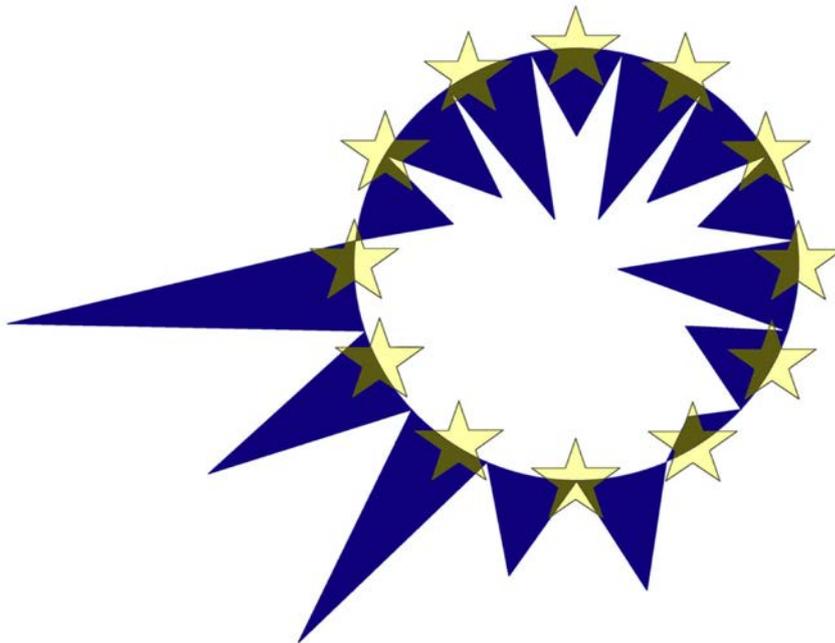


EUROMOD

WORKING PAPER SERIES



EUROMOD Working Paper No. EM 18/13

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Results: Often Easier than You Think. A Technical
Note**

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November 2013

Testing the Statistical Significance of Microsimulation Results: Often Easier than You Think. A Technical Note^{1,2}

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Abstract

In the microsimulation literature, it is still uncommon to test the statistical significance of results. In this paper we argue that this situation is both undesirable and unnecessary. Provided the parameters used in the microsimulation are exogenous, as is often the case in static microsimulation of the first-order effects of policy changes, simple statistical tests can be sufficient. Moreover, standard routines have been developed which enable applied researchers to calculate the sampling variance of microsimulation results, while taking the sample design into account, even of relatively complex statistics such as relative poverty, inequality measures and indicators of polarization, with relative ease and a limited time investment. We stress that when comparing

¹ This paper is also available as ImPRovE Methodological Paper N° 13/10 (<http://improve-research.eu/>). The ideas in this paper were presented during a workshop at the 3rd ImPRovE meeting in Urbino in Italy on 3 May 2013 (<http://improve-research.eu/>) and at the EUROMOD Research Workshop in Lisbon on 3 October 2013. We would like to thank the participants for their comments and questions which helped to develop the structure of the argument in this paper. In particular, we are grateful to Paola De Agostini, Jekaterina Navicke, Iva Tasseva and Holly Sutherland for very helpful comments and suggestions on a previous draft. This research benefited from financial support of the Flemish 'Methusalem' programme, and funding of IWT-Flanders in the framework of the SBO-project "*FLEMOsi: A tool for ex ante evaluation of socio-economic policies in Flanders*" (www.flemosi.be) and the European Union's Seventh Framework Programme (FP7/2012-2016) under grant agreement n° 290613 (project title: ImPRovE). The views expressed in this paper do not necessarily correspond to those of the funding agencies. All remaining errors and shortcomings are our own.

² This paper uses EUROMOD F3.0. The process of extending and updating EUROMOD is financially supported by the Directorate General for Employment, Social Affairs and Inclusion of the European Commission [Progress grant no. VS/2011/0445]. We make use of micro-data from national Lithuanian EU Statistics on Incomes and Living Conditions (EU-SILC) made available by the national statistical office. The usual disclaimers apply.

simulated and baseline variables, as well as when comparing two simulated variables, it is crucial to take account of the covariance between those variables. Due to this covariance, the mean difference between the variables can generally (though not always) be estimated with much greater precision than the means of the separate variables.

JEL Classification: C15, C83, C88, I32

Keywords: Statistical inference, significance tests, microsimulation, covariance, t-test, EUROMOD

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1. Introduction

When working with sample data, testing the significance of the results has become standard practice for a long time now. This is not only the case for articles in scientific journals, but also in reports of applied research for governments and other agencies. No doubt, the fact that standard errors and significance tests are routinely reported by the software packages most commonly used for this kind of empirical analysis (e.g. SAS, Stata, SPSS) plays an important role here. Also in the field of income distribution and poverty, where until recently many scientific publications ignored sampling variation, reporting standard errors and tests of statistical significance is becoming more and more common. Until fairly recently, most analysts carried out significance tests with the implicit assumption of simple random sampling. Statisticians have always insisted that it is important to take account of the sampling design when testing the statistical significance of results (e.g. Kish, 1965; for a recent discussion see Heeringa et al., 2010) and recent papers have shown that this is also the case for poverty and income distribution studies (e.g. Howes and Lanjouw 1998; Biewen and Jenkins 2006; Goedemé 2013).

At the same time, this trend has by and large not reached the microsimulation literature, despite some early examples (e.g. Pudney and Sutherland 1994). There may be a number of reasons for this situation, as discussed below. The purpose of this paper is to argue that this lack of attention to statistical inference is both unnecessary and undesirable. It is structured as follows. After a discussion of the background to the current situation, we show that the most straightforward of statistical tests are often sufficient to assess the statistical significance of microsimulation results. Even for less straightforward situations, software is available to calculate standard errors and significance tests with little effort. We illustrate these points with results from a recent microsimulation of family benefits in Lithuania using EUROMOD, and finish with some concluding remarks regarding statistical inference in the case of more complex microsimulation studies. We stress that in this paper we are not breaking new ground in either microsimulation models or statistical inference. Rather, it is a plea to microsimulation practitioners to use the statistical tools that are at hand, in order to enhance the quality of their work.

