion of deprived p_j. In the original specification, by contrast, they were simply 0 and 1 independently of the distribution of X_j. Also in the case of a binary indicator, then, the m.f. indicates the relative social position of any household in society. Besides a binary variable can be seen as a particular ordinal variable.

Denoting by p_j^A the proportion of households that manifest the *j*-th poverty symptom in situation *A* and by p_j^B the analogous proportion in situation *B*, the TFR deprivation index for *B* with reference *A* is given by:

$$P_j^{B|A} = (1 - p_j^B) \cdot g(x_{j(1)}) + p_j^B \cdot g(x_{j(2)}) \; = \; \frac{1}{2}(p_j^B - p_j^A + 1) \tag{2.3.2}$$

that appears to be the function of the difference between the proportions of those who manifest the symptom in the two situations[9]. Moreover we immediately obtain that $P_j^{A|A} = 1/2$.

Binary indicators are generally grouped so as to give a joint representation of a certain aspect of living conditions. From the methodological point of view such an aggregation can be done according to the scheme that we present in the following section.

3 Comparing Alternative Weight Functions for Aggregating Different Aspects of Poverty

TFR indices derive from a multidimensional approach to poverty measurement, where the different aspects of this phenomenon can be studied either one by one or fused together and

[9] It is opportune to underline the distinction between p_j and P_j: p_j represents the crisp (i.e. non fuzzy) proportion of those who manifest the *j*-th symptom; by contrast, $P_j = \overline{g(x_j)} = \sum_{k=1}^{m} h(x_{(k)}) \cdot g(x_{(k)})$ represents (not only in the binary case) the fuzzy proportion of the deprived with respect to X_j. While in the original specification p_j and P_j coincide, with the new one they no longer coincide.

measured by a single index. Once we have calculated the k m.fs. $g_1(x_{i1}),\ldots, g_k(x_{ik})$ relative to the k corresponding poverty indicators for the i-th household, we have to aggregate them so as to obtain a new m.f. which takes into account all the information jointly provided by the k items. Such a *global* m.f. can be defined as a weighted mean of the the specific m.fs. as follows:

$$f(\mathbf{x}_i) = f(x_{i1},\ldots,x_{iK}) = \frac{\sum_{j=1}^{K} w_j \cdot g_j(x_{ij})}{\sum_{j=1}^{K} w_j} \quad , \quad \text{where } w_j = w(P_j) = w\left(\frac{1}{n}\sum_{i=1}^{n} g_j(x_{ij})\right) \qquad (3.1)$$

that represents an *individual measure of global deprivation*. By averaging this measure over all the population analysed we obtain a *collective index of global deprivation* given by:

$$P = \frac{1}{n}\sum_{i=1}^{n} f(\mathbf{x_i}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{K} w_j \cdot g_j(x_{ij})}{\sum_{j=1}^{K} w_j}\right) = \frac{\sum_{j=1}^{K} w_j \cdot \left(\frac{1}{n}\sum_{i=1}^{n} g_j(x_{ij})\right)}{\sum_{j=1}^{K} w_j} = \frac{\sum_{j=1}^{K} w_j \cdot P_j}{\sum_{j=1}^{K} w_j} \qquad (3.2)$$

Each weight w_j in the preceding formula should be specified as a decreasing function of P_j. In fact, according to the relative concept of poverty, we base ourselves on the principle of giving more importance to the items that are more diffused (and for which, symmetrically, deprivation is lower) and therefore more representative of the lifestyle prevailing in society. The specification of the weights originally proposed with the TFR method and adopted in most of its applications is the one that follows:

$$w_j = \ln\frac{1}{P_j} \qquad (3.3)$$

According to this function, the graph of which is reported in Figure A2.1, w_j is minimum and equal to 0 for $P_j = 1$ (that is when everybody is deprived of item j), whereas w_j tends to grow to infinity as P_j approaches 0 (that is when item j is possessed by everybody). This specification appears preferable, for instance, to $w_j=1/P_j$ (see Figure A2.2) for two reasons: i) because its minimum value is equal to 0; ii) because the logarithm does not allow excessive importance to be given to extremely rare poverty symptoms. Besides, both functions display a drawback, namely they are not defined for $P_j = 0$. However, this is not a serious drawback, since the important thing is that the weight function is continuous in the open interval (0,1), whereas continuity in its extremes is just a merely formal question. In fact, items that are owned by everybody (for instance a pair of socks) or by nobody (for instance an aircraft carrier) would never be chosen as poverty indicators!

Of course there is an infinite number of possible alternative specifications for the weight function that are consistent with the relative deprivation principle mentioned above. However it is obvious that we can examine here only a very limited number of them, chosen for their simplicity. Perhaps the most natural and simplest alternative is of the linear type, in particular:

$$w_j = 1 - P_j \tag{3.4}$$

the graph of which is reported in Figure A2.3. This function takes values in the closed interval [0,1], and is continuous also in the extremes; moreover it implies that we give the same importance to any fixed variation ΔP_j of P_j independently on the size of P_j. On the contrary, the specification $w_j = 1 - \sqrt{P_j}$ (see Figure A2.4) gives more importance to ΔP_j for low values of P_j than for high values, whereas the function $w_j = 1 - P_j^2$ would produce the opposite effect (see Figure A2.5).

Another type of function that we can consider is the exponential one. The simplest decreasing function of this type is given by:

$$w_j = e^{-P_j}$$

that is continuous in the closed interval [0,1] and ranges from $1/e \approx 0.368$ to 1 (see Figure A2.6). Being quite flat, this function gives very little importance to small differences in P_j's values. In a way, weighting the various poverty indicators by means of this function is not very different from not weighting at all. If we look for a weight function of the exponential type that is more "sensitive" than the previous one, we can for instance take the one that follows:

$$w_j = e^{-4P_j} + e^{-2} \tag{3.5}$$

the graph of which is again reported in Figure A2.6.

