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Abstract

In discrete choice labour supply analysis, it is often reasonably expected that utility is increasing with income.
Yet, analyses based on discrete choice models sometimes mention that, when no restriction is imposed *a priori* in the statistical optimization program, the monotonicity condition is not fully satisfied *ex post*.
Obviously, the standard statistical optimization program might be completed with conditions (one per individual) imposing positive marginal utilities. Unfortunately, such a high-dimensional program most often appears to be rather time-consuming in order to be solved, if not practically unsolvable.
In order to overcome this drawback, some authors impose general parametric restrictions *a priori* (hence reducing *de facto* the dimension of the parameter set), which is sufficient to lead to positive marginal utilities *ex post*.
However, those restrictions might sometimes appear to be unnecessarily too severe and then generate a sub-optimal set of estimated values for the parameters of the utility function.
Alternatively, we show that it may be easy to avoid unnecessary restrictions. The high-dimensional program including conditions for positive marginal utilities for all can sometimes be equivalently replaced by a one-dimensional one. At the end, no observation is hopefully showing negative marginal utility anymore at optimum.

JEL Classification: C25, C61, H31, J22

Keywords: Labour supply, Discrete choice, Utility, Monotonicity condition

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[1] This paper uses EUROMOD version 31A and data from the PSELL/EU-SILC for 2004 (income 2003) made available by CEPS/INSTEAD. EUROMOD is continually being improved and updated and the results presented here represent the best available at the time of writing.

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1. INTRODUCTION

In discrete choice labour supply analysis, it is often claimed that quasi-concavity of the utility function is not obligatory, due to the fact that utility is maximized over a finite set, not requiring a tangency condition.

Nevertheless, the economic interpretation of the model is reasonably expecting a utility function increasing with income. This results from the assumption that everyone prefers consuming more, *ceteris paribus*, hence choosing a point on the frontier of the budget set, given that the income variable is continuous.

Yet, analyses based on discrete choice models sometimes mention that, when no restriction is imposed *a priori* in the statistical optimization program, the monotonicity condition is not fully satisfied *ex post*.

Obviously, the standard statistical optimization program might be completed with conditions (one per individual) imposing positive marginal utilities. Unfortunately, such a high-dimensional program most often appears to be rather time-consuming in order to be solved, if not practically unsolvable.

In order to overcome this drawback, some authors impose general parametric restrictions *a priori* (hence reducing *de facto* the dimension of the parameter set), which is sufficient to lead to positive marginal utilities *ex post*. Others avoid this by using for example a CES utility function.

However, those restrictions might sometimes appear to be excessively severe and then generate a sub-optimal set of estimated values for the parameters of the utility function.

Alternatively, we show that it may be easy to avoid unnecessary restrictions. The high-dimensional program including conditions for positive marginal utilities for all can sometimes be equivalently replaced by a one-dimensional one. At the end, no observation is hopefully showing negative marginal utility anymore at optimum.

The paper is organized as follows : *Section 2* introduces the modeling scene, *Section 3* is looking and solving for economic rationality (positive marginal utilities) and *Section 4* concludes.

2. THE DISCRETE CHOICE MODELLING SCENE

The discrete choice model underlying the formation of labour supply is based on the neoclassical consumer demand theory in which individuals make decisions about their hours worked (hence the time devoted to leisure) and consumption by maximizing their utility subject to a specific budget constraint and the total time endowment.

We describe the model and derive the likelihood function to be maximized. The model specification is following Berger *et al.* (2010).

2.1 THEORETICAL FRAMEWORK

We are considering individuals (either “single” or members of a “couple” household) maximizing their utility taking into account their own leisure and total household income.

The individual’s program can be written as:

$$\begin{gathered} \text{Max } U(y_i, h_i, z_i) \\ \textit{subject to} \\ y_i \leq w_i * h_i + W_i + A_i - t(w_i * h_i, W_i, A_i) \end{gathered} \tag{1}$$

where:

$U(.)$: well-being index (utility function)

i : individual’s index (i = 1, …, N)

y_i : net disposable income of the household

h_i : individual labour supply (in hours)
= total time endowment (T) – chosen level of leisure

w_i : gross wage per hour

z_i : (a vector of) characteristics of the household

W_i : non-labour income (all sources)

A_i : all kinds of allowances (positive transfers)

$t(w_i * h_i, W_i, A_i)$: (all kinds of) taxes on labour income, non-labour income, allowances

In the present paper, we are considering females only. We adopt the discrete choice approach (van Soest, 1995, Keane and Moffit, 1998, Blundell *et al.*, 2000, and many others) regarding the number of hours worked. These are to be chosen in a finite set of distinct values.