2. Problem statement

In the light of growing budgetary pressures, there is a rising demand for comprehensive evaluations of the effectiveness of current versus reformed public policies, which often requires microsimulation. For example, within the field of child poverty analysis, tax-benefit microsimulation has been used to assess different (actual and hypothetical) designs of transfers to families (recent examples are Levy, Morawski et al. 2009; Figari, Paulus et al. 2011). The usual way of evaluating the effectiveness of such policy options is by directly comparing point estimates (i.e. poverty measures, mean household income, total spending, etc.) obtained in the original and simulated settings.

Typically, microsimulation is based on sample data, implying it is important to check whether the simulated effects are statistically significant. Yet, perhaps surprisingly, this is not done routinely. There may be three reasons for this. First, some analysts may have the intuitive notion that sample variation does not play a role, since observed and simulated variables refer to the same sample. This notion is mistaken, because the measured effect of the simulation

will depend on who is selected into the sample. Second, some recent work on statistical inference in microsimulation has focused on changes in inequality, poverty and mobility indices, which often are non-linear functions of sample data (e.g. Osier 2009). Other authors look at statistical inference of microsimulation results where revenue-neutrality is imposed (Pudney and Sutherland 1994) or in the case of models involving uprating to future years, behavioural relations and dynamic microsimulation (Klevmarken 2002; Creedy, Kalb et al. 2007). These studies may have created the impression that testing the significance of microsimulation results requires substantial effort from analysts, either because the analytical derivation of the sampling variance is rather complex (e.g. Pudney and Sutherland) or because bootstrapping or some other kind of time-consuming replication-based technique has to be employed (e.g. Creedy, Kalb et al. 2007). However, many simulation results are simple linear functions of sample data (e.g. differences in means, sums or proportions). Calculating standard errors for those results is easily done with standard software. Furthermore, over the past ten years some software packages have been developed that make it much easier for applied researchers to perform statistical tests of changes in poverty and inequality measures. The third reason for the limited use of tests of statistical significance in microsimulation studies may be that most microsimulations are carried out with programs specially written for this purpose in computer languages such as Fortran and C. Commands performing significance tests are thus not readily available to microsimulators; doing such tests within these specific microsimulation packages requires either substantial programming, or the transfer of the simulated data to a statistical software package.

In order to reinforce our points, we give four examples of recent microsimulation studies, where statistical tests could and should have been employed, but were not, or in a way that was less useful than could have been possible. These studies explored, *inter alia*, the poverty reduction impacts of diverse policy reform scenarios. Generally, various poverty statistics were reported, but either no tests of statistical inference were performed, or if they were, the covariance between the baseline and reform scenario indicators was not taken into account. This warrants the question whether observed larger or smaller poverty reductions are indeed statistically significant, especially given that often sample sizes for particular groups are rather small and/or observed changes in poverty indicators are minor. The studies mentioned are diverse in terms of regional coverage, policy reforms, underlying household surveys and microsimulation model used. Several of these studies (Davies and Favreault 2004; Tanton, Vidyattama et al. 2009) used some kind of uprating to relevant recent or future years. Taking this into account could make the calculations of correct standard errors considerably more difficult. However, we take the position that it is better to perform statistical tests which fall short of the ideal than not to do any, provided the shortcomings are mentioned. We come back to this point in the concluding remarks.³

In our first example, (Tanton, Vidyattama et al. 2009) explore the poverty impacts among older single Australian people due to an increase in the single age-pension rate. The simulations at the national level are calculated using a static microsimulation model STINMOD, which runs on microdata collected from various ABS Surveys of Income and Housing Costs. One of the major study results points to a reduction in poverty rate for lone older persons from 46.5 percent to 36.5 percent, a 10-percentage point reduction. No standard

³ In taking these four studies as examples, we do not want to target special criticism to the authors. We have chosen these papers because they are typical (competent and interesting) applications of static microsimulation.

error or level of statistical significance is reported. As a second example, (Davies and Favreault 2004) analyse various potential US Social Security reforms. Simulations in the study are conducted using the microsimulation model MINT3; the database is drawn from the Survey of Income and Program Participation. The authors conclude that “Among the limited set of reform options we consider, Social Security minimum benefit plans would be more effective in reducing poverty among low-income beneficiaries.” However, depending on the poverty measure used, simulated poverty rates vary across reform options by as little as 3.9 to 4.8 percent for the strictest poverty measure, to 6.5 to 7.4 for a less strict one, and 14.1 to 17.1 for the most generous poverty measure. No statistical tests are reported which would make it possible to evaluate which of these results, if any are significantly different from one another”. (Notten and Gassmann 2008) use the Russia Longitudinal Monitoring Survey (RLMS) from 2000 to 2004 to analyse the impact of the Russian child allowance reforms and to simulate the effects of various means-tested and universal child benefit schemes. This study performs ad-hoc simulations, without any specific microsimulation model in use. Among various conclusions, the study suggests, that “the overall poverty reduction impact of a universal scheme along the current lines is modest”. In fact, it is only -0.3 percentage points, so its statistical significance is at least questionable given the sample size of 1079 households. Also, this paper suggests that “only a significant increase of the benefit level results in considerably higher poverty reduction impacts.” These impacts are -1.9 percentage points and -5.4 percentage points. Statistical tests whether these changes are statistically significant would be useful. In the last example, (Salanauskaite and Verbist 2009) evaluate the distributional impacts of a Lithuanian family allowance reform, using EU-SILC data. Similarly to the aforementioned studies, the authors note that the implemented reform has a limited capacity to alleviate poverty. The study estimates a 0.5 percentage point reduction in total poverty headcount due to an initial and a 1.5 percentage points reduction due to the implementation of a full design of the reform. They also remark that these differences are not statistically significant as indicated by the 95% confidence interval, which they show for each point estimate. Apparently, these authors did not calculate the confidence intervals of the differences in the poverty headcount, which might well have been statistically significant, as shown below.