In the empirical analysis we shall calculate the weights according to specifications (3.3), (3.4) and (3.5) that differ considerably from one another, in order to evaluate how much the results of the TFR analysis depend on the particular weighting system adopted.

4 Empirical Analysis

The fundamental aim of the empirical analysis is to compare results obtained: i) by applying the two specifications of the m.f. (the original and the alternative one) described in section 2 and ii) by using three different weight functions applied to the original specification of the m.f.. Comparisons are made on two levels: i) across years and ii) disaggregating the 1997 results by geographical macro-regions. Nevertheless, the results obtained, although not disaggregated enough, are surely interesting also from a substantive point of view.

The analysis has been conducted using the household data set of the British Household Panel Survey from 1991 to 1997 (for details on the BHPS see among others, Taylor, 1994; Taylor, 1998). The sample size ranges from the minimum of 4259 (in 1995) to the maximum of 4826 (in 1991)[10]. The TFR poverty measures were calculated on the basis of different types of indicators referring to lack of certain housing attributes, lack of certain durable goods (for details see Table A3.1), income deprivation (equivalent[11] net household income) and subjective appraisal of the household financial situation (for details see Table A3.3).

4.1 Comparing the two different m.f. specifications

Let us start from the analysis across years. Results obtained using the original m.f. specification (and the original weighting system) are reported in Table A3.1, whereas Table A3.2 contains results obtained using the alternative specification. The main consideration is that both specifications lead to identical conclusions concerning the temporal evolution of poverty. In particular, we observe a uniform decrease of poverty across years according to all specific indices (H, D, Y, S) and consequently also according to the global indices (HDY and HDYS) obtained by aggregating the specific ones. The only difference concerns index Y in Table A3.1 that shows an increase in 1997 which is not observed in Table A3.2. By contrast a similar pattern is found in the time series of Head Count Ratios (HCR) derived according to the International Standard of Poverty Line - ISPL - (Table A3.9). These three indices of income deprivation are plotted in Figure A3.3. The explanation of this difference lies in the fact that the specification (2.1.2bis) used in Table A3.1 gives more weight to the variations in the left

[10] Between 10 and 15 observations were deleted in each wave because of missing values in the variable relative to subjective appraisal of the financial situation of the household.

[11] According to McClements Equivalence Scales (McClements, 1977).

tail of the income distribution. Moreover, the fact that exponent is determined in relation to the HCR according to ISPL makes fuzzy index Y in Table A3.1 closer to the HCR than index Y of Table A3.2.

At this point it is important to point out that apart from the very similar results obtained, the new specification is theoretically better than the original one for one aspect and worse for another one. The positive property is the possibility of giving homogeneous interpretation to all the indices values. For example, the decreasing figures that we observe from 1991 to 1997 represent the proportion of households that are better off than the median one in the 1991 distribution. In other words, the median household in 1991 (our reference household) got relatively better year by year. Moreover, the fact that all the different indices have the same average magnitude in the reference situation given by 0.5 which represents a common term of comparison, makes the aggregation of different specific poverty measure more appropriate. On the contrary the drawback is the impossibility to weigh the different poverty indicators in relation to their corresponding fuzzy proportion of deprived. In fact, as these fuzzy proportions (average m.f.) in the reference year are all equal to 0.5, all the weights would coincide, that is the same as not weighting at all.

Let us consider now the disaggregated analysis by macro-regions. Results obtained using the original m.f. specification (and the original weighting system) are reported in Table A3.5, whereas Table A3.6 contains results obtained using the alternative specification. We created four macro-regions defined as follows: London (Inner London and Outer London); West (South West, West Midlands, Cornub., R. of West Midlands, Greater Manchester, Merseyside, Wales); East (R. of East, East Anglia, East Midlands); North (R. of North West, South Yorkshire, West Yorkshire, R. of York & Humber, Tyne & Wear, R. of North, Scotland). By comparing the two different m.f. specifications we observe an identical pattern for specific indices H, D, Y, S. Global indices HDY and HDYS behave more or less in the same way with only one exception that we shall comment briefly on below. These facts substantially confirm the conclusions we have drawn when considering the evolution of poverty over the years.

From a substantial point of view, let us start by comparing indices Y and S. Compared to other regions, London displays the lowest value of Y and the highest value of S, that means lowest income deprivation and highest subjective deprivation. This fact suggests that in London a fixed equivalent income amount allows a worse standard of living than elsewhere. This affirmation also seems to be confirmed by indices H and D that reveal the worst living condition with regard to housing attribute and durable goods.

In spite of the fact that all the specific indices agree for both specifications, when we compare the two global indices (HDY and HDYS) we notice some discrepancy. In particular, with the original specification London appears to be the poorest region according to both indices, whereas with the alternative specification the situation appears quite different. The Northern regions display the highest value of objective global deprivation, whereas the Western regions manifest the highest value of objective and subjective deprivations taken together (HDYS). According to both indices London is third in deprivation order and its values are below the UK average (0.5).

This discrepancy lead us to the “philosophical” question whether it is right and possible to aggregate different dimensions of the poverty phenomenon into a single index. In our opinion this problem is greater in the original specification than in the alternative one where all the specific indices have the same average magnitude as noted above. As far as the weight given to any specific index is concerned, both approaches appear arbitrary.

Besides it is important to underline that when we carry out a multidimensional poverty analysis, we focus on the different aspects of poverty and on their mutual relationship. Global indices similar to those that we have used here, with their power of synthesis, represent always a temptation for the researcher; however their effect is to reduce several incommensurable dimensions to a unique scale. Therefore, in the framework of multidimensional analysis like the present one, the relevant indices are the specific ones. Nevertheless, global measures may still be useful to give a more synthetic representation of the phenomenon when the specific indices fully agree.