The utility derived from leisure and income (hence "consumption", given our static framework) can be written as (the individual's index "*i*" is omitted for simplicity):

$$U(y,h,z) \triangleq \beta_y y + \beta_h(T-h) + \beta_{yy}y^2 + \beta_{hh}(T-h)^2 + \beta_{yh}y(T-h) + \varepsilon_j \qquad (2)$$

where :

β_y, β_h, β_{yy}, β_{hh}, β_{yh} are coefficients

$h = h_1, h_2, \ldots, h_J$ is the choice of labour supply, out of a finite set of possibilities

j is index of the choice of labour supply : $j = 1, \ldots, J$

ε_j is a random disturbance (*e.g.* error made in evaluating alternative *j*) : $\varepsilon_j \sim EV(I)$;

$EV(I)$ stands for "Type I extreme value distribution", with cumulative density $Prob[\varepsilon_j < \varepsilon] = \exp(-\exp(-\varepsilon))$, $\varepsilon \in \mathrm{R}$.

The utility *U(.)* is assumed to be increasing with consumption y. The total time endowment T is set to 4,000 hours per year. In this paper, we are considering 3 classes for the number of hours worked (J = 3) : non-workers (0 hour/year), part-time workers (1040 hours/year, which involves 0+ up to 1500 hours/year), and full-time workers (2080 hours/year, which is 1500+ hours/year). The labour supply by the partner in a couple is considered as exogenous.

Furthermore, to account for preference variations across individuals, we need to specify the nature of heterogeneity. For this, we assume that the preference parameters depend on the person's observed and unobserved characteristics. These characteristics are likely to influence the preference for leisure. Hence the leisure coefficient β_h is written as:

$$\beta_h = \sum_{c=1}^{C} \beta_{h,c} z_c + \theta \qquad (3)$$

where the first part of the right member is relating to observed characteristics and the second part θ refers to unobserved (latent) characteristics.

As unobserved heterogeneity (characteristics) θ is not observed, we specify a distribution for it. We choose the latent class approach proposed by Heckman and Singer (1984) and assume that there exists S different mass points for θ, each observed with probability π_s satisfying $0 <= \pi_s <= 1 \quad \forall \; s = 1, \ldots, S$ and $\sum_{s=1}^{S} \pi_s = 1$ ·

2.2 LIKELIHOOD FUNCTION

It can be shown that for any person i and given a mass point s (i = 1, …, N ; s = 1, …, S) :

$$P_{i,j}|\theta_s \triangleq Pr\left[U_{i,j} > U_{i,k}, \forall k \neq j, j = 1, \ldots, J \mid \theta_s\right] = \frac{\exp\left(U_{i,j} \mid \theta_s\right)}{\sum_{k=1}^{J} \exp\left(U_{i,k} \mid \theta_s\right)} \tag{4}$$

where $U_{i,j}$ is the *value* of the utility function for individual i, given his choice j for labour supply.

It follows that the contribution l_i of person i to the likelihood function is given by :

$$l_i \triangleq \sum_{s=1}^{S} \pi_s \left\{ \sum_{j=1}^{J} \left(P_{i,j}|\theta_s\right) \right\} \delta_j \tag{5}$$

where δ_j is an indicator (1 or 0) that the state (labour supply) is the one observed for the individual under consideration.

Practically, the analytical expression for $P_{i,j}|\theta_s$ is derived from the $U_{i,k} \mid \theta_s$ (k = 1, …, J) which in turn result from (2).