3. Statistical discussion

The typical situation in static microsimulation is that a simulated variable is compared with a corresponding variable that was observed or with another simulated variable, where both are quantitative (interval-level) variables. In many static simulations of the first-order effects of policy changes, the simulated variables are calculated using exogenous parameters (e.g. those describing a tax or benefit scheme) and possibly also observed variables (e.g. gross income). In those cases, the statistical issues are simple, as they involve a standard application of sampling theory (cf. Klevmarken 2002). It makes no difference whether an observed and a simulated variable, or two simulated variables are compared. A paired t-test can be used to assess the statistical significance of the difference of the means of the variables in the baseline and the reform scenario (Swinscow and Campbell 2002).⁴ A paired t-test takes account of the covariation between the two variables, by calculating the difference between the two variables

⁴ We refer purposefully to this very good but also very introductory text in order to underline our point that in the situations indicated the most basic of statistical techniques are sufficient to perform the appropriate tests of significance (disregarding complications introduced by the sampling design).

on the individual level, and performing a one-sample t-test on the average of these differences to evaluate whether it is significantly different from zero. Calculating the difference between the variables involved has the advantage that it becomes more straightforward to perform slightly more complicated tests, e.g. an F-test whether differences in the effect of the simulated measure vary significantly across groups. The equivalent of the paired t-test for qualitative (nominal) data is the equally simple but little used McNemar's test (Swinscow and Campbell 2002). The necessity of taking account of the sampling design may make the calculation of tests of statistical significance considerably more complicated (e.g. Wolter 2007; Heeringa, West et al. 2010), but this is also the case for any analysis of survey data. Our point is that the circumstance that we are dealing with microsimulation does not add further complications to these calculations. Furthermore, currently available software can perform this task with relatively little effort by the analyst, also in the case of distributive analyses for which freely available software packages have been developed (cf. Araar and Duclos 2007; Araar and Duclos 2009).

Why is it important to take the covariance into account by using the appropriate statistical tests? At this point it is useful to recall the formula of the sampling variance (VAR) of the difference in the mean (D) of two variables y and x with means Y and X (e.g. Heeringa, West et al. 2010):

$$\text{VAR}(D) = \text{VAR}(Y-X) = \text{VAR}(Y) + \text{VAR}(X) - 2*\text{COVAR}(Y,X) \quad (1)$$

As becomes clear from the formula, the sampling variance of a difference does not only depend on the variance of the two estimated averages, but also on their covariance. If this covariance is strongly positive, as is usually the case for microsimulation studies, the variance of the difference of the estimated averages can be much smaller than the variance of either of the averages of the original variables y and x . If two samples are independent, then the covariance is equal to zero.⁵ However, in the case of microsimulation studies usually two scenarios, or a scenario and the baseline, are compared based on one single sample. As a result, when comparing two scenarios, the dependence of estimates is very high and the covariance can be very strong.

Another way to present the same issue may be useful here. Suppose the variable in the baseline scenario is denoted x_i , and the variable in the reform scenario is denoted y_i , where the subscript i denotes the household or individual. Suppose also that the relation between x_i and y_i can be described by the following linear relationship:

$$y_i = a + b x \quad (2)$$

⁵ As has been stressed also in other fields of study: simply checking whether confidence intervals do not overlap in the case of independent samples is overly conservative. This is because $\text{VAR}(X)^{0.5} + \text{VAR}(Y)^{0.5}$ is larger than $(\text{VAR}(X) + \text{VAR}(Y))^{0.5}$. If confidence intervals are compared then the former formula (multiplied with a t-value) is applied, even though, as explained above, the second formula is the correct one Schenker, N. and J. F. Gentleman (2001). "On Judging the Significance of Differences by Examining the Overlap Between Confidence Intervals." *The American Statistician* **55**(3): 182-186, Wolfe, R. and J. Hanley (2002). "If we're so different, why do we keep overlapping? When 1 plus 1 doesn't make 2." *CMAJ: Canadian Medical Association Journal* **166**(1): 65-66, Cumming, G. (2009). "Inference by eye: Reading the overlap of independent confidence intervals." *Statistics in Medicine* **28**(2): 205-220, Afshartous, D. and R. A. Preston (2010). "Confidence intervals for dependent data: Equating non-overlap with statistical significance." *Computational Statistics & Data Analysis* **54**(10): 2296-2305..

where a and b are parameters from a microsimulation model.⁶ Then it is easily shown that the variance of the average difference between y_i and x_i is equal to (capital characters indicate variable means):

$$\text{VAR}(D) = \text{VAR}(Y-X) = (b-1)^2\text{VAR}(X) \quad (3)$$

Two features of this formula are noteworthy. First, the constant a does not appear, implying that the simulated result of a policy reform that increases income by the same fixed amount for every household or individual has no variance (variance zero). This makes intuitive sense. Imagine a simple policy reform that adds 100 EUR to all units in the population. In that case x_i corresponds to income in the baseline scenario and y_i is equal to x_i plus 100 EUR (the constant a). Whichever sample is drawn to test the effect of such a reform, the variance of the average income in the baseline scenario will equal the variance of average income after the reform and will be larger than zero (if there is some inequality in incomes). However, the difference between average income before the reform and average income after the reform will always be 100 EUR, whoever is selected in the sample. In other words, the sampling variance of the difference between the baseline scenario and the reform scenario will be equal to zero. In more general terms: the variance of a constant is always zero (by definition).

Secondly, in the case of most policy reforms b will be positive and will be close to one. This means that the variance of D is much smaller than the variance of X , and also much smaller than the variance of Y , which (given equation 2) is equal to $b^2\text{VAR}(X)$. For example, if $b = 1.2$, $\text{VAR}(D) = 0.04*\text{VAR}(X)$ and $0.028*\text{VAR}(Y)$, or, in words, the variance of the difference is only 4 per cent of the variance of the mean of the original variable, and 2.8 per cent of that of the simulated variable. If simulated policy reforms combine new (increased) taxes and benefits, households will re-rank and the covariance can be much lower. However, unless the reform completely overhauls the income distribution, which any remotely plausible policy reform is unlikely to do, the covariance will not become zero or negative. This means that for policy-relevant reforms, the variance of the difference will nearly always be smaller than the variance of the difference under the assumption of having two independent samples (one before and one after the reform). This will be less true if the analysis focuses on very specific income components and/or very specific subgroups. Note also that the covariance is zero if either the original or the simulated variable is a constant value (within the subgroup).