4.2 Comparing the different weighting systems

Table A3.8 reports the results obtained by applying the three different weighting systems. First of all we can notice that results do not seem to be substantially influenced by the weighting system used. Figures A3.1 and A3.2 outline this affirmation: the three analyses are coincident across years referring to the specific indices H, D and to the global indices HDY and HDYS. Also the alternative specification follows a similar pattern across years, even if no weighting system has been used in this case. The three different weighting systems produce coincident results also with respect to the disaggregated analysis by macro regions (see Table A3.8). Such finding constitutes evidence of the fact that the choice of a particular weight function, though arbitrary, has negligible influence on the substantial results. However, the empirical nature of this study makes this conclusion dependent on the particular data set that we considered. For this reason, a sensitivity analysis by simulation data would be useful in order to draw more general conclusions.

Other interesting developments of this analysis could consist in:

i) extending the comparison also to the weighting system proposed by Betti and Verma (2000) that takes into account the correlation among variables (in our analysis we have classified variables into homogeneous groups, so implicitly we have reduced this problem);
ii) studying the possibility to update the weights year by year, whereas at this stage the weigts must be fixed at a given year and cannot reflect the change in time of the distribution of the various items.

5 Final Remarks

In this paper we have dealt with two problems concerning i) the interpretability of TFR poverty indices and ii) a certain arbitrariness of the choice of their weighting system. As far as the first problem is concerned, we observed that TFR indices are ordinal measures that are effective for making cross section and intertemporal poverty comparisons. However their values have no intrinsic meaning and this fact limits both their interpretability and the possibility of comparing the indices that refer to different items (i.e. accounting for different aspects of poverty) to one another as well as the possibility of aggregating them in order to produce an index of global poverty. In particular, these limitations arise because the lack or possession of the various items are usually indicated by variables of a different kind (i.e. continuous or discrete) and we have showed that they may be overcome by adopting an alternative form of the membership function (m.f.). The alternative specification that we have proposed here is perfectly consistent with the TFR approach; in practice it produces a real change only with regard to the treatment of ordinal and binary variables. On the contrary, as far as continuous variables are concerned, the new and the original specifications coincide.

The second problem that we have dealt with concerns the weights by means of which we aggregate the information provided by different poverty indicators. The weight function originally adopted, namely $w_j=\ln(1/P_j)$, consistently with the relative concept of deprivation is decreasing with respect to the (fuzzy) proportion of deprived of item j, P_j. However, the fact that we may find infinite functions with this characteristic, makes this choice arbitrary. Here we examined few simple and "natural" alternative specifications and we selected two of them (in addition to the original one), for the empirical analysis.

It is interesting to have an overview of advantages and disadvantages of the alternative specification in respect of the original one. The main advantage is, as outlined before, that it allows an interpretation of the TFR poverty indices to be given independently of the variable used. In this way TFR indices calculated for different poverty indicators become homogeneous to one another, hence more suitable for aggregation so as to produce a synthetic measure of global poverty. Denoted by *A* the reference situation, the index value in situation *B* with respect to *A* ($P^{B|A}$) represents the proportion of households that are better off than the median household in *B* when this one is moved from *B* to *A*. Using other words, $P^{B|A}$ is the proportion of households in *A* that are less deprived than *B*'s median household. Obviously it holds that $P^{A|A}$ = 0.5, whereas $P^{B|A}$ < 0.5 indicates that *B*'s median household id less deprived (or more deprived if $P^{B|A}$ > 0.5) than *A*'s median household.

The first disadvantage of the new m.f. specification is the impossibility to give different weights to the various poverty symptoms in relation to their diffusion. However, from an opposite point of view, this fact might also be seen as an advantage since it implicitly removes the problem of choosing among different specifications of the weights. Another disadvantage of the new m.f. regards the fact that when a certain item is possessed by everybody or by nobody, it would seem desirable and intuitive that the corresponding index assumed values 0 and 1 respectively. With the old specification this fact is always verified, whereas with the new one it is often unverified.

In the empirical analysis, carried out on BHPS data from 1991 to 1997, we compared the results obtained by applying i) the original versus the new m.f. specification and ii) three different weight functions. All these sets of results derived according to different methodological variations of the TFR method substantially coincide and this constitutes empirical evidence of the robustness of the method itself to the mentioned changes. In particular it suggests that both the arbitrary choice of the weight function and the preference for the original or the alternative m.f. are not crucial problems, since they do not seem to condition the results of the analysis.

Of course, the empirical nature of this study makes this conclusion dependent on the particular data set that we considered. For this reason, a sensitivity analysis by simulation data would be useful in order to draw more general conclusions. Nevertheless, the empirical evidence that we found here is corroborated and complemented by the conclusions of the research carried out by Lelli (2001). A part of this remarkable work regards the comparison of various methods for fuzzy analysis of poverty and well-being. Here the Author applied several m.f. specifications as well as two different weighting systems[12] to the same set of data (the Belgian section of the European Community Household Panel – 1998 wave) and obtained substantially coincident results.

[12] The m.f. specifications analysed are: the quadratic sigmoid, the logistic, the linear, the trapezoidal (proposed by Cerioli and Zani, 1990) and the original TFR (as in Cheli and Lemmi, 1995). The two weighting systems used are the logarithmic and the one composed of all equal weights (or equivalently by no weight at all).

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Appendix 1

Here we give the proof of Proposition 1 in the case $g(x)=\widetilde{H}(x)$. In the other case represented by $g(x)=1-\widetilde{H}(x)$ the proof is exactly the same and we shall omit it.