Finally, the likelihood function L can be written as:

$$L\left(\beta_y, \beta_{h,c}, \beta_{yy}, \beta_{hh}, \beta_{yh}, \pi_s, \theta_s \ ; \ \mathrm{c} = 1, \ldots, \mathrm{C}, s = 1, \ldots, S\right) = \prod_{i=1}^{N} l_i \tag{6}$$

2.3 STANDARD STATISTICAL OPTIMIZATION PROGRAM "P1"

The model is estimated through maximum-likelihood method :

$$\underset{\Phi}{\text{Max}} \quad L(\Phi) \tag{P1}$$

where $\Phi \triangleq \beta_y, \beta_{h,c}, \beta_{yy}, \beta_{hh}, \beta_{yh}, \pi_s, \theta_s \ ; \ \mathrm{c} = 1, \ldots, \mathrm{C}, \ s = 1, \ldots, S$

Maximizing equation (6) yields estimates for the unknown coefficients of utility function which, under general regularity assumptions, are consistent and asymptotically normal.

2.4 DATA AND ESTIMATION

We are considering PSELL3/EU-SILC survey data collected during the year 2004, which include information on income for 2003[1]. To evaluate the budget set at different levels for the hours worked, the EUROMOD tax-benefit static microsimulation model[2] is used.

In the present paper, we launch the analysis of the labour supply in Luxembourg regarding females in couple. Moreover, the analysis is targeting residence households with the simplest structure and then concentrates on a sub-sample only. These limitations drive us to a target population of 533 "couple" households involving 1,766 persons (including partners and dependents, mainly children). The final (optimal) results below are based on equation (2) where the parameters are replaced by their estimated values shown in *Table A.1* in the *Appendix*.

3. SOLVING FOR ECONOMIC RATIONALITY

The economic interpretation of the model is reasonably expecting a utility function increasing with income :

$$\frac{\partial U}{\partial y} = \beta_y + 2\,\beta_{yy}\,y + 2\,\beta_{yh}(T-h) > 0 \tag{7}$$

This comes from the assumption that everyone prefers consuming more, *ceteris paribus*, hence choosing a point on the frontier of the budget set.

3.1 THE UNCONSTRAINED PROGRAM AND COMPLEMENTARY RESTRICTIVE APPROACHES

In our results based on program "P1", this condition is not satisfied for all. For example, around 17% of sample observations for females in couple do not satisfy the monotonicity condition (see *Figure 3.1*, blue/bottom curve). Similar shortcoming is found in many other papers (see, for example, Labeaga et al., 2008, Van Soest and Das, 2001, and Vlasblom, 1998).

In order to overcome this drawback, some authors like Van Soest and Das (2001) impose general parametric restrictions *a priori*, hence reducing *de facto* the dimension of the parameter set (see *Figure 3.2* for illustration), which is sufficient to lead to positive marginal utilities *ex post*. Vlasblom (1998) avoids this by using a CES utility function.