4. Application using EUROMOD and Lithuanian SILC data

To illustrate the importance of estimating the sampling variance of the difference between a baseline and a reform scenario (and between various reform scenarios), we further elaborate on an example borrowed from a study by (Salanauskaite and Verbist 2013). In this example we calculate the effect on mean equivalent disposable household income, poverty and inequality of a policy reform that first abolishes family benefits in Lithuania, and subsequently implements the Estonian system of family transfers. We calculate equivalent household disposable income using the modified OECD scale (cf. Atkinson, Cantillon et al. 2002; Decancq, Goedemé et al. 2013) and we re-estimate net disposable household income after the

⁶ It is important that the coefficients a and b are not interpreted as sample estimates (e.g. least-square estimates), since that would imply that they are not exogenously given.

policy reforms using the microsimulation model EUROMOD⁷. By doing so, we do not simply subtract family transfers from disposable income, but we deduct gross family transfers from gross household income and recalculate net incomes by applying all relevant tax and benefit regulations to the new gross household income. Consequently, we obtain a more realistic first-order estimate of net income without family transfers (respectively Estonian family transfers implemented in Lithuania), although without taking behavioural effects into account. This type of analysis is quite common in the literature, and issues regarding variance estimation are not different from those when estimating the effect of many other, more complicated policy reforms. Below we discuss cases in which variance estimation is less straightforward. In this exercise, we ignore 'simulation error' and potential errors introduced by uprating samples for aligning them with 'policy years' (see below). Simulation error is the error that is due to the fact that observed data are compared with simulated data, where the former may incorporate measurement error, and the latter may be approximations if the microsimulation model does not include all relevant tax and benefit rules (see Pudney and Sutherland, 1994 for a discussion of this issue).⁸

In this illustration we use Lithuanian data, which is derived from the EU-SILC 2006 survey (Ivaškaitė-Tamošiūnė, Lazutka et al. 2010). The income reference year is 2005 and the analysed policy year is 2008. As the income reference date is "older" than analysed policies, EUROMOD utilises a number of country-specific adjustment factors to update income levels to the corresponding policy year⁹. The chosen data and policy years are aligned with the assumptions of the (Salanauskaite and Verbist 2013) study, where this as well as other examples of microsimulation reform scenarios are discussed in more detail.

The family benefits included in this example are two major non-contributory benefits, namely a birth grant and a universal child benefit. In addition, the income effect of the Lithuanian tax allowance is taken into account. This implies that in the microsimulation scenario of

⁷ The used EUROMOD version is F3.0. More details on the EUROMOD model are available in e.g. Sutherland, H. and F. Figari (2013). "EUROMOD: the European Union tax-benefit microsimulation model." *International Journal of Microsimulation* 6(1): 4-26.. More information on the simulation of Lithuanian policies in EUROMOD is available in Ivaškaitė-Tamošiūnė, V., R. Lazutka, et al. (2010). EUROMOD Country Report: Lithuania 2005-2008. Essex, ISER: 100..

⁸ Simulation error and measurement error are of a rather different kind than sampling error. The latter is the consequence of the random selection of a limited number of sample units from a larger population, while the first refer to a difference between the measured or simulated value of a particular observation and its real value in some sense. In ignoring simulation and measurement error we follow current practice in inferential statistics in survey analysis. This does not mean that such error does not have an impact on the estimated standard errors and significance levels, but the size and direction of the impact depend on the kind of error and the assumptions that are made regarding its properties. In general, a source of variation that affects one variable but not another one, will reduce the covariance between those variables (and thus increase the standard error of the average difference of those variables). If the baseline variable is directly observed, while the reform scenario variable is simulated using tax-and-benefit rules only, measurement error will only be present in the former variable, reducing the covariance between the baseline and reform scenario variables. On the other hand, if both variables are simulated with the same microsimulation model, any simulation error in those variables is likely to be correlated, possibly increasing the covariance between the two variables, leading to a standard error of the average difference that is biased downwards. (The difference itself may also be biased in the presence of simulation error.) A full discussion of these issues is far outside the scope of this paper.

⁹ In some cases this may also influence the sampling variance. However, in this paper we focus on the principal sources of the sampling variance that can relatively easily be taken into account. Further research is necessary to evaluate how various forms of uprating can most easily be taken into account and to estimate what their potential impact is on the sampling variance.

‘removing family benefits’, the special tax allowance available to families with children is taken away, while other tax allowances, such as a basic tax allowance available to all personal income tax payers, are applied.

The Lithuanian sample contains information on 12,098 individuals and 4,660 households¹⁰. The Lithuanian EU-SILC sample has a single-stage stratified sample design. Within each of the seven strata a simple random sample of persons is drawn and the entire household of each selected person is included in the sample (Statistics Lithuania 2010). Therefore, we take account of clustering at the household level, but unfortunately we lack information on stratification in the data. As a result, the standard errors are likely to be slightly over-estimated (e.g. Kish 1965). All variance estimates are based on Taylor first order linearization and make use of Stata standard estimation procedures and the DASP module developed for Stata (Duclos and Araar 2006; Araar and Duclos 2007). The advantage of DASP is that it includes standard estimation commands for typical distributive analyses in relation to poverty, inequality and polarization. DASP is also available as a stand-alone free software package under the name of DAD (Araar and Duclos 2009). For all statistical tests presented below, we made use of ready-made routines that require very little effort in programming and in computation time. Once all income variables are prepared, running the computations for the results presented in the table below takes less than 15 seconds with Stata/SE 11.2.

Our estimates presented in Table 1 illustrate the importance of three issues. First of all, the variance of point estimates cannot be ignored as even for estimates referring to the total population on the basis of a relatively large sample, standard errors and confidence intervals are quite substantial. Second, especially in the case of this type of analyses, it is crucial to take the covariance between the baseline scenario and the reform scenario into account: not doing so would result in a very misleading interpretation of the statistical significance of distributive effects of reforms. Third, when using a floating (relative) poverty line, it is important to take this relativity into account. In what follows we will shortly discuss the results, while explaining these three principal messages.