$$
\begin{aligned}
P = \overline{g(x)} &= \sum_{k=1}^{m} h(x_{(k)}) \cdot g(x_{(k)}) = \\
&= \sum_{k=1}^{m} h(x_{(k)}) \left[H(x_{(k-1)}) + \frac{1}{2} h(x_{(k)}) \right] = \\
&= \sum_{k=1}^{m} h(x_{(k)}) H(x_{(k-1)}) + \frac{1}{2} \sum_{k=1}^{m} h(x_{(k)})^2 = \\
&= h(x_{(2)}) h(x_{(1)}) + h(x_{(3)}) \left[h(x_{(1)}) + h(x_{(2)}) \right] + \\
&\quad + \ldots + h(x_{(m)}) \left[h(x_{(1)}) + h(x_{(2)}) + \ldots + h(x_{(m-1)}) \right] + \frac{1}{2} \sum_{k=1}^{m} h(x_{(k)})^2 \\
&= \frac{1}{2} h(x_{(1)}) \left[h(x_{(2)}) + h(x_{(3)}) + \ldots + h(x_{(m)}) \right] + \frac{1}{2} h(x_{(2)}) \left[h(x_{(1)}) + h(x_{(3)}) + \ldots + h(x_{(m)}) \right] + \\
&\quad + \ldots + \frac{1}{2} h(x_{(m)}) \left[h(x_{(1)}) + h(x_{(2)}) + \ldots + h(x_{(m-1)}) \right] + \frac{1}{2} \sum_{k=1}^{m} h(x_{(k)})^2 \\
&= \frac{1}{2} \sum_{k=1}^{m} h(x_{(k)}) \left[h(x_{(1)}) + h(x_{(2)}) + \ldots + h(x_{(m)}) \right] = \\
&= \frac{1}{2} \sum_{k=1}^{m} h(x_{(k)}) = \frac{1}{2}
\end{aligned}
$$

Appendix 2 (figures)