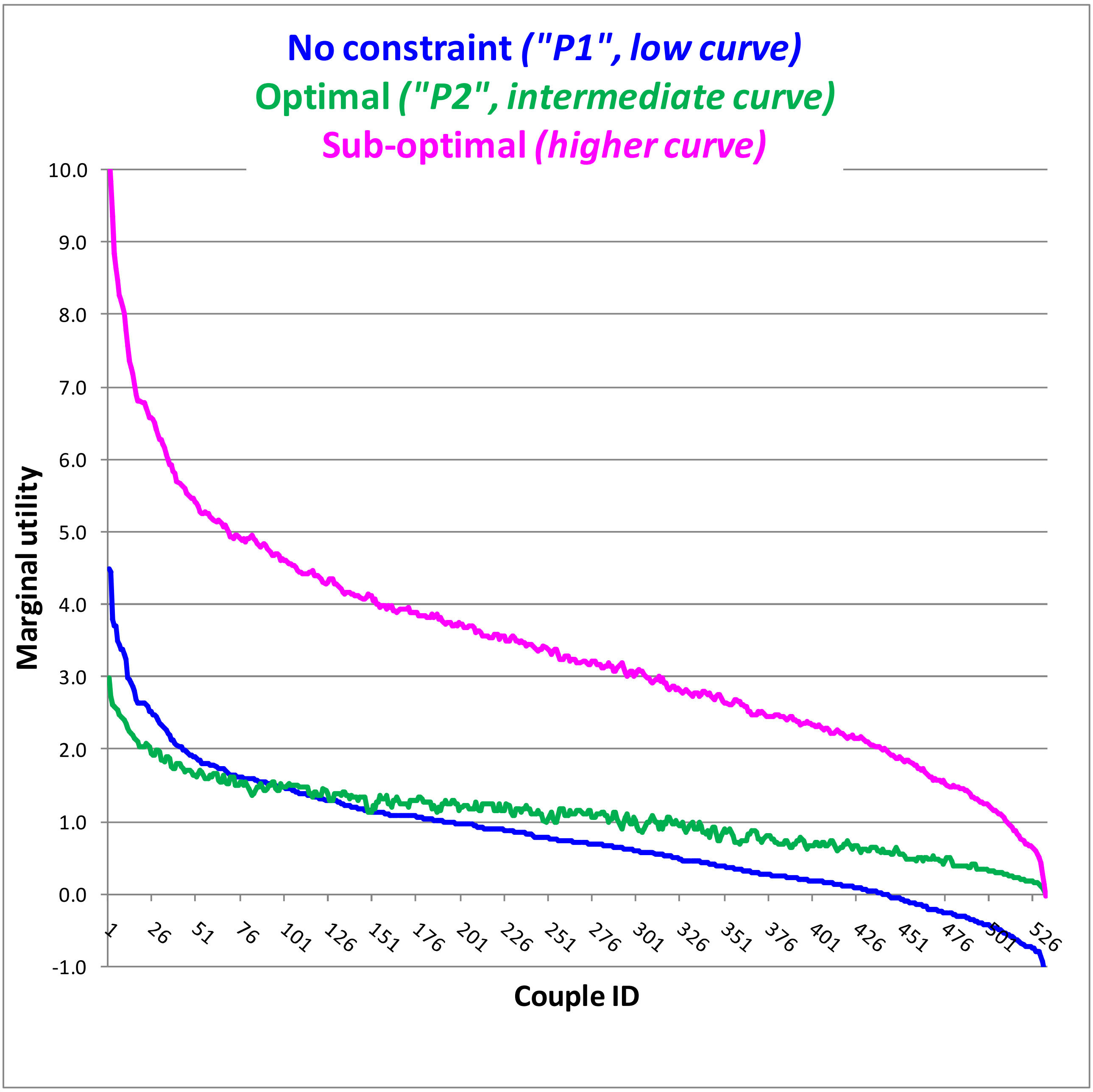
[1] The initial objective was to assess the impact of an important tax reform in Luxembourg, spread over the years 2001 and 2002. See Berger *et al.* (2010) for details.

[2] EUROMOD is an integrated European benefit-tax model for the EU Member States of the European Union. See http://www.iser.essex.ac.uk/msu/emod/ and Sutherland (2007).

However, those restrictions might sometimes appear to be unnecessarily too severe and then generate a sub-optimal set of estimated values for the parameters of the utility function.

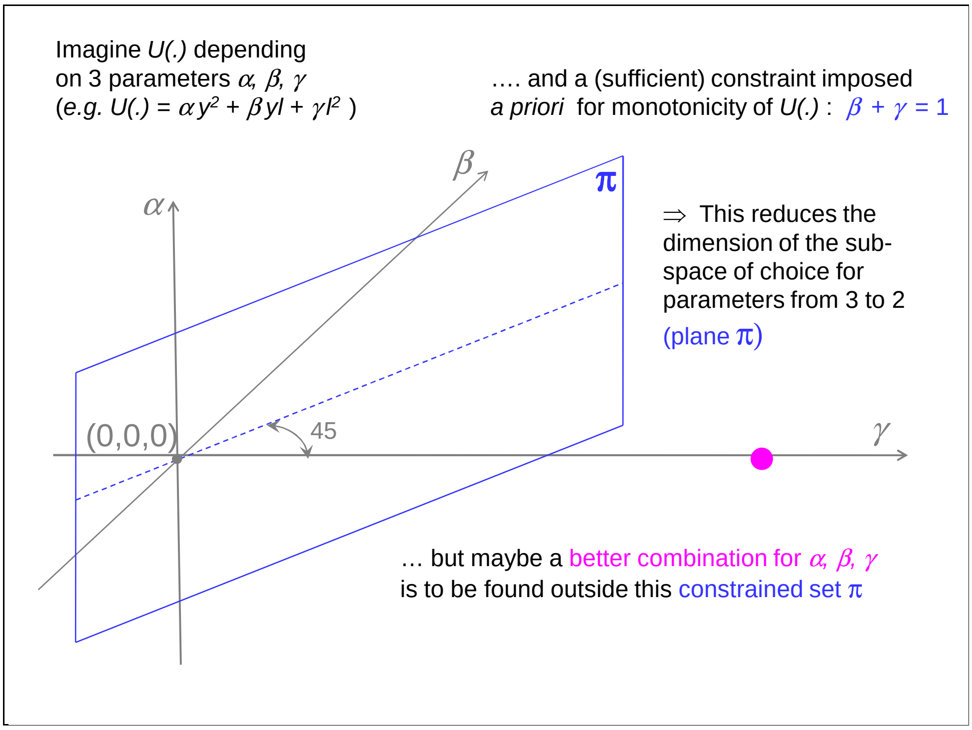
Figure 3.1 **: Marginal Utilities given several framework for optimization**