¹⁰ In comparison to the original EU-SILC data, observations of 36 children born in the year of survey collection are dropped. Information on newborns in 2006 is actually available only until the survey collection time (May-June, 2006). By dropping this group, we align income and demographic references to the calendar year of 2005.

Table 1: The effect of family transfers on equivalent disposable household income in Lithuania, EU-SILC 2006

Parameter	Scenario	Estimate	Standard error	95% confidence interval	
				lower bound	upper bound
Mean	baseline (1)	1519.70	24.42	1471.82	1567.58
	without family transfers (2)	1493.29	24.36	1445.54	1541.04
	Estonian family transfers (3)	1520.23	24.38	1472.43	1568.02
	difference (2)-(1)	-26.41	0.86	-28.11	-24.71
	difference (3)-(1)	0.53	0.45	-0.35	1.41
	difference (3)-(2)	26.94	0.99	25.00	28.88
Percentage poor (fixed poverty line)	baseline (1)	20.25	0.91	18.47	22.03
	without family transfers (2)	21.57	0.93	19.75	23.39
	Estonian family transfers (3)	20.08	0.90	18.30	21.85
	difference (2)-(1)	1.32	0.26	0.81	1.83
	difference (3)-(1)	-0.18	0.17	-0.51	0.16
	difference (3)-(2)	-1.49	0.30	-2.08	-0.91
Percentage poor (floating poverty line assumed to be fixed for variance estimation)	baseline (1)	20.25	0.91	18.47	22.03
	without family transfers (2)	20.79	0.92	18.99	22.59
	Estonian family transfers (3)	20.11	0.90	18.34	21.88
	difference (2)-(1)	0.54	0.20	0.15	0.93
	difference (3)-(1)	-0.14	0.17	-0.47	0.20
	difference (3)-(2)	-0.68	0.25	-1.17	-0.19
Percentage poor (floating poverty line, also for variance estimation)	baseline (1)	20.25	0.78	18.72	21.78
	without family transfers (2)	20.79	0.78	19.26	22.32
	Estonian family transfers (3)	20.11	0.78	18.59	21.64
	difference (2)-(1)	0.54	0.28	-0.00	1.08
	difference (3)-(1)	-0.14	0.18	-0.50	0.22
	difference (3)-(2)	-0.68	0.32	-1.30	-0.06
Gini	baseline (1)	34.95	0.59	33.79	36.11
	without family transfers (2)	35.49	0.60	34.32	36.66
	Estonian family transfers (3)	34.90	0.59	33.75	36.05
	difference (2)-(1)	0.54	0.04	0.47	0.61
	difference (3)-(1)	-0.05	0.03	-0.10	0.00
	difference (3)-(2)	-0.59	0.05	-0.69	-0.49
Decile ratio	baseline (1)	19.52	0.70	18.14	20.89
	without family transfers (2)	18.97	0.71	17.57	20.36
	Estonian family transfers (3)	19.41	0.71	18.01	20.81
	difference (2)-(1)	-0.55	0.24	-1.02	-0.08
	difference (3)-(1)	-0.11	0.21	-0.51	0.30
	difference (3)-(2)	0.44	0.30	-0.14	1.03

Reading note: The poverty line is calculated as 60 per cent of the median equivalent disposable household income. In the case of a fixed poverty line, the poverty line is kept constant for incomes with and without family transfers. In the case of a floating poverty line, the poverty line is equal to 60 per cent of the median equivalent disposable household income, with the median income recalculated in every reform scenario.

Source: EU-SILC 2006 UDB, EUROMOD, own calculations.

The first six rows show average equivalent disposable household income in the baseline scenario and the two reform scenarios (the first three rows), as well as a t-test of the difference between mean income in the three scenarios (the subsequent three rows). For all three income definitions, the standard error of the average income is about 24 EUR and the width of the 95% confidence interval is close to 100 EUR. The 95% confidence intervals considerably overlap: for average income in the baseline and in the third scenario it ranges from 1472 EUR to 1568 EUR and for average income without family transfers it ranges between 1446 and 1541. The question now is: do the reforms of family benefits result in a significant change in

average equivalent disposable income? A common, but mistaken, approach is to simply check whether confidence intervals overlap. In that case, the conclusion clearly would be that abolishing family transfers (scenario 2) has no significant effect on average equivalent disposable household income (or at least: the power of the sample would be insufficient to show such an effect). However, the fourth and sixth row clearly show that such an approach can be very misleading. Indeed, even though average income is decreased by just over 26 EUR, the difference is highly significant, with the 95% confidence interval ranging between minus 28 EUR and minus 25 EUR when the baseline and the second scenario are compared. Apparently, the effect on average incomes can be estimated with a high degree of precision even though the confidence intervals around average incomes are rather substantial. This is because the covariance between mean income in the baseline and in the reform scenario is so strong: it is equal to 594.5, somewhat smaller than the variance of average income in the baseline scenario and slightly larger than the variance of the average in the reform scenario; the correlation is 0.9995. This is a clear illustration of the importance of taking account of the covariance, as discussed above. Please note that we would come to exactly the same conclusion if we would calculate first the difference between income in the reform and in the baseline scenario and subsequently estimate the confidence interval of the average of the difference¹¹. Even though the implementation of the Estonian family benefit system (scenario 3) leads to significant changes in mean income for several subpopulations, on average we do not find neither a substantial nor a significant effect. Nonetheless, we include this scenario in the example to show that it is easy to also compute a standard error and confidence interval for the difference between two reform scenarios. On the basis of the sixth row, it can be seen that the implementation of Estonian family benefits in Lithuania would lead to a significant increase in average equivalent disposable household income compared to reform scenario 2, even though confidence intervals of average income of both scenarios strongly overlap. Also in this case, a strong covariance exists between mean income in the two reform scenarios that can easily be taken into account by using the same t-test as for rows four and five.