Figure A2.1

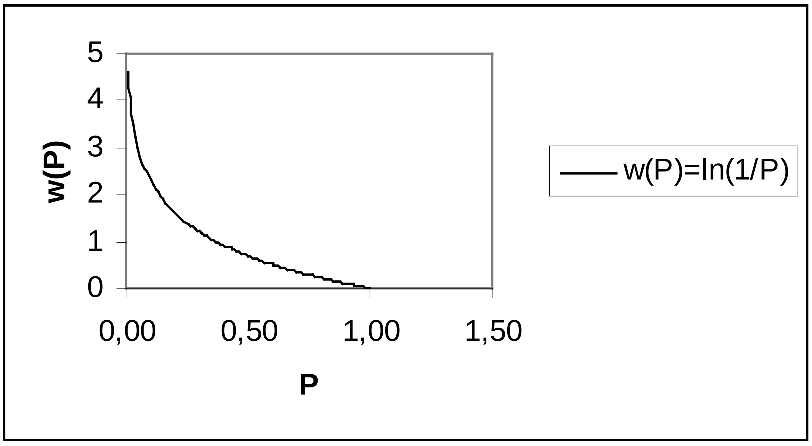


Figure A2.2

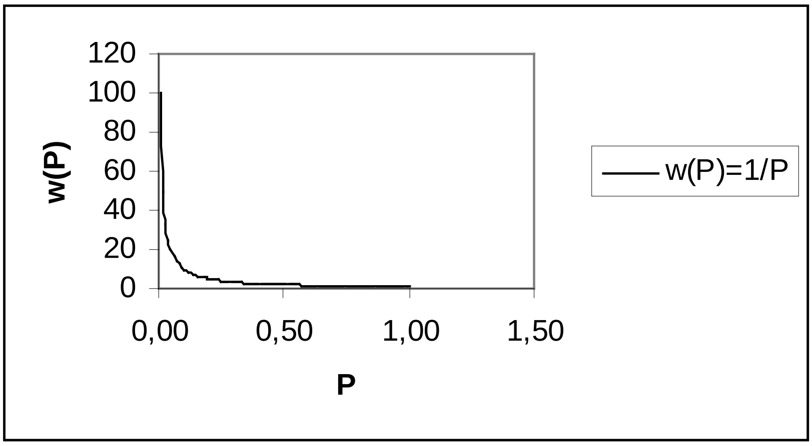


Figure A2.3

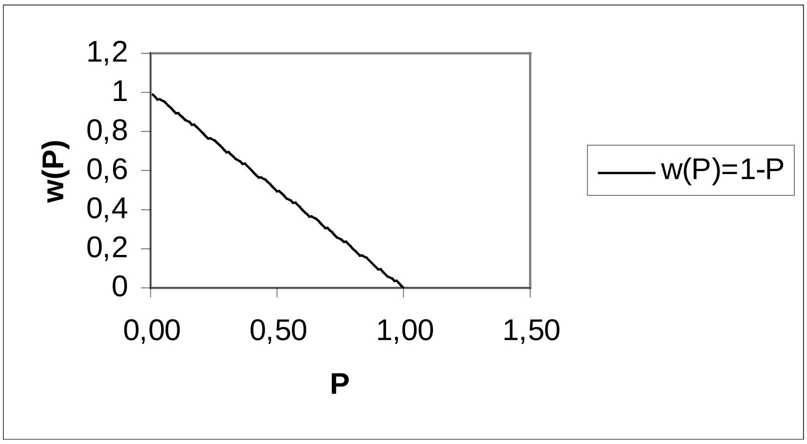


Figure A2.4

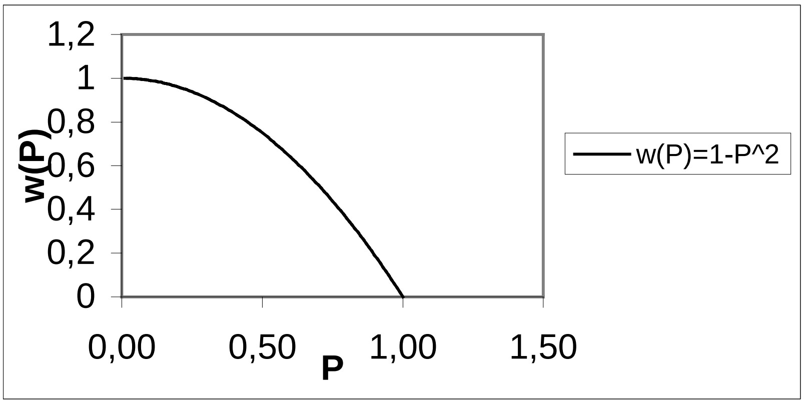


Figure A2.5

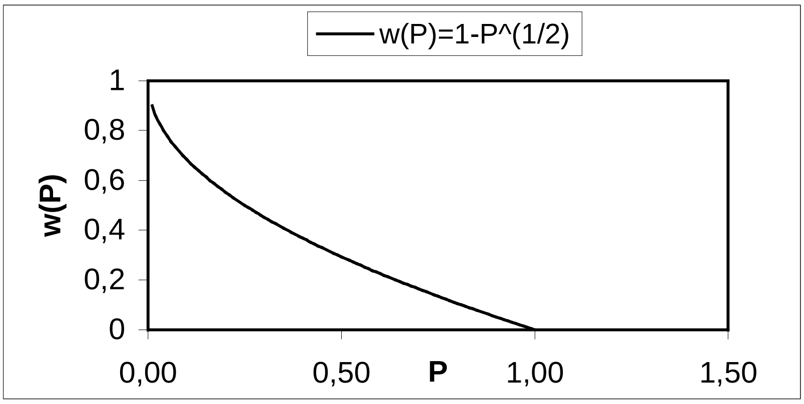
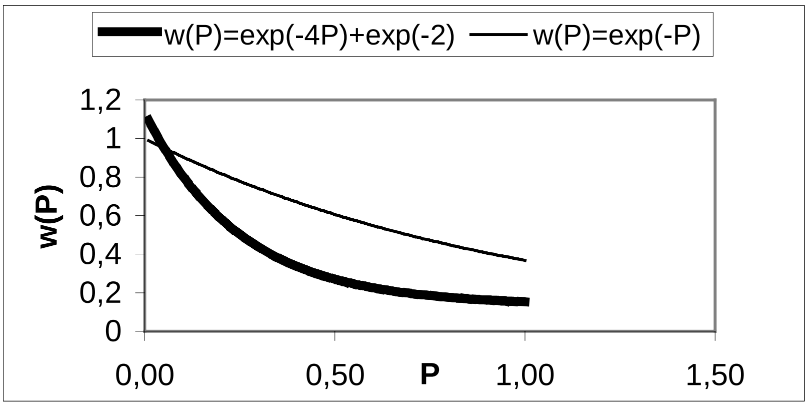


Figure A2.6



Appendix 3 (Tables)

Table A3.1. TFR Poverty Indices (reference year 1991) according to the original specification (weights: ln $(1/P_j)$)

Poverty Indicator	1991		1992	1993	1994	1995	1996	1997
	Average m.f.	Weight ln $(1/P_j)$	Average m.f.	Average m.f.	Average m.f.	Average m.f.	Average m.f.	Average m.f.
Housing Attributes (H)	**0.2443**	***1.4093***	**0.2360**	**0.2271**	**0.2208**	**0.2169**	**0.2048**	**0.1940**
Lack of central heating	0.3461	*1.0609*	0.3516	0.34589	0.3478	0.3538	0.3433	0.3338
House not owned	0.1811	*1.7085*	0.1643	0.1533	0.1418	0.1320	0.1188	0.1072
Durable Goods (D)	**0.2468**	***1,3992***	**0.2391**	**0.2275**	**0.2137**	**0.2054**	**0.1881**	**0.1769**
Lack of:								
Colour TV	0.0568	*2.8680*	0.0501	0.0472	0.0415	0.0402	0.0329	0.0261
VCR	0.3343	*1.0957*	0.3121	0.2896	0.2679	0.2538	0.2303	0.2097
Washing. machine	0.1450	*1.9306*	0.1558	0.1523	0.1408	0.1410	0.1188	0.1182
Microwave oven	0.4639	*0.7682*	0.4313	0.3869	0.3442	0.3165	0.2828	0.2561
Home computer	0.7879	*0.2383*	0.7887	0.7703	0.7573	0.7558	0.7447	0.7097
CD player	0.7377	*0.3043*	0.6762	0.6199	0.5683	0.5093	0.4532	0.4218
Dish washer	0.8547	*0.1570*	0.8434	0.8297	0.8165	0.8095	0.7952	0.7881
Car/van	0.3461	*1.0610*	0.3488	0.3487	0.3484	0.3389	0.3309	0.3195
Income deprivation (Y)	**0.2004**	***1.6074***	**0.1799**	**0.1747**	**0.1718**	**0.1583**	**0.1505**	**0.1529**
Subjective deprivation (S)	**0.4962**	***0.7010***	**0.4925**	**0.4872**	**0.4737**	**0.4690**	**0.4359**	**0.4094**
HDY	**0.2291**	***1.4736***	**0.2166**	**0.2082**	**0.2007**	**0.1919**	**0.1798**	**0.1736**
HDYS	**0.2657**	***1.3254***	**0.2544**	**0.2464**	**0.2381**	**0.2299**	**0.2148**	**0.2059**

Table A3.2. TFR Poverty Indices (reference year 1991) according to the alternative specification