(Females in couple)



Note : Females are ranked (1-533) based on their marginal utilities resulting from the unconstrained program "P1" ; all curves smoothed (3-point moving average), only for clarity reasons

Figure 3.2 **: Effect of general (*ad hoc*) restrictions on parameters *a priori***



3.2 The comprehensive constrained program "P2"

Obviously, the standard statistical optimization program might be completed with conditions (one per individual) imposing positive marginal utilities :

$$\underset{\Phi}{\text{Max}} \quad L(\Phi)$$

under the *constraints* that $\frac{\partial U}{\partial y} \geq 0$ for *all females* at their observed level of income y_i^* (P2)

Given Lagrange multipliers λ_i associated with each constraint, a Lagrangian is then defined :

$$\pounds(\Phi, \lambda_i \,; i = 1, \ldots, n) \triangleq L(\Phi) + \sum_{i=1}^{n} \lambda_i \left. \frac{\partial U_i}{\partial y} \right|_{y_i = y_i^*} \qquad (8)$$

and the following first-order Kuhn-Tucker necessary conditions[3] must be solved for an optimum (maximum) :

[3] In addition to a "qualification constraint" which is here satisfied

$$\frac{\partial £}{\partial \Phi} = 0 \tag{9}$$

$$\lambda_i \left.\frac{\partial U_i}{\partial y}\right|_{y=y^*} = 0 \quad ; \quad i = 1, \dots, n \tag{10}$$

$$\lambda_i \geq 0 \quad ; \quad i = 1, \dots, n \tag{11}$$

Unfortunately, such a high-dimensional program "P2" most often appears to be rather time-consuming in order to be solved, if not practically unsolvable, given the number of constraints involved[4].

3.2 An alternative program "P3"

Alternatively, we show that the high-dimensional program "P2" can sometimes be equivalently replaced by a one-dimensional one, while avoiding unnecessary restrictions[5].

Individuals are re-ranked following marginal utilities resulting from the unconstrained program "P1". The female with the lowest (negative) marginal utility is marked as "l". Then, the following program is solved :

$$\underset{\Phi}{\text{Max}} \quad L(\Phi)$$

under the *unique constraint* that $\frac{\partial U}{\partial y} \geq 0$ for female l at her observed[6] level of income y_l^* (P3)

Given the new Lagrangian $£_l$:

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[4] In practice, n Lagrange multipliers (and their constraints) are introduced in the computer program, and a first set of values chosen for them (starting with 0, progressively increasing). The empirical method is to check alternative vectors of values for the multipliers, until the Kuhn-Tucker conditions are fulfilled. Of course, more sophisticated algorithms (Newton-Raphson, *etc.*) for converging towards the optimum might be introduced, but these are rather demanding in programming terms and, indeed, unnecessary, as we show *infra*.

[5] A tempting "short cut" would be to solve "P2" through a Lagrange multiplier λ common for all females. This would result, in terms of marginal utilities, in the top (red) curve in *Figure 3.1*. Clearly, this "solution" (which indeed does not properly fulfills Kuhn-Tucker conditions) is sub-optimal, by comparison to the intermediate/green curve (introduced *infra*), leading to both higher marginal utilities at "optimum" than needed and a worse value for the likelihood function (495.130, to be compared to the optimal value 487.607 shown in *Table A1*).