The same observation holds true for the proportion of individuals living in a household with an equivalent disposable household income below the poverty threshold. In the second set of rows the poverty threshold is equal to 60 per cent of median equivalent disposable household income in the baseline scenario, but is assumed to be known from an exogenous source that is not subject to sampling variance. When family transfers are deducted, the poverty rate rises from 20.2 to 21.6 per cent of the Lithuanian population. Here again, it is clear that even though 95% confidence intervals of both percentages strongly overlap, the difference between them is strongly significant, with the standard error of the difference being substantially smaller than the standard error of the estimated percentages being poor. Similar conclusions can also be drawn for more complex, 'non-smooth' indicators such as the gini coefficient and the ratio of the tenth and the nineteenth percentile (i.e. the decile ratio): even though the sampling variance of estimated inequality measures are non-negligible, the difference between inequality in the baseline scenario and the reform scenarios (and between both reform scenarios) can be estimated with a high degree of precision. Please note that for some reforms the covariance may be much smaller (this could be the case if a large amount of re-ranking takes place), so that even if the aggregate difference between the baseline and simulated variable is not negligible, it could be non-significant.

¹¹ In fact, this is the way that Stata calculates a paired t-test StataCorp (2009). [Stata Base Reference Manual, Release 11](#). College Station, TX, Stata Press..

Finally, we would like to draw attention to the fact that it is important to take as much as possible account of the characteristics of the measure of interest. With *DASP* it is possible to take many complexities of so-called non-smooth indicators into account. This is illustrated in the third and fourth set of rows. In some cases, one may be interested in the effect of a policy reform on poverty estimated with a floating poverty line. In this example, we set the poverty line equal to 60 per cent of the median equivalent disposable household income of the baseline, respectively the reform scenario. Since in this case the poverty threshold is floating, it moves downward with median income in reform scenario 2 and, consequently, the effect of deducting family transfers from household incomes does not affect so much the poverty rate as in the case of a fixed poverty line: the percentage below the poverty line increases from about 20.2 to 20.8 per cent of the population. If we would assume that in both cases the poverty line would not be subject to sampling variance (third set of rows), we would still be able to observe with a relatively high degree of precision an increase in poverty. However, in reality, the poverty line is subject to sampling variance as the median is estimated on the basis of the sample. If we take this into account, as we should when working with a poverty line estimated on the basis of the sample, the standard error of the separate poverty estimates decreases (for an explanation, see Preston 1995; Berger and Skinner 2003; Goedemé 2012), but the standard error of the difference between the baseline and the reform scenario increases. As a result, the 95% confidence interval includes zero and the difference is no longer significant with this level of confidence (in a two-sided test). Similar observations apply when comparing with reform scenario 3. However, in this case the difference between the two reform scenarios still remains significant, even though the upper bound of the confidence interval is much closer to zero than in the case of poverty measured with a fixed poverty line.

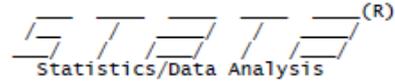
5. Concluding remarks

As we have shown in this article, the sampling variance cannot be ignored in microsimulation studies working with sample data. In many situations, the most elementary of statistical techniques suffice to perform the appropriate test of significance (though the sample design may complicate matters). Furthermore, standard routines have been developed which make it possible for applied researchers to calculate the sampling variance, while taking the sample design into account, of relatively complex statistics such as relative poverty, inequality measures and indicators of polarization, with relative ease and a limited time investment. Very helpful in this regard, is the software developed at the Université Laval (Duclos and Araar 2006; Araar and Duclos 2007; Araar and Duclos 2009). Therefore, there is no excuse for not using these routines and estimating and reporting standard errors, confidence intervals and other indicators of the statistical reliability of point estimates. As Klevmarken (2002: 264) has written "The credibility of [microsimulation models] with the research community as well as with users will in the long run depend on the application of sound principles of inference in the estimation, testing and validation of these models."

At the same time however, we would like to stress that when comparing baseline and reform scenarios, as well as when comparing two reform scenarios, it is crucial to take account of the covariance which will generally, though not always, result in a high degree of precision of estimates of the effect of a reform, even though the sampling variance of the separate point estimates may be substantial. Furthermore, also the characteristics of the indicator of interest and the structure of the sample design should be properly taken into account.

Nonetheless, we are also aware that more research is needed for estimating confidence intervals of the effects of more complex policy reforms, and especially the development of software to enable microsimulation practitioners to perform proper statistical tests for complex cases with relative ease. For instance, in the case of budget-neutral scenarios the size of the benefits in the reform scenario depends on the estimated total amount that is currently spent in the baseline scenario (e.g. Clauss and Schubert 2009; Levy, Morawski et al. 2009; Salanauskaite and Verbist 2013). This induces dependence between the baseline and the reform scenario that could affect the covariance in an unpredictable way. The same is true for estimating the effect of reform scenarios in a dynamic model that incorporates behavioural effects, or any other reform that includes some stochastic element (e.g. Immervoll, Kleven et al. 2007; Ericson and Flood 2012; Navicke, Rastrigina et al. 2013). It would be very useful if future research would analyse whether, for instance, bootstrapping the effect would result in an accurate estimate of the sampling variance and whether more naive estimates of the variance, ignoring this dependence, result in strongly biased variance estimates or not. Previous papers have already addressed parts of these questions (e.g. Pudney and Sutherland 1994; Pudney and Sutherland 1996; Klevmarken 2002; Creedy, Kalb et al. 2007), but have so far not resulted in universally applicable solutions and user-friendly software. For the time being however, we hope to have shown that in many cases the sampling variance can relatively easily be estimated, such that researchers have little excuse for not estimating and reporting the sampling variance of the estimated effect of policy reforms, at least in a simplified form. In other words, we are well aware that many microsimulation studies involve complex estimation procedures. However, this cannot be an excuse for not making and reporting tests of statistical significance. Reporting a less than ideal test (and mentioning the shortcomings) is still far better than not testing at all. While new research is necessary to develop user-friendly software and procedures that accommodate all kinds of complexities, microsimulation researchers should at least make use of the user-friendly software for statistical tests that is already available to them.