Poverty Indicator	1991				1992	1993	1994	1995	1996	1997
	p_j	$g(x_{j(1)})$	$g(x_{j(2)})$	Average m.f	Average m.f	Average m.f	Average m.f	Average m.f	Average m.f	Average m.f
Housing Attributes (H)				**0.5**	**0.4959**	**0.4914**	**0.4882**	**0.4863**	**0.4802**	**0.4748**
Lack of central heating	0.3461	0.3269	0.8269	0.5	0.5027	0.4999	0.5004	0.5038	0.4986	0.4938
House not owned	0.1811	0.4094	0.9094	0.5	0.4916	0.4861	0.4804	0.4754	0.4688	0.4630
Durable Goods				**0.5**	**0.4961**	**0.4903**	**0.4834**	**0.4793**	**0.4706**	**0.4650**
Lack of:										
Colour TV	0.0568	0.4716	0.9716	0.5	0.4967	0.4952	0.4923	0.4917	0.48805	0.4846
VCR	0.3343	0.3328	0.8328	0.5	0.4889	0.4777	0.4668	0.4598	0.4479	0.4377
Washing. Machine	0.1450	0.4275	0.9275	0.5	0.4944	0.5036	0.4979	0.4979	0.4868	0.4866
Microwave oven	0.4639	0.2680	0.7680	0.5	0.4837	0.4615	0.4402	0.4263	0.4095	0.3961
Home computer	0.7879	0.1060	0.6060	0.5	0.5004	0.4912	0.4847	0.4839	0.4784	0.4609
CD player	0.7377	0.1312	0.6312	0.5	0.4693	0.4411	0.4153	0.3858	0.3578	0.3421
Dish washer	0.8547	0.0726	0.5726	0.5	0.4944	0.4875	0.4809	0.4774	0.4703	0.4667
Car/van	0.3461	0.3269	0.8269	0.5	0.5013	0.5013	0.5011	0.4963	0.4924	0.4867
Income Deprivation (Y)				**0.5**	**0.4833**	**0.4746**	**0.4742**	**0.4569**	**0.4455**	**0.4421**
Subjective Deprivation (S)				**0.5**	**0.4961**	**0.4912**	**0.4807**	**0.4769**	**0.4534**	**0.4319**
HDY				**0.5**	**0.4918**	**0.4854**	**0.4819**	**0.4742**	**0.4654**	**0.4606**
HDYS				**0.5**	**0.4928**	**0.4869**	**0.4819**	**0.4748**	**0.4624**	**0.4534**

Table A3.3. Distribution and Membership Function of the Subjective Indicator (reference year 1991) according to the original specification

Subjective appraisal of Financial Situation	1991		1992	1993	1994	1995	1996	1997
X	**h(x)**	**g(x)**	**h(x)**	**h(x)**	**h(x)**	**h(x)**	**h(x)**	**h(x)**
Living comfortably	0.2617	0	0.2635	0.2622	0.2720	0.2666	0.2913	0.3254
Doing alright	0.2611	0.3566	0.2621	0.2615	0.2818	0.2999	0.3159	0.3191
Just abt getting by.	0.3282	0.8012	0.3345	0.3336	0.3260	0.3199	0.2962	0.2710
Finding it quite dif	0.0919	0.9260	0.0907	0.0914	0.0801	0.0764	0.0646	0.0529
Finding it very diff	0.0571	1	0.0492	0.0383	0.0391	0.0372	0.0308	0.0314
Average m.f. (S)	**0.4962**		**0.4925**	**0.4872**	**0.4737**	**0.4690**	**0.4359**	**0.4094**

Table A3.4. Distribution and Membership Function of the Subjective Indicator (reference year 1991) according to the alternative specification

Subjective appraisal of Financial Situation	1991		1992	1993	1994	1995	1996	1997
X	**h(x)**	**g(x)**	**h(x)**	**h(x)**	**h(x)**	**h(x)**	**h(x)**	**h(x)**
Living comfortably	0.2617	0.1308	0.2635	0.2622	0.2720	0.2666	0.2913	0.3255
Doing alright	0.2610	0.3922	0.2621	0.2615	0.2818	0.2999	0.3159	0.3191
Just abt getting by.	0.3282	0.6868	0.3345	0.3336	0.3260	0.3198	0.2962	0.2710
Finding it quite dif	0.0919	0.8997	0.0907	0.0914	0.0809	0.0764	0.0646	0.0529
Finding it very diff	0.0571	0.9714	0.0492	0.0383	0.0392	0.0371	0.0309	0.0314
Average m.f. (S)	**0.5**		**0.4961**	**0.4912**	**0.4807**	**0.4769**	**0.4534**	**0.4319**

Table A3.5. TFR Poverty Indices (year 1997) disaggregated by geographical macro -regions[13] -original specification (weights: ln $(1/P_j)$)

Poverty Indicator	UK		London	West	East	North
	Average m.f	Weight	Average m.f	Average m.f	Average m.f	Average m.f
Housing Attributes (H)	**0.1818**	***1.7048***	**0.2251**	**0.1941**	**0.1520**	**0.1851**
Lack of central heating	0.3338	*1.0972*	0.4163	0.3239	0.2915	0.3588
House not owned	0.1071	*2.2333*	0.1311	0.1303	0.0834	0.0997
Durable Goods (D)	**0.1934**	***1.6430***	**0.2237**	**0.2074**	**0.1730**	**0.1894**
Lack of:						
Colour TV	0.0261	*3.6463*	0.0305	0.0333	0.0198	0.0236
VCR	0.2097	*1.5618*	0.2243	0.2317	0.1836	0.2099
Washing. Machine	0.1182	*2.1353*	0.1741	0.1465	0.1028	0.0837
Microwave oven	0.2561	*1.3622*	0.3287	0.2570	0.2560	0.2268
Home computer	0.7097	*0.3429*	0.6618	0.7257	0.67588	0.7488
CD player	0.4218	*0.8632*	0.4278	0.4475	0.3949	0.4221
Dish washer	0.2381	*2.1353*	0.8182	0.8094	0.7388	0.8087
Car/van	0.3195	*1.1409*	0.3982	0.3220	0.2427	0.3712
Income deprivation (Y)	**0.1366**	***1.9907***	**0.1078**	**0.1399**	**0.1256**	**0.1565**
Subjective deprivation (S)	**0.4700**	***0.7570***	**0.5042**	**0.4806**	**0.4621**	**0.4557**
HDY	**0.1695**	***1.7749***	**0.1809**	**0.1780**	**0.1486**	**0.1758**
HDYS	**0.2059**	***1.5803***	**0.2210**	**0.2155**	**0.1874**	**0.2104**

[13] **London**= Inner London+Outer London; **West**=South West+West Midlands Cornub+R. of West Midlands+ Greater Manchester+Merseyside+Wales; **East**=R. of East+ East Anglia+East Midlands; **North**=R. of North West+South Yorkshire+West Yorshire+ R. of Yorks & Humber+Tyne & Wear+R. of North+Scotland.