[6] All programs are solved based on "observed" values of labor supply and income, given that an optimum can be derived only as soon as parameters –to be estimated through the present program- are known. However, after check, it comes out that the condition of positive marginal utility is fulfilled for all *at optimum* as well, what was of course not guaranteed *a priori* . Had we been less "lucky", additional constraints might have been needed in the program.

$$\pounds_l(\Phi,\lambda) \triangleq L(\Phi) + \lambda_l \left.\frac{\partial U_l}{\partial y}\right|_{y_l = y_l^*} \tag{12}$$

the Kuhn-Tucker conditions for an optimum[7] can be re-written as follows :

$$\frac{\partial \pounds_l}{\partial \Phi} = 0 \tag{13}$$

$$\lambda_l \left.\frac{\partial U_l}{\partial y}\right|_{y_l = y_l^*} = 0 \tag{14}$$

$$\lambda_l \geq 0 \tag{15}$$

The outcome of the program, expressed in terms of marginal utilities, is shown in *Figure 3.1* (green/intermediate curve).

Even if one constraint only was imposed here, it appears that the outcome of program "P3" is also a solution for the comprehensive constrained program "P2". The set of Lagrange multipliers λ_l derived from "P3" for female l and $\lambda_i = 0 \ \forall \ i \neq l$ fulfills condition (11). Moreover, conditions (10) are satisfied with the same Lagrange multipliers, given (14). Finally, it is easy to show that condition (9) holds, given (13).

6. CONCLUSIONS

The economic interpretation of the discrete choice modeling of labour supply is reasonably expecting a utility function increasing with income. Yet, analyses based on discrete choice models sometimes mention that such a monotonicity condition is not fully satisfied *ex post*, if not imposing *a priori* parametric restrictions on the shape of utility function, hence reducing *de facto* the dimension of the parameter set.

However, those restrictions might sometimes appear to be unnecessarily too severe and then generate a sub-optimal set of estimated values for the parameters of the utility function.

Alternatively, we show that the high-dimensional statistical optimization program imposing explicitly positive marginal utilities for all individuals at optimum can sometimes be easily replaced by a one-dimensional one. Re-ranking individuals on the basis of their marginal utilities of income resulting from the unconstrained framework, and imposing a unique constraint (on marginal utility) to the so-revealed "worse" person, can lead to a mathematical program which is both

[7] In practice, the value of the unique Lagrange multiplier is set to 0 and progressively increased until the marginal utility resulting from the program "P3" for female l, which is initially negative, reaches zero. After check, it comes out that marginal utilities are then positive or null for all (female "l" remains under "P3" the one getting lowest marginal utility).

manageable in technical terms and a proper solution to our general problem.

At the end, no observation is hopefully showing negative marginal utility anymore at optimum.

APPENDIX

Table A.1 **: Estimated Parameters for Females in Couple**

(optimal outcome : one single constraint for female with the lowest marginal utility under unconstrained program "P1")

	Coefficient	Estimate	S.E.
	Preference for leisure		
Observed heterogeneity			
Nb of children in the household	$\beta h1$	0.262	0.067
Nb of children [0-5] in the household	$\beta h2$	0.744	0.157
Age of female / 10	$\beta h3$	0.571	0.111
Female-head with University degree	$\beta h4$	-0.359	0.102
Female-head with Higher non-university degree	$\beta h5$	-0.505	0.175
Female-head is Portuguese	$\beta h6$	-0.994	0.202
Female-head is other EU-15 (out of Luxembourg)	$\beta h7$	-0.691	0.188
Male partner's labour supply	$\beta h8$	-0.216	0.092
Male partner with University degree	$\beta h9$	-0.071	0.026
Unobserved heterogeneity error			
Type 1	$\theta 1$	-3.408	0.463
Type 2	$\theta 2$	-5.562	0.578
Probability of unobserved heterogeneity error			
Type 1	$\pi 1$	0.799	
Type 2	$\pi 2$	0.201	
	Other utility parameters		
Income	βy	-1.291	0.390
Income square	βyy	0.167	0.039
Leisure square	βhh	0.268	0.077
Income * leisure	βyh	0.485	0.090
Log likelihood function *(c)*	L	487.607	
Nb of observations	N	533	

Notes : *a) We keep only those available variables which are significant.*

b) The variables have been rescaled in the following way :

Income = (Disposable income in euros)/ 10,000 ;
Hours worked = (Yearly hours worked)/ 1000 ; Age = Age/ 10.

c) The Log likelihood function is showing a higher value, compared to the one resulting from the unconstrained program "P1" (486.812)

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