Appendix: Output of estimations using Stata®



```

User: Statistical Significance Microsimulation[space -18]
.
.   name: SignificanceMicrosim
.   log:  C:\Analyse\Stata files\LogFiles\VarianceEuromod-final.smcl
. log type: smcl
. opened on: 30 Aug 2013, 18:49:00
.
. local start=c(current_time)
.
. *Calculations for the following paper:
. *** "Testing the Statistical Significance of Microsimulation Results" ***
.
. *The input data for this exercise are the EU-SILC microdata,
. ***... after the Euromod simulations have been performed
.
. cap rename eq_dpi_baseline inc0
. cap rename eq_dpi_before inc1
. cap rename eq_dpi_EEreform inc2
.
. *** inc0 is equivalent disposable household income in the baseline scenario
. *** inc1 is equivalent disposable household income without family transfers
. *** inc2 is equivalent disposable household income after application of
. ***... the Estonian family transfer system
.
. * The next command indicates the sample design:
. ***...PSUS are identified with idhh and the weight is dwt
.
. svyset idhh [pw=dwt]
.
.   pweight: dwt
.           VCE: linearized
. Single unit: missing
. Strata 1: <one>
.   SU 1: idhh
.   FPC 1: <zero>
.
.
. *1. Mean income
. *****
.
. svy: mean inc0 inc1 inc2 // we estimate average income in the various scenarios
. (running mean on estimation sample)
.
. Survey: Mean estimation
.
. Number of strata =      1      Number of obs   =   12098
. Number of PSUs   =   4660     Population size = 3374517
.                               Design df       =    4659

```

	Mean	Linearized Std. Err.	[95% Conf. Interval]	
inc0	1519.698	24.42289	1471.818	1567.579
inc1	1493.288	24.35687	1445.537	1541.039
inc2	1520.225	24.38033	1472.428	1568.022

```
. vce // this command shows the variance-covariance matrix of the previous estimation
Covariance matrix of coefficients of mean model
```

e(V)	inc0	inc1	inc2
inc0	596.47735		
inc1	594.4931	593.257	
inc2	595.3385	593.34031	594.4007

```
. *** lincom can be used to test linear combinations of estimated parameters
. lincom [inc1]-[inc0]
```

```
( 1) - inc0 + inc1 = 0
```

Mean	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	-26.41068	.8649577	-30.53	0.000	-28.1064 -24.71495

```
. lincom [inc2]-[inc0]
```

```
( 1) - inc0 + inc2 = 0
```

Mean	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	.5266357	.4483853	1.17	0.240	-.3524118 1.405683

```
. lincom [inc2]-[inc1]
```

```
( 1) - inc1 + inc2 = 0
```

Mean	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	26.93731	.9884761	27.25	0.000	24.99943 28.87519

```
.
.
. *2. Proportion poor
. *****
.
. * Fixed poverty line
.
. *** we first estimate median income in the baseline scenario,
. *** subsequently we create a dummy variable indicating those with
. *** an income below 60% of the median in the baseline scenario
.
. sum inc0 [aw=dwt], de // we calculate the median of inc0
```

equivalised hh disposable income

Percentiles	Smallest		
1%	256.5	-4.6301	
5%	429.3553	0	
10%	548.0386	0	Obs 12098
25%	819.9982	0	Sum of Wgt. 3374517.01
50%	1241.578		Mean 1519.698
		Largest	Std. Dev. 1088.119
75%	1856.589	11961.46	
90%	2808.286	11961.46	Variance 1184004
95%	3592.361	12523.06	Skewness 2.348878
99%	5715.678	12523.06	Kurtosis 12.10698

```

. local med=r(p50)
. forvalues x=0/2 {
2. cap drop poor`x'
3. gen poor`x'=(inc`x'<0.6*`med') // all individuals with inc lower than ///
4.          /// 60% of the median of inc0 are considered to be poor
> }

```

```

. ta poor0 poor1 // simple non-weighted cross-tabulation of poor0 and poor1

```

poor0	poor1		Total
	0	1	
0	9,765	158	9,923
1	0	2,175	2,175
Total	9,765	2,333	12,098

```

. ta poor0 poor2 // simple non-weighted cross-tabulation of poor0 and poor2

```

poor0	poor2		Total
	0	1	
0	9,894	29	9,923
1	44	2,131	2,175
Total	9,938	2,160	12,098

```

. ta poor1 poor2 // simple non-weighted cross-tabulation of poor1 and poor2

```

poor1	poor2		Total
	0	1	
0	9,753	12	9,765
1	185	2,148	2,333
Total	9,938	2,160	12,098

```

. svy: prop poor0 poor1 poor2 // we estimate the proportion of poor
(running proportion on estimation sample)

```

Survey: Proportion estimation

```

Number of strata =      1      Number of obs   =   12098
Number of PSUs   =   4660      Population size = 3374517
Design df        =          Design df   =   4659

```

		Proportion	Linearized Std. Err.	[95% Conf. Interval]	
poor0	0	.7974793	.0090773	.7796836	.8152751
	1	.2025207	.0090773	.1847249	.2203164
poor1	0	.7843107	.0092825	.7661126	.8025087
	1	.2156893	.0092825	.1974913	.2338874
poor2	0	.7992416	.0090363	.7815262	.8169569
	1	.2007584	.0090363	.1830431	.2184738

```
. vce
```

```
Covariance matrix of coefficients of proportion model
```

e(V)		poor0		poor1		poor2	
		0	1	0	1	0	1
poor0	0	.0000824					
	1	-.0000824	.0000824				
poor1	0	.00008092	-.00008092	.00008616			
	1	-.00008092	.00008092	-.00008616	.00008616		
poor2	0	.00008056	-.00008056	.00007948	-.00007948	.00008165	
	1	-.00008056	.00008056	-.00007948	.00007948	-.00008165	.00008165

```
. lincom [poor1]1-[poor0]1 // t-test of the difference between poor0 and poor1
```

```
( 1) - [poor0]1 + [poor1]1 = 0
```

Proportion	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	.0131686	.002593	5.08	0.000	.0080851 .0182522

```
. lincom [poor2]1-[poor0]1 // t-test of the difference between poor0 and poor2
```

```
( 1) - [poor0]1 + [poor2]1 = 0
```

Proportion	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	-.0017623	.0017099	-1.03	0.303	-.0051144 .0015899

```
. lincom [poor2]1-[poor1]1 // t-test of the difference between poor1 and poor2
```

```
( 1) - [poor1]1 + [poor2]1 = 0
```