Table A3.6. TFR Poverty Indices (year 1997) disaggregated by geographical macro-regions - alternative specification

Poverty Indicator	UK				London	West	East	North
	p_j	$g(x_{j(1)})$	$g(x_{j(2)})$	Average m.f	Average m.f	Average m.f	Average m.f	Average m.f
Housing Attributes (H)				**0.5**	**0.5216**	**0.5061**	**0.4851**	**0.5016**
Lack of central heating	0.3338	0.3331	0.8331	0.5	0.5413	0.4950	0.4789	0.5125
House not owned	0.1072	0.4464	0.9464	0.5	0.5120	0.5116	0.4881	0.4963
Durable Goods (D)				**0.5**	**0.5152**	**0.5070**	**0.4898**	**0.4980**
Lack of:								
Colour TV	0.0261	0.4869	0.9869	0.5	0.5022	0.5036	0.4969	0.4988
VCR	0.2097	0.3951	0.8951	0.5	0.5073	0.5109	0.4869	0.5001
Washing. Machine	0.1182	0.4409	0.9409	0.5	0.5279	0.5141	0.49228	0.4828
Microwave oven	0.2561	0.3719	0.8719	0.5	0.5363	0.5004	0.4999	0.4854
Home computer	0.7097	0.1452	0.6452	0.5	0.4761	0.5080	0.4831	0.5196
CD player	0.4218	0.2891	0.7891	0.5	0.5029	0.51287	0.4866	0.5002
Dish washer	0.7881	0.1059	0.6059	0.5	0.5106	0.51063	0.4753	0.5103
Car/van	0.3195	0.3402	0.8402	0.5	0.5393	0.5013	0.4616	0.5258
Income Deprivation (Y)				**0.5**	**0.4223**	**0.5185**	**0.4809**	**0.5290**
Subjective Deprivation (S)				**0.5**	**0.5269**	**0.5060**	**0.4944**	**0.4905**
HDY				**0.5**	**0.4874**	**0.5098**	**0.4832**	**0.5121**
HDYS				**0.5**	**0.4973**	**0.5089**	**0.4860**	**0.5067**

Table A3.7. Distribution and Membership Function of the Subjective Indicator (year 1997) original and alternative specification

Subjective appraisal of Financial Situation		**Original spec.**	**Altenative spec.**
X	**h(x)**	**g(x)**	**g(x)**
Living comfortably	0.3254693	0	0.1627347
Doing alright	0.3191366	0.4731239	0.4850376
Just abt getting by.	0.2710246	0.8749212	0.7801182
Finding it quite dif	0.0529578	0.9534316	0.9421094
Finding it very diff	0.0314118	1	0.9842941
Average m.f. (S)		**0.4700**	**0.5**

Table A3.8. TFR Poverty Indices (original specification) using alternative weighting system

Temporal Analysis							
Weight : w(P)= 1-P							
	1991	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
Housing Attributes **(H)**	0.2544	0.2474	0.2388	0.2333	0.2304	0.2185	0.2078
Durable Goods **(D)**	0.3182	0.3077	0.2924	0.2758	0.2644	0.2448	0.2309
Global Deprivation **(HDY)**	0.2299	0.2176	0.2090	0.2015	0.1929	0.1806	0.1742
Global Deprivation **(HDYS)**	0.2776	0.2668	0.2589	0.2503	0.2423	0.2263	0.2163
Weight : w(P)= exp(-4P)+exp(-2)							
	1991	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
Housing Attributes **(H)**	0.2444	0.2361	0.2272	0.2208	0.2170	0.2049	0.1941
Durable Goods **(D)**	0.3022	0.2931	0.2797	0.2647	0.2551	0.2368	0.2243
Global Deprivation **(HDY)**	0.2291	0.2166	0.2081	0.2007	0.1919	0.1797	0.1736
Global Deprivation **(HDYS)**	0.2680	0.2569	0.2487	0.2404	0.2322	0.2170	0.2079

Disagregated Anlysis

Weight : w(P)= 1-P					
	UK	**London**	**West**	**East**	**North**
Housing Attributes **(H)**	0.2040	0.2530	0.2130	0.1723	0.2104
Durable Goods **(D)**	0.2563	0.2890	0.2710	0.2313	0.2557
Global Deprivation **(HDY)**	0.1699	0.1840	0.1796	0.1496	0.1766
Global Deprivation **(HDYS)**	0.2226	0.2402	0.2325	0.2045	0.2256
weight: w(P)= exp(-4P)+exp(-2)					
	UK	**London**	**West**	**East**	**North**
Housing Attributes **(H)**	0.1833	0.2270	0.1954	0.1534	0.1868
Durable Goods **(D)**	0.2290	0.2597	0.2437	0.2066	0.2264
Global Deprivation **(HDY)**	0.1674	0.1781	0.1766	0.1478	0.1750
Global Deprivation **(HDYS)**	0.2091	0.2231	0.2185	0.1912	0.2137

Table A3.9. Head Count Ratio (HCR) of the Poor according to the International Standard of Poverty Line

Year	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
HCR	0.2004	0.1658	0.1631	0.1586	0.1430	0.1329	0.1380

Figure A3.1.[14] Global Poverty Indices (HDY) across years using different weighting system