Proportion	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
(1)	-.0149309	.0029764	-5.02	0.000	-.020766 -.0090958

```
.
.
.
. * floating poverty line
.
. *-> assuming the poverty line is exogeneous
.
. *** we create dummy variables as before,
. *** ...but recalculate median income in each scenario
.
. forvalues x=1/2 {
2.   cap drop poor`x'
3.   sum inc`x' [aw=dwt], de
4.   gen poor`x'=(inc`x'<0.6*r(p50))
5. }
```


. vce

Covariance matrix of coefficients of **proportion** model

e(V)		poor0		poor1		poor2	
		0	1	0	1	0	1
poor0	0	.0000824					
	1	-.0000824	.0000824				
poor1	0	.00008135	-.00008135	.00008427			
	1	-.00008135	.00008135	-.00008427	.00008427		
poor2	0	.00008057	-.00008057	.00007984	-.00007984	.00008166	
	1	-.00008057	.00008057	-.00007984	.00007984	-.00008166	.00008166

. lincom [poor1]1-[poor0]1 // t-test of the difference between poor0 and poor1

(1) - [poor0]1 + [poor1]1 = 0

Proportion	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
(1)	.0054116	.0019888	2.72	0.007	.0015126	.0093106

. lincom [poor2]1-[poor0]1 // t-test of the difference between poor0 and poor2

(1) - [poor0]1 + [poor2]1 = 0

Proportion	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
(1)	-.0013927	.0017082	-0.82	0.415	-.0047415	.001956

. lincom [poor2]1-[poor1]1 // t-test of the difference between poor2 and poor1

(1) - [poor1]1 + [poor2]1 = 0

Proportion	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
(1)	-.0068044	.0024996	-2.72	0.007	-.0117047	-.001904

.

. **-> correct inference taking relativity of poverty line into account
 . *** ...The following commands of the DASP module correctly take the random character
 . *** ...of the poverty line into account when estimation the variance of the proportion
 . *** ... of poor and calculating the difference between poor0, poor1 and poor2

. difgt inc0 inc1, alpha(0) op11(median) prop1(60) op12(median) prop2(60)

Variable	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]		Pov. line
inc0	.2025207	.0077972	25.9735	0.0000	.1872345	.2178069	744.9469
inc1	.2079323	.0077963	26.6706	0.0000	.1926479	.2232167	730.2515
diff.	.0054116	.0027684	1.95478	0.0507	-.0000158	.010839	---

. difgt inc0 inc2, alpha(0) opl1(median) prop1(60) opl2(median) prop2(60)

Variable	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]	Pov. line
inc0	.2025207	.0077972	25.9735	0.0000	.1872345 .2178069	744.9469
inc2	.2011279	.0077878	25.826	0.0000	.1858601 .2163957	745.7956
diff.	-.0013927	.001835	-.758965	0.4479	-.0049902 .0022048	---

. difgt inc1 inc2, alpha(0) opl1(median) prop1(60) opl2(median) prop2(60)

Variable	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]	Pov. line
inc1	.2079323	.0077963	26.6706	0.0000	.1926479 .2232167	730.2515
inc2	.2011279	.0077878	25.826	0.0000	.1858601 .2163957	745.7956
diff.	-.0068044	.0031517	-2.15896	0.0309	-.0129832 -.0006256	---

. *3. Effect on inequality
 . *****

. * Gini (DASP command)
 . digini inc0 inc1

Index	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]
GINI_Dis1	.3494926	.0058965	59.2712	0.0000	.3379327 .3610525
GINI_Dis2	.3548816	.0059538	59.6059	0.0000	.3432093 .3665539
diff.	.0053889	.0003685	14.6239	0.0000	.0046665 .0061113

. digini inc0 inc2

Index	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]
GINI_Dis1	.3494926	.0058965	59.2712	0.0000	.3379327 .3610525
GINI_Dis2	.3489958	.0058816	59.3369	0.0000	.3374651 .3605265
diff.	-.0004969	.0002713	-1.83155	0.0671	-.0010288 .000035

. digini inc1 inc2

Index	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]
GINI_Dis1	.3548816	.0059538	59.6059	0.0000	.3432093 .3665539
GINI_Dis2	.3489958	.0058816	59.3369	0.0000	.3374651 .3605265
diff.	-.0058858	.0005114	-11.5092	0.0000	-.0068884 -.0048832

.
 . * Decile ratio (DASP command)
 . dinineq inc0 inc1, p1(0.1) p2(0.9)

Difference: Quantile ratio index of inequality
 Lower rank : p1 = .1
 Higher rank : p2 = .9

Index	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]
Dist1	.1951506	.0069941	27.9022	0.0000	.1814389 .2088623
Dist2	.1896536	.0071036	26.6982	0.0000	.1757272 .20358
diff.	-.005497	.0023934	-2.29673	0.0217	-.0101892 -.0008048

```
. dinineq inc0 inc2, p1(0.1) p2(0.9)
```

```
    Difference: Quantile ratio index of inequality  
    Lower rank   : p1 = .1  
    Higher rank  : p2 = .9
```

Index	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]	
Dist1	.1951506	.0069941	27.9022	0.0000	.1814389	.2088623
Dist2	.1940892	.0071344	27.2047	0.0000	.1801024	.208076
diff.	-.0010614	.0020513	-.517428	0.6049	-.0050829	.0029601

```
. dinineq inc1 inc2, p1(0.1) p2(0.9)
```

```
    Difference: Quantile ratio index of inequality  
    Lower rank   : p1 = .1  
    Higher rank  : p2 = .9
```

Index	Estimate	Std. Err.	t	P> t	[95% Conf. Interval]	
Dist1	.1896536	.0071036	26.6982	0.0000	.1757272	.20358
Dist2	.1940892	.0071344	27.2047	0.0000	.1801024	.208076
diff.	.0044356	.0029933	1.48184	0.1384	-.0014327	.0103039

```
.  
.  
. local end=c(current_time)  
  
. di "start of the do-file: `start'"  
start of the do-file: 18:49:00  
  
. di "end of the do-file: `end'"  
end of the do-file: 18:49:11  
  
. cap log close  
  
.
```

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