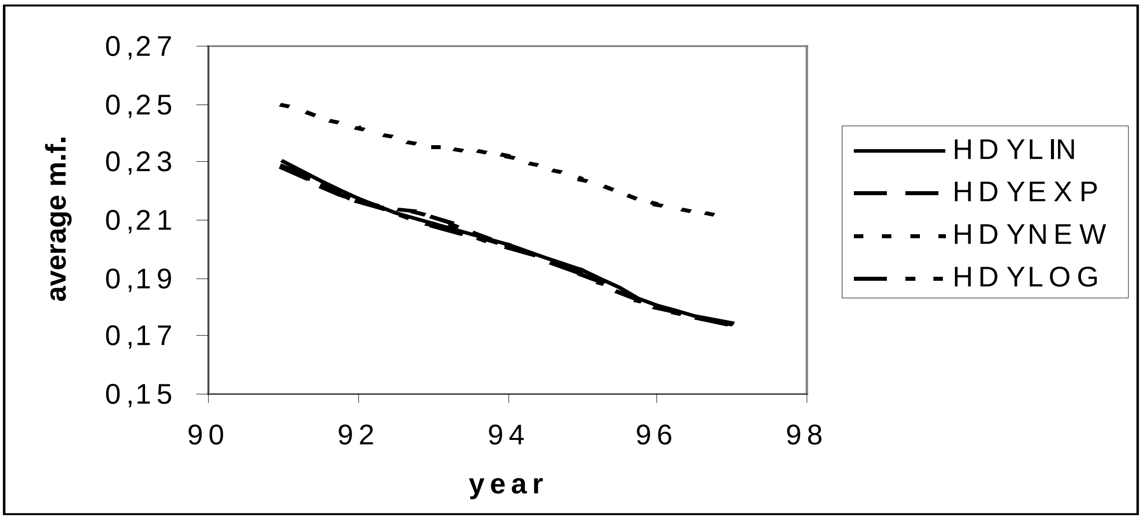


Figure A3.2[15]. Global Poverty Indices (HDYS) across years using different weighting system

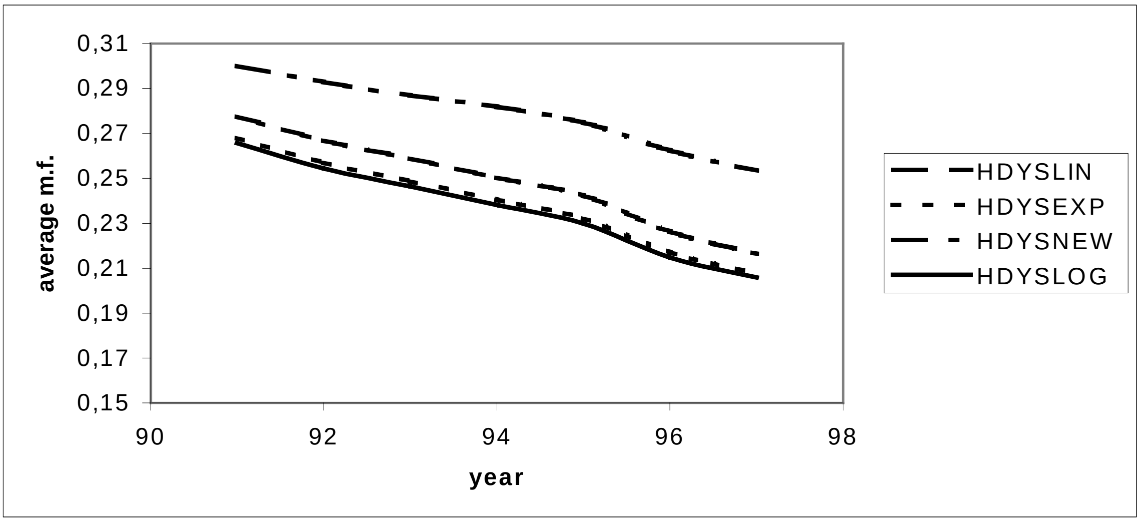
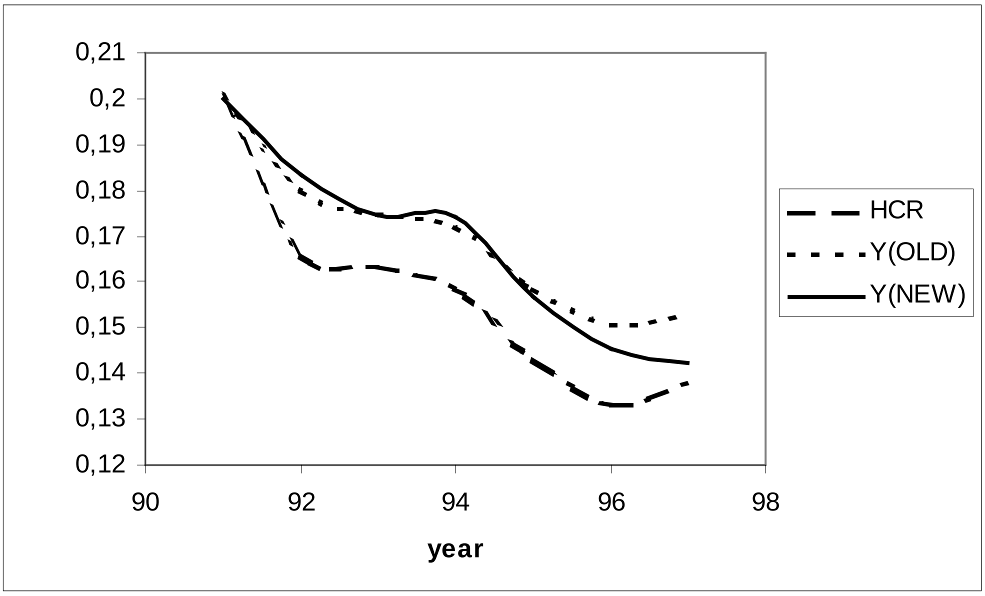


Figure A3.3. Indices Income Deprivation across years

[14] HDYLIN and HDYSLIN calculate using linear weighting system (***w(P)= 1-P***), HDYLIN and HDYSLIN calculate using exponential weighting system (*W(P)=**exp(-4P)+exp(-2))***, HDYLIN and HDYSLIN calculate using logarithmic weighting system (*W(P)=ln(1/P)*)**.** The value of HDYNEW has been decreased by 0.25 in order to fit it in the figure.

[15] The value of HDYSNEW has been decreased by 0.25 in order to fit it in the figure.



0,21
0,2
0,19
0,18
0,17
0,16
0,15
0,14
0,13
0,12
90
92
94
96
98
year
HCR
Y(OLD)
Y(NEW)